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Spatial Correlation Variability in Multiuser Systems

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Abstract—Spatial correlation across an antenna array is known to be detrimental to the terminal signal–to–interference–plus–noise–ratio (SINR) and system spectral efficiency. For a downlink multiuser multiple–input multiple–output system (MU–MIMO), we show that the widely used, yet overly simplified, correlation models which generate fixed correlation patterns for all terminals tend to underestimate the system performance. This is in contrast to more sophisticated, yet physically motivated, remote scattering models that generate variations in the correlation structure across multiple terminals. The remote scattering models are parameterized with measured data from a recent 2.53 GHz urban macrocellular channel measurement campaign in Cologne, Germany. Assuming spatially correlated Ricean fading, with maximum–ratio transmission (MRT) precoding, tight closed–form approximations to the expected (average) SINR, and ergodic sum spectral efficiency are derived. The expressions provide clear insights into the impact of variable correlation patterns on the above performance metrics. Our results demonstrate the sensitivity of the MU–MIMO performance to different correlation models, and provide a cautionary tale of its impact.

I. INTRODUCTION

In multiuser multiple–input multiple–output (MU–MIMO) systems, an antenna array at a cellular base station (BS) serves a multiplicity of user terminals [1]. Electromagnetic propagation between the array and the terminals is typically characterized by the deterministic far–field specular wavefronts, superimposed on a set of diffuse multipath components (MPCs) [2]. Depending on the severity of scattering and the relative physical separation of two terminals, the MPCs often arrive at two terminals via the same clusters of scatterers, and thus are spatially correlated. Indeed, it is well known in the MU–MIMO literature, that spatial correlation is detrimental to the signal–to–interference–plus–noise–ratio (SINR) of a given terminal, and the system spectral efficiency (see e.g., [3–7]). In fact, this finding is routinely reported when each terminal has the same correlation matrix.

In sharp contrast to the above statement, a different line of investigations has identified that correlation can enhance MU–MIMO performance [8–11]. The critical observation from these studies is that the departing spread of energy from the BS can arrive at a given terminal via a (partially) different set of clusters located within the vicinity of the terminal. This contributes to variations in the statistics of the channels seen by the terminals. Fundamentally, such variations depend on the geometry of local scattering, as well as the antenna spacing at the BS. To capture the above physical manifestations, remote scattering models, such as one–ring correlation, have been proposed [8–11]. These models are characterized in terms of the mean azimuth angle–of–arrival (AoA) at a terminal, departing azimuth angular spread, as well as the antenna spacing at the BS. The work of [8, 10] utilized the one–ring model to group the terminals with similar correlation characteristics.

Moreover, [11] reports that if the terminal correlation matrices span orthogonal subspaces, the fundamental impairment of pilot contamination vanishes.

Nevertheless, all of the above studies neglect the presence of line–of–sight (LoS) components with correlated diffuse MPCs. Moreover, it remains to be seen just how much performance gain is available with variable correlation, in comparison to the case when each terminal has the same correlation matrix. Even more critically, almost all of the analytical results predicting MU–MIMO performance with linear signal processing and variable correlation, are left in terms of numerical fixed point algorithms, making it extremely difficult to gain any practical insights (see e.g., [7, 9, 10, 12]). To gain a fundamental understanding of MU–MIMO performance with and without variable correlation, it is desirable to have a set of insightful and simple downlink performance metrics. This is missing from the current literature, and in this paper we close this gap. As the remote scattering models rely on spatial parameters of the propagation channel, for most accurate parameterization, we extract the required parameters from a recent 2.53 GHz MU–MIMO measurement campaign in Cologne, Germany. To the best of the authors’ knowledge, there are very few studies which use measured multipath parameters to investigate the variability of correlation in multiuser systems. We note that the authors in [2, 13] make initial investigations into characterizing the commonality of scattering clusters along with the LoS components at different terminal locations.

Specifically, our main contributions are as follows:

• We derive closed–form approximations to the expected per–terminal SINR and ergodic sum spectral efficiency with maximum–ratio transmission (MRT) precoding. Assuming spatially correlated Ricean fading, the approximations provide clear insights into the impact of various system parameters, such as the number of BS antennas, unequal correlation structure, Ricean $K$–factor, and...
average downlink signal-to-noise-ratio (SNR). To the best of the authors’ knowledge, such general, and simple analyses are missing from the literature.

