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Secure Massive MIMO with the Artificial Noise-Aided Downlink Training

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Abstract—This paper considers a massive MIMO network that includes one multiple-antenna base station, one multiple-antenna eavesdropper, and $K$ single-antenna users. The eavesdropper operates in passive mode and tries to overhear the confidential information from one of the users in the down-link transmission. In order to secure the confidential information, two artificial noise (AN)-aiding schemes are proposed. In the first scheme, AN is injected into the downlink training signals to prevent the eavesdropper from obtaining the correct channel state information of the eavesdropping link. In the second scheme, AN is deployed in both downlink training phase and payload data transmission phase to further degrade the eavesdropping channel. Analytical expressions and tight approximations of the achievable secrecy rate of the considered systems are derived with taking imperfect channel estimation and two types of precoding, i.e., maximum-ratio-transmission and zero-forcing, into consideration. Optimization algorithms for power allocation are proposed to enhance the secrecy performance of the proposed AN-aiding schemes. The results reveal that deploying AN in the downlink training phase of massive MIMO networks does not affect the downlink channel estimation process at users while enabling the system to suppress the downlink channel estimation process at eavesdropper. As a consequence, the proposed AN-aided schemes improve the system performance significantly. Furthermore, implementing AN in both phases allows the considered system having a flexible solution to maximize its secrecy performance at the price of higher complexity.

Index Terms—Physical layer security, massive MIMO, artificial noise.

I. INTRODUCTION

The booming of wireless communication demands huge efforts in securing information. Exploiting the broadcast nature of wireless channels, adversary can easily intercept the confidential messages. The conventional method for security is to implement cryptographic encryption in the application layers. However, this approach is potentially vulnerable to malicious attack because it is based on assumptions of computational complexity [1]. Recently, physical layer security (PLS) [2] has attracted a broad attention from the research community as a complement to the traditional cryptographic encryption. PLS takes advantage of the randomness of wireless channels to enhance the secrecy performance of wireless communication.

PLS concept was first outlined in [3] in which a network consisting of an information source, an intended receiver, and a passive eavesdropper was considered. It has shown that the perfect secured transmission between Alice and Bob can be achieved as long as the condition of the legitimate channel is better than that of the eavesdropping channel. Usually, eavesdroppers operate in the passive mode to prevent the legitimate side from obtaining their channel state information (CSI) [4]. There have been methods to improve the condition of the legitimate channels and degrade the eavesdropping channel’s quality, e.g., using multiple antennas and/or using AN-aiding source [5]. Combining multiple-antennas and AN can further enhance the secrecy performance when precoding is implemented to cancel the interference at the legitimate users while suppressing the illegitimate channels [5]–[7].

Recently, massive multiple-input multiple-output (MIMO) network has attracted a lot of attention from the research community and has become the key candidate for next generation wireless networks [8]–[10]. By using a very large number of antennas to serve several users simultaneously in the same frequency band, massive MIMO network offers great power efficiency and spectral efficiency [11]–[13]. In massive MIMO, with time-duplex division (TDD), the number of antennas at the base station (BS) does not affect the resources needed for the channel estimation. Therefore, TDD is more preferable than frequency-duplex division (FDD). In TDD operation, the BS and users exchange their CSI over uplink and dowlink training phases. In uplink training, the users send their pilots to the BS. The BS estimates the channel based on the knowledge of the pilots and then creates the precoding matrix. In dowlink training, the BS processes beamforming pilot to users and then users estimate the effective channels. These effective channels are used in decoding information in the payload data transmission phase. However, these processes enable the illegitimate side to gain CSI of the eavesdropping channels. As a consequence, the eavesdropper can take advantage on this knowledge to successfully decode the transmitted information from the base station in the downlink transmission. Therefore, protecting the training phases from exposing to the illegitimate side is crucial [14], where deploying AN in protecting downlink training phase of massive MIMO network is promising.
A. Related Works

