Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method


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Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method

6th European CFD conference - ECCOMAS 2014

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1. Overview
2. Motivation
3. Aeroelastic-Harmonic Balance
4. Validation
5. Results
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Motivation

**Figure**: F-16 Flight Test Variability

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1 Thomas et al., AIAA paper 2010-2632
Hence, for such problems we need:

- Physics Based (nonlinear) Simulations $\rightarrow$ High-Fidelity Analysis
- Uncertainty & Variability $\rightarrow$ Probabilistic/Possibilistic
- Large Parameter Spaces $\rightarrow$ Efficient Methods
- Integration of Available Measurements $\rightarrow$ Model Updating and Calibration
Hence, for such problems we need:

- **Physics Based (nonlinear) Simulations** $\rightarrow$ **High-Fidelity Analysis**
- Uncertainty & Variability $\rightarrow$ Probabilistic/Possibilistic
- **Large Paramenter Spaces** $\rightarrow$ **Efficient Methods**
- Integration of Available Measurements $\rightarrow$ Model Updating and Calibration
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1. Overview
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6. Conclusions
• Consider a generic structural system described by the equation of motion:

\[
M\ddot{x} + C\dot{x} + Kx = F \\
\dot{\dot{W}} + R = 0
\]
Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

\[ M \ddot{x} + C \dot{x} + K x = F \\
\dot{W} + R = 0 \]

- Assuming the solution is periodic, it is approximated by a truncated Fourier series of \( N_H \) harmonics and fundamental frequency \( \omega \)

\[ x(t) \approx \hat{x}_0 + \sum_{n=1}^{N_H} (\hat{x}_{2n-1} \cos(n\omega t) + \hat{x}_{2n} \sin(n\omega t)) \]
Consider the general form of the fluid Euler equations:

\[
\frac{\partial W(t)}{\partial t} + R(t) = 0
\]

Consider the general form of the fluid Euler equations:

$$\frac{\partial W(t)}{\partial t} + R(t) = 0$$

Apply Fourier expansion to $W$ & $R$ and re-write eq. in the frequency domain:

$$\omega \hat{W} + \hat{R} = 0$$

---

\(^2\text{Hall et al.}, “Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique,” AIAA J., 40(5)\)
Consider the general form of the fluid Euler equations:

\[
\frac{\partial \mathbf{W}(t)}{\partial t} + \mathbf{R}(t) = 0
\]

Apply Fourier expansion to \( \mathbf{W} \) & \( \mathbf{R} \) and re-write eq. in the frequency domain:

\[
\omega \hat{\mathbf{A}} \hat{\mathbf{W}} + \hat{\mathbf{R}} = 0
\]

Recast the Fourier coefficients in the time domain and solve:\(^2\)

\[
\frac{\partial \tilde{\mathbf{W}}}{\partial \tau} + \omega \mathbf{D} \tilde{\mathbf{W}} + \tilde{\mathbf{R}} = 0
\]

\(\text{pseudo—time}\)  
Unsteady Source  
residual

---

where:

\[ D = E^{-1} AE, \quad E = \begin{bmatrix}
1/2 & 1/2 & \cdots & 1/2 \\
\cos t_0 & \cos t_1 & \cdots & \cos t_{2N_H} \\
\sin t_0 & \sin t_1 & \cdots & \sin t_{2N_H} \\
\cos 2t_0 & \cos 2t_1 & \cdots & \cos 2t_{2N_H} \\
\sin 2t_0 & \sin 2t_1 & \cdots & \sin 2t_{2N_H} \\
\vdots & \vdots & \cdots & \vdots \\
\cos N_H t_0 & \cos N_H t_1 & \cdots & \cos N_H t_{2N_H} \\
\sin N_H t_0 & \sin N_H t_1 & \cdots & \sin N_H t_{2N_H}
\end{bmatrix}, \quad \tilde{W} = \begin{bmatrix}
W(t_0) \\
W(t_1) \\
\vdots \\
W(t_{2N_H})
\end{bmatrix} \]

\[ t_i = \frac{i2\pi}{2N_H + 1}, \quad i = \{0, 1, 2, \ldots, 2N_H\} \]
Consider a generic, undamped, structural system described by the equation of motion:

\[ M\ddot{x} + Kx = F(W) \]
Consider a generic, undamped, structural system described by the equation of motion:

\[ M\ddot{x} + Kx = F(W) \]

Recast in state-space form

\[ \dot{y} = A_s y + B_s F \]

where:

\[ A_s = \begin{bmatrix} 0 & I \\ M^{-1}K & 0 \end{bmatrix} \quad B_s = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \quad y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \]
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

Even CFD unsteady problems can be computed efficiently, as long as \( N_H \) does not become excessive and \( \omega \) is known.
However, we don’t know $\omega_{LCO}$
However, we don’t know $\omega_{LCO}$
However, we don’t know $\omega_{LCO}$