- We prove that for a fixed average correlation matrix for all terminals, equal correlation increases the expected interference power, in contrast to variable correlation. As a result, equal correlation provides a useful lower limit on the performance of such systems.
- Our numerical findings suggest that the choice of a particular correlation model has a significant impact on the expected SINR and ergodic spectral efficiency. Physically motivated models, such as one-ring, give enhanced performance over non-physical models, such as the exponential and Clerckx correlation models [14]. To parameterize the one-ring model, we utilize measured angular spreads and mean AoA distributions at 2.53 GHz and is denoted by \( A \).

For the remainder of the paper, without loss of generality, we assume that terminal 1, obtained from column 1 of \( G \), the composite \( M \times L \) un-normalized precoding matrix. The data symbol on the performance of such systems.

**Notation.** Upper and lower boldface letters represent matrices and vectors. The \( M \times M \) identity matrix is denoted as \( I_M \). Transpose, Hermitian transpose, inverse and trace operators are denoted by \((\cdot)^T\), \((\cdot)^H\), \((\cdot)^{-1}\), and \(\text{tr}\{\cdot\}\), respectively. \(\parallel \cdot \parallel_F\) and \(\parallel \cdot \parallel\) denote the Frobenius and scalar norms. We use \( h \sim CN(m,Q) \) to denote a complex Gaussian distribution for \( h \) with mean \( m \) and covariance matrix \( Q \). Similarly, \( h \sim U[a,b] \) is used to denote a uniform random variable for \( h \) taking on values from \( a \) to \( b \). Finally, \( \mathbb{E}\{\cdot\} \) denotes the statistical expectation of a random variable.

**II. SYSTEM AND PROPAGATION MODELS**

We consider the downlink of a single-cell MU-MIMO system in an UMa environment. The BS is located at the center of a circular cell with radius \( R_c \) and is equipped with a ULA of \( M \) transmit antennas. The BS simultaneously serves \( L \) single-antenna terminals (\( M \geq L \)) in the same time-frequency resource. Channel knowledge is assumed at the BS with narrow-band transmission and uniform power allocation. For the remainder of the paper, without loss of generality, terminal 1 will be considered as the desired terminal, while terminals 2, 3, \ldots, \( L \) are considered as interfering terminals.

**A. Propagation Model**

The \( 1 \times M \) propagation channel to terminal 1 from the BS array is assumed to follow a spatially correlated Ricean fading distribution, and is denoted by

\[
h_1 = \sqrt{\frac{K_1}{K_1 + 1}} \mathbf{h}_1 + \sqrt{\frac{1}{K_1 + 1}} \mathbf{h}_1 \mathbf{R}_2 = \sqrt{\frac{K_1}{K_1 + 1}} \mathbf{h}_1 + \sqrt{\frac{1}{K_1 + 1}} \mathbf{h}_1 \mathbf{R}_2, \quad (1)
\]

Note that the \( 1 \times M \) specular and diffuse MPCs are denoted by \( v_1 \) and \( w_1 \), respectively. Moreover, \( K_1 \) denotes the ratio between the power of the specular and diffuse MPCs for terminal 1, and is known as the Ricean \( K \)-factor. Note that \( K_1 \) is unique to terminal 1, and is a function of the local scattering in its proximity. Further to this, \( h_1 \sim CN(0, \mathbf{I}_M) \), \( \mathbf{h}_1 = [1, e^{j2\pi d \cos(\phi_1)}, \ldots, e^{j2\pi d (M-1) \cos(\phi_1)}] \). Here \( d \) is the antenna spacing between successive elements normalized by \( \lambda \), the wavelength associated with the operating carrier frequency, \( f_c \). Note that \( \phi_1 \) is the azimuth angle-of-departure (AoD) for terminal 1. In addition to the LoS components, we consider correlated MPCs. To this end, unlike previous works (see e.g., [3–7]), we define a \( M \times M \) spatial correlation matrix for terminal 1 as \( \mathbf{R}_1 \). Naturally, \( \mathbf{R}_1 \) is a function of the spatial parameters of the propagation channel, such as the angular spread and the mean AoA [8–11,13]. Further discussion on the possible structures of \( \mathbf{R}_1 \) is given in Section V.