There have been several studies on PLS in massive MIMO network using AN. In [15], a hybrid spatial and temporal AN scheme to protect the confidential information in a massive MIMO OFDM system was proposed. In this work, the spatial AN is injected to the precoded signal and also transmitted into the orthogonal direction of the information vector. The paper [16] investigated the secrecy performance of a massive MIMO system in the presence of one multi-antennas eavesdropper and hardware impairment effects. A generalized null-space AN in payload data transmission phase was proposed to deal with the effect of hardware impairment. In [17], the authors proposed a low complexity joint data and AN precoding scheme in a massive MIMO network with limited number of RF chains. Various AN schemes for secrecy enhancement in massive MIMO systems with distributed antennas were proposed in [18]. The authors in [19] suggested a symbol phase rotated scheme to protect the massive MIMO system from a massive MIMO eavesdropper. The results have shown that the proposed scheme can prevent the massive MIMO eavesdropper from recovering most of the transmitted symbols. In [20], various data precoders and AN precoders were proposed to secure the multi-cell massive MIMO system when the eavesdropper’s CSI is unavailable. In [21], a phase-only zero-forcing (ZF) AN scheme was studied to reduce the complexity in computing the conventional ZF AN for securing the massive MIMO network. In [22], the authors investigated the performance of an AN jamming-aided scheme at the transmitter over Rician channels. Furthermore, the authors in [23] used null-space and random AN generated from the spare antennas at the BS for securing the multi-cell massive MIMO system. However, to the best of the authors’ knowledge, there is no work investigating the AN-based scheme in the downlink training phase of massive MIMO.

B. Contributions

In this paper, we consider a massive MIMO downlink in the presence of a multiple-antenna eavesdropper. The main contributions of this paper are as follows:

- We propose two AN-aiding schemes, i.e., downlink training phase AN-aiding scheme and both phases (downlink training phase and payload data transmission phase) AN-aiding scheme to secure the massive MIMO network in the presence of a multiple-antenna eavesdropper. Furthermore, the effect of imperfect channel estimation is also considered.
- In order to study the behaviors of the proposed AN-aiding schemes on the massive MIMO network, we develop analytical expressions and tight approximations of the achievable secrecy rate of the considered system in two cases where the system uses maximum ratio transmission (MRT) precoding and implements ZF precoding while taking into consideration the influence of imperfect channel estimation.
- Optimization algorithms of extremely low-complexity for power allocation are also developed to enhance the secrecy performance of the two proposed AN-aiding schemes. By applying these algorithms, we show that the secrecy performance of the considered system is significantly enhanced. The results show that deploying AN in the downlink training phase of massive MIMO networks can confuse the eavesdropper’s channel estimation while staying harmless to the channel estimation process at the users. In addition, using AN in both phases offers the system a flexible solution to maximize its secrecy performance at the price of a higher complexity in the power allocation process.

The rest of this paper is organized as follows. The system and channel models are described in Section II. The analytical expressions and tight approximations for the achievable secrecy rate of the considered system with the two proposed AN-aiding schemes are developed in Section III and Section IV. In Section V, optimization algorithms for power allocation are proposed. The numerical results based on Monte-Carlo methods and discussions are presented in Section VI. Finally, we conclude our paper in Section VII.

Notations: Subscripts (·)T, (·)*, and (·)H stand for the transpose, the conjugate, and the conjugate transpose, respectively. The expectation operation, variance operation, and Euclidean norm are denoted by \( \mathbb{E}\{\cdot\} \), \( \text{var}(\cdot) \), and \( \|\cdot\| \), respectively. \( \mathbb{C}^{M \times N} \) represents the space of all \( M \times N \) matrices with complex-valued elements. \( I_N \) denotes the \( N \)-dimensional identity matrix. \( \mathcal{CN}(\mu, \sigma^2) \) indicates complex normal distribution with \( \mu \) mean and \( \sigma^2 \) variance.

II. System and Channel Models

Consider a massive MIMO network which includes one \( M \)-antenna BS, \( K \) single-antenna end users (EUs), and one \( N \)-antenna eavesdropper as described in Fig. 1.