New approach based on frequency updating
Define the following residual:

\[ R(\omega, y) = \omega D_s y - (A_s y + B_s f) \]

From the residual, define the following figure of merit:

\[ L = \frac{1}{2} R^T R = \frac{1}{2} [\omega D_s y - (A_s y + B_s f)]^T [\omega D_s y - (A_s y + B_s f)] \]
We then propose the following algorithm to determine LCOs:

Find \([\omega, F]\) that minimizes \(R(\omega, y)\) using:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

\[
\frac{\partial L_n}{\partial \omega} = \left( Dy - B_s \frac{\partial f}{\partial \omega} \right)^T [\omega Dy - (A_s y + B_s f)]
\]
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Forced Motion Predictions

AGARD CT-6

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_\infty$</th>
<th>$\alpha_m$</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>$x_m$</th>
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<td>0.796</td>
<td>0</td>
<td>1.01</td>
<td>0.202</td>
<td>0.25</td>
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Figure: NACA 64A010
Forced Motion Predictions
Forced Motion Predictions
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Problem widely studied in the community\textsuperscript{3}:

\[
\begin{align*}
    m \ddot{h} + S_\alpha \ddot{\alpha} + K_h &= -q_\infty c C_l \\
    S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha &= q_\infty c^2 C_m
\end{align*}
\]

\textsuperscript{3}Thomas et al., AIAA paper 2010-2632
Problem widely studied in the community:

\[
M = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}, \quad K = \begin{bmatrix} \left( \frac{\omega h}{\omega_\alpha} \right)^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix}
\]

\[
f = \begin{bmatrix} -C_l \\ 2C_m \end{bmatrix}, \quad y = \begin{bmatrix} \frac{h}{b} \\ \frac{\alpha}{\alpha} \end{bmatrix}, \quad V = \frac{U_\infty}{\sqrt{\mu \omega_\alpha b}}
\]
Pitch - Plunge Aerofoil
Solver Convergence

Frequency Convergence

(a) $V_s = 0.725$

(b) $V_s = 0.80$

A-HB initialised at flutter conditions
Pitch - Plunge Aerofoil
Solver Convergence

Frequency Convergence ($V_s = 0.80$)

- Displacement $L_2$ Norm
- Residual $L_2$ Norm

Number of Iteration

Dr. Simão Marques — LCO Prediction with A-HB
Position-Velocity Diagram \( (V_s = 0.80) \)
Pitch - Plunge Aerofoil
Solver Convergence

LCO Amplitude

- Graphs showing the LCO Amplitude for different parameters and solvers (HB_LCO, time marching).

Dr. Simão Marques — LCO Prediction with A-HB
Generic Fighter Delta Wing
Delta Wing

Structural Model based on 2D shell elements

3.94Hz 12.88Hz
15.82 27.56Hz
Delta Wing

LCO Prediction - $M_\infty = 0.91$; $\alpha = 0^\circ$; $q_{\text{flutter}} = 0.759q_{\text{sl}}$

Figure: $q_{\text{LCO}} = 0.85q_{\text{sl}}$
Position-Velocity Diagram ($q_{LCO} = 0.85q_{sl}$)
Delta Wing
## Solver Efficiency

<table>
<thead>
<tr>
<th></th>
<th>Aerofoil Wall Clock [s]</th>
<th>Speed-Up</th>
<th>Delta Wing Wall Clock [h]</th>
<th>Speed-Up</th>
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<tbody>
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<td>204</td>
<td>1.0</td>
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<td>2 Harmonics</td>
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<td>3 Harmonics</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>4 Harmonics</td>
<td>1224</td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
</tbody>
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Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
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• a new formulation for LCO prediction based on a Harmonic Balance method has been presented.

• LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences.
Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences
- the new approach shows significant speed-ups w.r.t. time-domain, without compromising accuracy
Future Work

- Include stronger nonlinearities.
- Quantify uncertainties
Thank you for your attention

Questions Welcome
- Grid size: 160k
- NACA 65A004
Flutter Calculations:

**Eigenvalue Problem**

\[
\begin{bmatrix}
A_{ff} & A_{fs} \\
A_{sf} & A_{ss}
\end{bmatrix} \mathbf{p} = \lambda \mathbf{p}
\]

This can be reduced to a small nonlinear eigenvalue problem, by applying the Schur Complement Method\(^4\)

**Schur Formulation**

\[
S(\lambda)\mathbf{p}_s = \lambda \mathbf{p}_s
\]

where

\[
S(\lambda) = (A_{ss} - \lambda I) - A_{sf}(A_{ff} - \lambda I)^{-1}A_{fs}
\]

Flutter Calculations:

Figure: Delta Wing Flutter response with Volterra Series 808th ROM