**B. Downlink Received Signal Model**

The received signal at terminal 1 can be written as

\[
y_1 = \sqrt{\frac{\beta_1}{\eta}} \mathbf{h}_1 \mathbf{g}_1 s_1 + \sum_{i=2}^{L} \sqrt{\frac{\beta_i}{\eta}} \mathbf{h}_1 \mathbf{g}_i s_i + n_1, \quad (2)
\]

where \( \beta_1 \) denotes the received power at terminal 1 from the BS (discussed later in the text). Moreover, \( g_1 \) is the \( M \times 1 \) un-normalized downlink precoding vector from the BS array to terminal 1, obtained from column 1 of \( G \), the composite \( M \times L \) un-normalized precoding matrix. The data symbol \( s_1 \), \( s_i \), respectively. Moreover, \( \eta \) denotes the ratio of the expected SINR and ergodic spectral efficiency.

Mathematically, the received power at terminal 1 is denoted by \( s_1 \), such that \( \mathbb{E}\{|s_1|^2\} = 1 \), and \( n_1 \sim CN(0,\sigma^2) \) models the additive white Gaussian noise at terminal 1. Note that \( \sigma^2 \) is fixed for all terminals \( 1,2,\ldots,L \). Following [3], \( \eta = \|G\|_F^2/L \) is the precoding normalization parameter such that \( \mathbb{E}\{\|g_1\|^2\} = 1 \), for \( \ell = 1,2,\ldots,L \) (discussed further in the text). This ensures that the average transmit power at the BS remains unchanged.

The received power at terminal 1, \( \beta_1 = \rho A \zeta_1 (r_0/r_1)^\alpha \), is composed of the average downlink transmit power, \( \rho \), with large-scale propagation effects. In particular, \( A \) is the unitless constant for geometric attenuation at a reference distance \( r_0 \), \( r_1 \) is the link distance between the BS and terminal 1, \( \alpha \) is the attenuation exponent and \( \zeta_1 \) models the effects of shadow fading which follows a lognormal distribution, i.e., \( 10 \log_{10}(\zeta_1) \sim N(0,\sigma^2_\zeta) \). For the rest of the paper, we assume that \( \sigma^2 = 1 \). Thus, the average downlink SINR is equivalent to the average downlink transmit power, \( \rho/\sigma^2 = \rho \). In line with [15], we employ a probability based approach to determine if a given terminal experiences LoS or non-LoS (NLoS) propagation. The LoS and NLoS probabilities are a function of the link distance, from which the LoS and NLoS geometric attenuation and other link characteristics are obtained. The terminal dependent \( K \)-factors are assumed to follow a lognormal distribution with the mean and variance provided in [15]. For the sake of consistency, we delay the discussion of the above mentioned approach and other large-scale parameters to Section V.

**C. MRT SINR and Ergodic Sum Spectral Efficiency**

With MRT, \( \mathbf{g}_1 = \mathbf{g}_{1,\text{MRT}} = \mathbf{h}_1^H \), which forms the first column of the composite \( M \times L \) MRT precoding matrix, \( \mathbf{G}_{\text{MRT}} = \mathbf{H}^H \). Note that \( \mathbf{H} = [\mathbf{h}_1^H, \mathbf{h}_2^H, \ldots, \mathbf{h}_L^H] \) is the \( L \times M \) composite matrix containing channels for all \( L \) terminals. From (2), the MRT SINR for terminal 1 is given by

\[
\text{SINR}_{1,\text{MRT}} = \frac{\beta_1}{\sigma^2 + \frac{\beta_1}{\eta} \sum_{i=2}^{L} \|h_1 g_i,\text{MRT}\|^2}, \quad (3)
\]
where $\eta_{\text{MRT}} = \|G_{\text{MRT}}\|^2/L$. The received SINR with MRT in (3) can be translated into an ergodic sum spectral efficiency (in bits/sec/Hz) for all $L$ terminals. This is denoted as

$$R_{\text{MRT}} = \mathbb{E}\left\{ \sum_{\ell=1}^{L} \log_2 \left(1 + \text{SINR}_{\ell,\text{MRT}} \right) \right\}. \tag{4}$$

In (4), the statistical expectation is performed over an ensemble of the diffuse MPCs in the propagation channel.

### III. ANALYTICAL RESULTS AND IMPLICATIONS

#### A. Expected MRT SINR and Ergodic Sum Spectral Efficiency

From (3), the expected SINR for terminal 1 can be obtained by computing $\mathbb{E}\{\text{SINR}_{1,\text{MRT}}\}$. Exact evaluation of $\mathbb{E}\{\text{SINR}_{1,\text{MRT}}\}$ is extremely challenging, due to the ratio of expectations [3, 5, 16]. To overcome this difficulty, we employ the commonly used first–order Laplace expansion [3, 16] to approximate $\mathbb{E}\{\text{SINR}_{1,\text{MRT}}\}$, allowing us to write

$$\mathbb{E}\{\text{SINR}_{1,\text{MRT}}\} \approx \frac{\beta_1}{\sigma_1^2} \mathbb{E}\left\{ \left| h_1 g_{1,\text{MRT}} \right|^2 \right\}. \tag{5}$$

In (5), $\beta_1 = \mathbb{E}\{\text{SINR}_{1,\text{MRT}}\}$.