In this system, the BS transmits information to the \( k \)-th EU, \( U_k \), over channel vector \( \sqrt{\beta_k} h_k \), where \( h_k \in \mathbb{C}^{M \times 1} \) is the small-scale fading vector, \( h_k \sim \mathcal{CN}(0, I_M) \), and \( \beta_k \) models the large-scale fading. We assume that all the \( K \) users share the same time-frequency resource.

The eavesdropper tries to eavesdrop the confidential information from the BS through the channel matrix \( \sqrt{\alpha} G \in \mathbb{C}^{N \times M} \).
$C^{M \times N}$, where the elements of $G$ follow i.i.d. $CN(0,1)$, and $\beta$ models the large-scale fading.

### A. Uplink training

All $K$ EUs simultaneously send their orthogonal pilots to the BS for CSI acquisition. The received signal at the BS is

$$Y_{BS} = \sum_{k}^{K} \sqrt{\rho \tau_u \beta_k} h_k \omega_k^H + N_{BS},$$

(1)

where $\omega_k \in C^{\tau_x \times 1}$ are the orthogonal pilot sequences of the length $\tau_x$, i.e., $\omega_k^H \omega_k = 1$, $\omega_k^H \omega_j = 0$ when $j \neq k$, $\rho$ is the transmit power at EU, and $N_{BS} \in C^{M \times \tau_x}$ is the AWGN at the BS with its elements following $CN(0, \sigma_0^2)$. Being known at the BS, $\omega_k$ is used to get $\hat{y}_k$ such that

$$\hat{y}_k = \sqrt{\rho \tau_u \beta_k} h_k \omega_k + N_{BS} \omega_k,$$

(2)

where $n_{BS} \sim CN(0, \sigma_0^2 I_{\tau_x})$.

From (2), the BS deploys the MMSE technique to estimate the channel between the BS and $U_k$. The channel estimate $\hat{h}_k$ is given by

$$\hat{h}_k = \Lambda_{\gamma_k, y}^{-1} y_{BS} \Lambda_{\gamma_k, y}^{-1} y_{BS},$$

(3)

where $\Lambda_{x,y}$ is the covariance matrix of random vectors $x$ and $y$. The estimation error is defined as

$$\tilde{h}_k = h_k - \hat{h}_k.$$

(4)

From the MMSE’s properties, $\tilde{h}_k$ and $\tilde{h}_k$ are mutually independent. Besides, we also have $\tilde{h}_k \sim CN(0, I_M(1 - \sigma_{u,k}^2))$, $\tilde{h}_k \sim CN(0, I_M(1 - \sigma_{u,k}^2))$, and $\sigma_{u,k} = \frac{\rho \tau_u \beta_k}{\sqrt{\nu_0 \tau_u \beta_k + 1}}$ where $\gamma_k = \frac{\alpha_u}{\sigma_0}$.

### B. Downlink Data Transmission

After performing the channel estimation, BS uses the estimated channels to decode the symbols intended for all $K$ users. To enhance the secrecy performance, AN is injected into the channel between the BS and $U_k$. The channel estimate $\hat{h}_k$ is

$$\hat{h}_k = \Lambda_{\gamma_k, y}^{-1} y_{BS} \Lambda_{\gamma_k, y}^{-1} y_{BS},$$

(5)

where $\alpha^H = \text{transmit power ratio for data}$, $\alpha^T = \text{transmit power ratio for AN}$, $W = [w_1...w_K] \in C^{M \times K}$ is the precoding matrix, $J = [j_1,j_K] \in C^{M \times K}$ is the AN matrix, $\|J\|_1 = 1$, $x = [x_1...x_K]$, $\|x\|_2 = 1$, is the vector of $K$ symbols intended for the $K$ users, and $\Lambda = [\lambda_1...\lambda_K]$, where $\|\lambda_k\|_2 = 1$, is the AN vector which is independent of $x$. The AN matrix lies in the null-space of the estimated legitimate channels, i.e., $\hat{H}^T J = 0$.