**Remark 1.** The approximation in (5) is a first–order Laplace expansion, and is of the form $\mathbb{E}\{X/Y\} \approx \mathbb{E}\{X\}/\mathbb{E}\{Y\}$. As shown in [3, 16], the accuracy of such approximations relies on $Y$ having a small variance relative to its mean. This can be seen by applying a multivariate Taylor series to $X/Y$ around $\mathbb{E}\{X\}/\mathbb{E}\{Y\}$. The quadratic forms in (5) are well suited to this approximation, especially when $M$ and $L$ start to grow, since the averaging implicit in the quadratic forms gives the variance reduction required. For further discussion, we refer the interested reader to [3, 16]. In the sequel, the expectations on the numerator and denominator of (5) are derived.

#### Lemma 1

With a spatially correlated Ricean channel to terminal 1, the expected value of the desired signal power using MRT is given by

$$\delta_1 = (\bar{\kappa}_1)^2 M^2 + 2 M \bar{\kappa}_1^2 (\bar{\kappa}_1^2) + 2 (\bar{\kappa}_1^2) \bar{h}_1 R_1 \bar{h}_1^H$$

$$+ (\bar{\kappa}_1^2) \mathbb{E}\left\{ \left| h_1 g_{1,\text{MRT}} \right|^2 \right\}. \tag{6}$$

**Proof:** From (1), we know that $h_1 = \bar{h}_1 + \bar{h}_1 R_1^2 = v_1 + w_1$. Then, via first principles, one can state

$$\delta_1 = \mathbb{E}\left\{ \left| h_1 g_{1,\text{MRT}} \right|^2 \right\} = \mathbb{E}\left\{ \left| h_1 h_1^H \right|^2 \right\} = \mathbb{E}\left\{ \left( v_1 + w_1 \right) \left( v_1^H + w_1^H \right) \right\}. \tag{7}$$

Expanding (7), taking the expectation over $\bar{h}_1$ in $w_1$, and simplifying allows one to write

$$\delta_1 = (v_1 v_1^H) + 2 M (\bar{\kappa}_1^2) v_1 v_1^H$$

$$+ 2 (\bar{\kappa}_1^2) v_1 R_1 v_1^H + (\bar{\kappa}_1^2) \mathbb{E}\left\{ \left( \bar{h}_1 \bar{h}_1^H \right)^2 \right\}. \tag{8}$$

Via an eigenvalue decomposition, $R_1 = X_1 A_1 X_1^H$, and as a result $\mathbb{E}\{h_1 R_1 \bar{h}_1^H\} = \mathbb{E}\{\sum_{j=1}^{L} A_{1,j} \bar{h}_1 (\bar{h}_1^j)^H\}$. The notation $A_{1,j}$ denotes the $(j,j)$–th entry of $A_1$, and $\bar{h}_1 (\bar{h}_1^j)^H$ denotes the $j$–th entry of $\bar{h}_1$. Taking the expectation over $\bar{h}_1$ and simplifying permits us to write $\mathbb{E}\{h_1 R_1 \bar{h}_1^H\} = \mathbb{E}\{h_1 R_1 \bar{h}_1^H\} = (\text{tr} \{R_1\})^2 + \text{tr} \{R_1\}$. Recognizing that $\text{tr} \{R_1\} = M$, $\mathbb{E}\{h_1 R_1 \bar{h}_1^H\} = M^2 + \text{tr} \{R_1\}$. Substituting the right–hand side of (9) into $\delta_1$ in (8), recognizing that $v_1 v_1^H = (\bar{\kappa}_1^2) M$, and extracting the constants gives the desired result in Lemma 1.

**Lemma 2.** Under the same conditions as Lemma 1, the expected interference power to terminal 1 from transmission to terminal $i$ is given by

$$\chi_{1,i} = (\bar{\kappa}_1^2) (\bar{\kappa}_i^2) \mathbb{E}\{h_i R_i \bar{h}_i^H\} + (\bar{\kappa}_i^2) \mathbb{E}\{h_i R_i \bar{h}_i^H\} = (\bar{\kappa}_1^2) (\bar{\kappa}_i^2) \mathbb{E}\{h_i R_i \bar{h}_i^H\} + (\bar{\kappa}_i^2) \mathbb{E}\{h_i R_i \bar{h}_i^H\}. \tag{10}$$

Note that $i = 2, 3, \ldots, L$, and are the interfering terminals.

**Proof:** Similar to the proof of Lemma 1, we know that $h_1 = \bar{h}_1 + \bar{h}_1 R_1^2 = v_1 + w_1$. This allows us to express the interference to terminal 1 as

$$\chi_{1,i} = \mathbb{E}\{h_1 g_{1,\text{MRT}}^i\} = \mathbb{E}\{v_1 + w_1\} \left( v_i^H + w_i^H \right). \tag{11}$$

Further expansion and simplifications permits us to write

$$\chi_{1,i} = \mathbb{E}\{w_i w_i^H v_i v_i^H\} + \mathbb{E}\{v_i v_i^H v_i v_i^H\} \tag{12}$$

Noticing that $\mathbb{E}\{w_i w_i^H v_i v_i^H\} = \mathbb{E}\{\bar{h}_i R_i^2 \bar{h}_i R_i^2 \bar{h}_i^H \bar{h}_i^H\} = (\bar{\kappa}_i^2) R_i$, and substituting back the definitions of $w_i$, $v_i$, $w_i$, and $v_i$ into (12), and extracting the relevant constants gives

$$\chi_{1,i} = (\bar{\kappa}_1^2) (\bar{\kappa}_i^2) \mathbb{E}\{h_i R_i \bar{h}_i^H\}$$

$$+ (\bar{\kappa}_i^2) \mathbb{E}\{h_i R_i \bar{h}_i^H\} \tag{13}$$

Taking the expectation of the traces and simplifying gives the desired result, concluding the proof.

**Lemma 3.** Under the same condition as Lemmas 1 and 2, $\hat{\eta}_{\text{MRT}}$ is given by

$$\hat{\eta}_{\text{MRT}} = \mathbb{E}\{\text{SINR}_{1,\text{MRT}}\} = M. \tag{14}$$

**Proof:** By definition, $\hat{\eta}_{\text{MRT}} = (1/L) \mathbb{E}\{\text{tr}\{\sum_{i=1}^{L} h_i h_i^H\}\}$. Substituting $h_i$ from (1) for any terminal $j$, taking the expectation over $\bar{h}_i$, and extracting the relevant constants yields the desired result. Note that only a sketch of the proof is given here as it relies on straightforward algebraic manipulations.

**Theorem 1.** The expected SINR for terminal 1 with MRT processing can be approximated as

$$\mathbb{E}\{\text{SINR}_{1,\text{MRT}}\} \approx \frac{\beta_1}{\sigma_1^2} \mathbb{E}\left\{ \left| h_1 g_{1,\text{MRT}} \right|^2 \right\} \tag{15}$$

where $\delta_1, \chi_{1,i}$, and $\hat{\eta}_{\text{MRT}}$ are as in (6), (10), and (14).

**Proof:** Substituting Lemmas 1, 2, and 3 for $\delta_1, \chi_{1,i}$, and $\hat{\eta}_{\text{MRT}}$ gives the desired result.

### B. Implications of Theorem 1

To the best of the authors’ knowledge, the result in Theorem 1 is the first closed–form approximation for an average SINR of an arbitrary terminal, under a fully heterogeneous channel consisting of variable correlation, variable LoS levels, and variable link gains. Several important insights can be obtained by inspecting the individual structures of $\delta_1$ and $\chi_{1,i}$ in Lemmas 1 and 2. It can be seen that both $\delta_1$ and $\chi_{1,i}$ contain quadratic forms of the type $\bar{h}_i R_i \bar{h}_i^H$. Via the Rayleigh
quotient result, such quadratic forms are maximized when \( \hat{h}_1 \) is aligned (parallel) with the direction of the maximum eigenvector of \( R_1 \). Alignment of the desired LoS component, \( \hat{h}_1 \) with the desired correlation matrix \( R_1 \) amplifies the expected signal power. On the other hand, alignment of \( \hat{h}_1 \) with \( R_1 \), \( \hat{h}_1 \) with \( R_i \), and \( \hat{h}_1 \) with \( \hat{h}_1 \) increases the expected multiuser interference. In the same fashion, if \( R_1 \) and \( R_i \) are aligned, the overall preferential directions of the two propagation channels become similar and degrade the expected SINR. The global phenomenon is that the SINR decreases by virtue of channel similarities of various types (LoS and correlation), and increases if the channels are more diverse. The result in Theorem 1 also lends itself to many special cases, for instance in pure NLoS conditions, with and without variable correlation, as well as under Ricean fading with fixed correlation matrices. Due to space constraints, we omit presenting all possible special cases, but demonstrate an important insight into the impact of variable spatial correlation for fully NLoS (correlated Rayleigh) fading below.