From (5), the transmit power condition for downlink is

$$E\{ \| \bar{S}_{BS} \|^2 \} \leq \rho_d.$$  

As a result,

$$\alpha^2 E\{ tr(WW^H) \} + \alpha_n^2 E\{ tr(JJ^H) \}$$

$$\alpha_p \alpha_n E\{ tr(WJ^H) \} + \alpha_p \alpha_n E\{ tr(JW^H) \} \leq 1.$$  

(6)

Note that $\alpha^T_0 = 0$ corresponds to the case where no AN is added in the data transmission phase.

1) **At End Users:** The received signals at $U_k$ is

$$y_k = \sqrt{\beta_k} h_k^T S_{BS} + n_k,$$

$$= \sqrt{\rho \alpha_p} h_k^T (\alpha_p^T W x + \alpha_n^T J \lambda) + n_k,$$

$$= \sqrt{\rho \alpha_p} h_k^T (\alpha_p^T u_k + \alpha_n^T j_k),$$

$$+ \sum_{l \neq k} \sqrt{\rho \beta_k} h_k^T (\alpha_p^T w_l, x_l + \alpha_n^T j_l) + n_k,$$

$$= \sqrt{\rho \alpha_p} (\alpha_p^T \tilde{u}_k + \alpha_n^T \tilde{j}_k),$$

$$+ \sum_{l \neq k} \sqrt{\rho \beta_k} (\alpha_p^T \tilde{w}_k, x_l + \alpha_n^T \tilde{j}_l) + n_k,$$

(7)

where $a_k = h_k^T w_k, a_k = h_k^T \tilde{w}_l, e_k = \tilde{h}_k^T j_k, e_k = \tilde{h}_k^T \tilde{j}_k$, and $n_k \sim CN(0, \sigma_0^2)$ is AWGN at $U_k$. Since $\|j_l\|_2 = 1, e_k$ and $e_k$ follow $CN(0, 1 - \sigma_{u,k}^2)$.

2) **At the Eavesdroppers:** The received signal at the eavesdropper is

$$y_k = \sqrt{\rho \beta_E} G^T S_{BS} + n_E,$$

$$= \sqrt{\rho \beta_E} G^T (\alpha_p^T w_k, x_k + \alpha_n^T j_k),$$

$$+ \sum_{l \neq k} \sqrt{\rho \beta_E} G^T (\alpha_p^T w_l, x_l + \alpha_n^T j_l) + n_E,$$

$$= \sqrt{\rho \beta_E} (\alpha_p^T \tilde{c}_k, x_l + \alpha_n^T \tilde{j}_k),$$

$$+ \sum_{l \neq k} \sqrt{\rho \beta_E} (\alpha_p^T \tilde{c}_l, x_l + \alpha_n^T \tilde{j}_l) + n_E,$$

(8)

where $c_k = G^T w_k, c_l = G^T w_l, u_k = G^T \tilde{j}_k, u_l = G^T \tilde{j}_l$, and $n_k \sim CN(0, \sigma_0^2)$ is the noise at the eavesdropper. Since $\|j_l\|_2 = 1, u_k$ and $u_l$ follow $CN(0, I_N)$.

### C. Downlink Training

To detect the desired signal, the $k$-th UE needs to estimate the effective channel gain $\alpha_{kk}$. Typically, the BS beamforms the pilots using the precoding matrix in Section II-B. Then from the received pilot signals, each EU will estimate its corresponding effective channel gain $\alpha_{kk}$. During this phase, the eavesdropper can also estimate its desired effective channel (between the BS and the eavesdropper) in order to improve its ability of eavesdropping the confidential information from the BS. In this work, we propose to use AN during the training phase to enhance the secrecy performance. With our proposed scheme, AN is added into the training signal to contaminate the channel estimation at the eavesdropper. More precisely, the pilot signal beamformed from the BS is given by

$$X_{BS} = \sqrt{\rho_d \sigma} (\alpha_p W + \alpha_n Z) \Phi^H,$$

(9)

where $\Phi \in C^{\tau_x \times K}$ represents pilot sequences for $K$ EUs, each of them has length $\tau_x, \Phi = [\phi_1...\phi_K], \Phi^H \Phi = I_K, Z = [z_1...z_K] \in C^{M \times K}, \|z_k\|_2 = 1$, is the AN matrix, $\alpha_p$ is the transmit power ratio for pilot, $\alpha_n$ is the transmit power ratio for AN, and $\rho_d$ is the transmit power at the BS. The
AN matrix is implemented in the null-space of the estimated downlink channels, where $z_k$ is chosen to satisfy
\[ \hat{H}_k^T z_k = 0. \]  
(10)