**Corollary 1.** Under fully NLoS conditions, with unequal MPCs statistics to each terminal, the expected MRT SINR for terminal 1 can be approximated as

\[
E\{\text{SINR}_{1,\text{MRT}}\} \approx \frac{\beta_1}{M} \left\{ M^2 + \text{tr}\left\{ (R_1)^2 \right\} \right\} \frac{1}{\sigma^2 + \frac{1}{M} \sum_{i=2}^{L} \text{tr}\{R_i R_i^H\}}. \quad (16)
\]

**Proof:** Setting \( \hat{\kappa}_1 = \hat{\kappa}_1 = 0 \) in Lemmas 1, 2, and 3 yields the desired result.

**Remark 2.** Note that the interference power (second term on the denominator of (16)) can also be written as \( (\beta_1/M)(L - 1)\text{tr}\{R_1 R_{-1}\} \), where \( R_{-1} = (\sum_{i=2}^{L} R_i)/(L - 1) \) is the average correlation matrix of all interfering terminals \( i = 2, 3, \ldots, L \). While the numerator grows quadratically with increasing \( M \), and the size of \( R_1 \), it is straightforward to observe that when \( \text{tr}\{R_1 R_{-1}\} \) increases, the expected interference power also increases. To this end, we study the impact of variation in the correlation structures on the expected SINR derived in (16). The case of variable correlation results in \( R_1 \neq R_{-1} \). In order to make a fair comparison with the distinct values of \( R_1 \) and \( R_{-1} \), for the fixed correlation case, a common value given by \((1/2)(R_1 + R_{-1})\) is assumed for both correlation matrices. Therefore, the interference term relies on \( \text{tr}\{R_1 R_{-1}\} \) for the variable correlation case, and \((1/2)(R_1 + R_{-1})^2 \) for the fixed (common) correlation case. Beginning with the fact that \( \text{tr}\{A^2\} \geq 0 \) for any Hermitian matrix \( A \), one can write

\[
\text{tr}\left\{ \left( \frac{R_1 - R_{-1}}{2} \right)^2 \right\} \geq 0, \quad (17)
\]

\[
\text{tr}\left\{ \frac{R_1^2}{4} + \frac{R_{-1}^2}{4} - \frac{R_1 R_{-1}}{4} - \frac{R_{-1} R_1}{4} \right\} \geq 0, \quad (18)
\]

\[
\text{tr}\left\{ \frac{R_1^2}{4} + \frac{R_{-1}^2}{4} + \frac{R_1 R_{-1}}{4} + \frac{R_{-1} R_1}{4} \right\} \geq 0, \quad (19)
\]

\[
\text{tr}\left\{ \left( \frac{R_1 + R_{-1}}{2} \right)^2 \right\} \geq \text{tr}\{R_1 R_{-1}\}. \quad (20)
\]

Hence, if \( R_1 \neq R_{-1} \), the case for fixed (common) correlation matrices, terminal 1 will have a higher total expected interference power in comparison to the variable correlation case where \( R_1 \neq R_{-1} \). Since this holds across all terminals, it can be concluded that fixed correlation matrices result in the lowest SINR, given a fixed overall average correlation matrix. As a result, such a scenario provides a useful lower bound for the performance of spatially correlated MU-MIMO channels.

We note that (15) can be further translated to approximate the ergodic sum spectral efficiency of the system by

\[
E\{R_{\text{MRT}}\} \approx \log_2 \left( 1 + E\{\text{SINR}_{\ell,\text{MRT}}\} \right). \quad (21)
\]

**IV. PROPAGATION CHANNEL MEASUREMENTS**

**A. Measurement Environment**

The propagation measurements were conducted in the old-town city center in Cologne, a moderate–sized city in Germany with a typical European layout. The investigated area was mostly made up of buildings with similar heights and multiple floors (ranging between 4–8). The BS array (referred to as TX) was mounted on a rooftop of a 30 m high–rise building with the terminals (referred to as RX) placed on the rooftop of a car at approximately 2.5 m above ground. The TX and RX placements in the environment are shown in Figs. 1 and 2. The measurements were conducted in 45 random terminal positions throughout the environment. The TX array was fixed at a given location throughout the measurement campaign. To avoid probable interference at the desired frequencies, the German service provider, Deutsche Telekom, agreed to switch off their BSs operating at the same or adjacent frequencies throughout the duration of the measurements.

**B. Measurement Setup**

The propagation measurements were performed with a wideband MIMO MEDAV RUSK channel sounder, operating at a center frequency of 2.53 GHz. The RUSK channel sounder is based on the switched array principle, and has been used in a number of previous measurement campaigns (see e.g., [17, [18]).
As a result, only a single transmit and receive chains exist. The transmit signal is connected via a fast electronic switch, to the elements of the TX array sequentially. The RX side operates in a similar manner as the TX, allowing for sequential measurement of the propagation channel transfer function between all combinations of TX and RX elements. So long as the measurement sequence occurs within a timescale shorter than the channel coherence time, such a measurement is equivalent to a truly real-time measurement with parallel TX/RX chains for all antenna elements. The precise channel sounder settings are provided in Table I.

The measurement setup utilizes cylindrical array structures at both TX and RX ends of the RUSK sounder. The array structures guarantee a truly 3D (azimuth and elevation) channel measurement and ensures that MPCs from all directions in the urban environment could be easily captured. Both elevation and azimuth parameters are needed to be estimated to obtain accurate results, even though in this paper only the azimuth parameters are needed to populate the correlation models. A synthetic array was used at the TX end in the channel sounding setup and was constructed such that an 8-element (2 ports per-element) polarimetric uniform linear patch array (PULPA)\(^1\) was placed on a programmable positioner. The positioner was rotated in the azimuth domain with an angular range from \(-180^\circ\) to \(180^\circ\), in 6 degree step-sizes creating 60 virtual positions and imitating a cylindrical structure. Overall, this results in \(60 \times 8 \times 2\) TX channels. This is equivalent to a 480 element cylindrical array operating with two polarization states. At the RX, a stacked polarimetric uniform circular patch array (SPUCPA) with 2 (vertical) \(\times 8\) (circumference) \(\times 2\) (polarization) antenna ports were employed. Further discussion on the above is provided in [19].

C. Parameter Extraction

The RUSK sounder provides a 4-dimensional channel transfer function matrix, \(H(s,f,t,r)\), where \(s\) denotes the measurement snapshot index, \(f\) is the measured frequency index, while \(t\) and \(r\) denote TX and RX element indices, respectively. In total, \(s = 10\) snapshots were recorded for each measured transfer function to improve the measurement SNR when averaged. The impulse response, of the propagation channel with a given \((t,r)\) pair was obtained via an inverse fast Fourier transform. To extract the spatial parameters of the propagation channel, a high resolution parameter estimation algorithm known as RIMAX was utilized [20]. RIMAX provides a complete double-directional description of the propagation channel and extracts the spatial parameters to obtain an antenna independent characterization of the channel. This means that the spatial parameters of the channel remain independent to the type of the antenna array that is used to make the measurement. Due to space limitations, we omit presenting the procedure for identifying MPCs which belong to a particular clusters of scatterers. More details on this can be found in [19]. Leveraging this property, from the MPCs, we extract the root mean square azimuth angular spread, as well as the mean AoA distributions across all 45 terminal positions. These form the basis for parameterizing the spatial correlation structures considered in the paper (See Section V-A). Even though this is an approximation, it is able to offer significantly greater insights than a purely numerical calculation of the correlation structure, as done routinely in the MU-MIMO literature. The extracted results are presented in Fig. 3 as cumulative distribution functions (CDFs). It can be observed that the azimuth AoD spread has a degree of symmetry and spans over \(40^\circ\). The variability in the angular spread is due to variability in the local scattering, which can be modeled as \(\mathcal{N}(14.02, (6.45^2)^\circ)\) (Gaussian fit on the fig.). In contrast to this, the mean AoA is \(\mathcal{U}([-180^\circ, 180^\circ])\) (uniform fit on the fig.), primarily reflecting the distribution of the terminals in the measurement environment.

V. Numerical Results

Unless otherwise specified, the parameters described below are utilized for all numerical results, and are obtained from [15]. A cell radius, \(R_c = 100\) m was chosen with a reference distance \(r_0 = 10\) m, such that the terminals are randomly located outside \(r_0\), and inside \(R_c\), following \(\mathcal{U}([-180^\circ, 180^\circ])\). The LoS and NLoS attenuation exponents, \(\alpha_i\), are given by 2.2 and 3.67, respectively. Furthermore, the unit–less constant for geometric attenuation, \(A\), is chosen such that the fifth–percentile of the instantaneous SINR with MRT processing at terminal 1 is \(0\) dB, when \(\rho = 0\) dB with \(M = 64\) and \(L = 8\).

### Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>2.52 GHz – 2.54 GHz</td>
</tr>
<tr>
<td>No. of frequency points</td>
<td>257</td>
</tr>
<tr>
<td>Number of channels</td>
<td>900 (\times 32)</td>
</tr>
<tr>
<td>Total time syn. aperture</td>
<td>Appro. 10 mins</td>
</tr>
<tr>
<td>Tx ports, Rx ports</td>
<td>900 ports, 32 ports</td>
</tr>
<tr>
<td>Azimuth range</td>
<td>([-180^\circ) to (180^\circ)]</td>
</tr>
<tr>
<td>Elevation range</td>
<td>([90^\circ) to (-90^\circ)]</td>
</tr>
</tbody>
</table>

\(^1\)The PULPA contains more than 8–elements. The other elements are used as dummy elements to assuage fringe effects caused by mutual coupling.
Furthermore, let $L = 8$.

### B. Impact of Variable Correlation Structures

The O.R. model are specified in each subsequent result.

Naturally, $P_{dB}$ and $6$ dB. The probability of terminal 1 experiencing LoS is given by $P_{\text{LoS},1}(r_1) = (\min(18/r_1, 1)(1-e^{-r_1/36})) + e^{-r_1/36}$. Naturally, $P_{\text{NLoS},1}(r_1) = 1 - P_{\text{LoS},1}(r_1)$. For each subsequent result, $10^5$ Monte–Carlo realizations were generated with an inter–element spacing, $d = 0.5\lambda$ at the BS.

### A. Spatial Correlation Models

As a baseline case, we model fixed correlation to each terminal with the widely used exponential model, where the $(i,j)$–th element of $R_1$ is expressed as $[R_1]_{i,j} = \xi^{i-j}$, for any $i, j$ in $1, 2, \ldots, M$ with $0 \leq \xi \leq 1$ [3]. Unless otherwise specified, $\xi = 0.9$ is used throughout the evaluation. With variable correlation, we employ two models, namely Clerckx [14], and one–ring (O.R.) [9, 10] correlation. For the Clerckx correlation model, $[R_1]_{i,j} = \xi e^{\phi}$, where $\xi = |\xi|e^{j\Delta_1}$. Here, $|\xi| = \xi$, as in the exponential model, and is the same for each terminal. However, a terminal specific phase, $\Delta_1$, is assumed to be uniformly distributed on a subset of $-180^\circ$ to $180^\circ$. This is used to differentiate the terminal locations. In each result, the range of $\Delta_1$ is specified. We refer to the Clerckx model as Clerckx Corr. and the exponential model as Exp. Corr. In contrast to the above, the O.R. model for terminal 1 states $\Delta_1 + \phi_1 = \frac{1}{2\Delta_1} \int_{-\Delta_1+\phi_1}^{\Delta_1+\phi_1} e^{j2\pi d(i,j)\sin(\phi_1)} d\phi_1$, where $\Delta_1$ denotes the azimuth angular spread for terminal 1, $\phi_1$ denotes the mean AoA, $\phi_1$ is the actual AoA, uniformly distributed within the angular spread around the mean AoA. Furthermore, $d(i,j)$ captures the normalized antenna spacing between the $i$–th and $j$–th elements. The precise values of $\Delta_1$ for the O.R. model are specified in each subsequent result.

### B. Impact of Variable Correlation Structures

Fig. 4 depicts the CDFs of the expected SINR with $M = 64$, $L = 8$, and $\rho = 10$ dB. Each density is obtained by averaging over the diffuse MPCs, with the CDFs representing the variations resulting from the link gains and the $K$–factors.
correlation plays a less significant role, as the resulting performance from each correlation matrix is almost identical. The proposed approximations are seen to remain tight across all the considered models, and SNR values. This result shows the profound impact variable angular spread has on MU–MIMO performance from each correlation matrix is almost identical. The authors’ knowledge, such an evaluation of a MU–MIMO system is unique and emphasizes that its performance is ultimately governed by the respective correlation model in use.

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REFERENCES


Fig. 6. Ergodic sum spectral efficiency CDFs for various LoS and NLoS scenarios with $M = 64$, $L = 8$, and $\rho = 10$ dB.