Using (9), the transmit power condition for downlink pilot transmission is $\mathbb{E}\left\{\|X_{BS}\|^2\right\} \leq \tau_d \rho_d$. As a result,
\[ \alpha_p^2 \mathbb{E}\left\{\text{tr}(WW^H)\right\} + \alpha_n^2 \mathbb{E}\left\{\text{tr}(ZZ^H)\right\} + \alpha_p \alpha_n \mathbb{E}\left\{\text{tr}(WZ^H)\right\} + \alpha_p \alpha_n \mathbb{E}\left\{\text{tr}(ZW^H)\right\} \leq 1. \]  
(11)

Note that $\alpha_n = 0$ corresponds to the conventional case where no AN is added [12].

1) At End Users: The received signal at $U_k$ is
\[ \begin{align*}
Y_{d,k}^T &= \sqrt{\alpha_p} h_k^T X_{BS} + n_k^T \\
&= \rho_d \tau_d \beta_k h_k^T (\alpha_p W + \alpha_n Z) \Phi^H + n_k^T,
\end{align*} \]  
(12)

where $n_k \sim \mathcal{CN}(0, \sigma_n^2 I_{r_a})$ is the noise at $U_k$.

Since $\phi_k$ is known at $U_k$, $U_k$ can process the received signal as follows:
\[ \begin{align*}
\hat{y}_{d,k}^T \phi_k &= \frac{\sqrt{\rho_d \tau_d \beta_k} h_k^T (\alpha_p W + \alpha_n Z) \Phi^H + n_k^T \phi_k}{n_k} \\
&= \frac{\rho_d \tau_d \beta_k \alpha_p w_k + \rho_d \tau_d \beta_k \alpha_n z_k + n_k}{n_k},
\end{align*} \]  
(13)

where $w_k = h_k^T z_k$, $w_k$ follows $\mathcal{CN}(0, 1 - \sigma_n^2 I_{r_a})$. $U_k$ estimates the effective channel $\alpha_k$ for decoding confidential messages in IT phase. $E_k$ estimates the channel $\alpha_{kk}$ for decoding confidential messages in IT phase. $E_k$ estimates the channel $\alpha_{kk}$ for decoding confidential messages in IT phase.

\[ \hat{a}_{kk} = \mathbb{E}\{a_{kk}\} + \Lambda_{\hat{a}_{kk}, \hat{y}_{d,k}} \Lambda_{\hat{y}_{d,k}, \hat{y}_{d,k}}^{-1} (\hat{y}_{d,k} - \mathbb{E}\{\hat{y}_{d,k}\}), \]  
(14)

where
\[ \Lambda_{\hat{y}_{d,k}, \hat{y}_{d,k}} = \rho_d \tau_d \beta_k [\alpha_p \mathbb{E}\{a_{kk}\} + \alpha_n \mathbb{E}\{a_{kk} b_{kk}^*\} - \alpha_n \mathbb{E}\{a_{kk}\} \mathbb{E}\{b_{kk}\}]. \]  
(15)

and
\[ \Lambda_{\hat{a}_{kk}, \hat{y}_{d,k}} = \rho_d \tau_d \beta_k [\alpha_p \mathbb{E}\{a_{kk}\} + \alpha_n \mathbb{E}\{a_{kk} b_{kk}^*\} - \alpha_n \mathbb{E}\{a_{kk}\} \mathbb{E}\{b_{kk}\}]. \]  
(16)

2) At Eavesdropper: During the downlink training, the eavesdropper intercepts the training information. The received signal at the eavesdropper is given as
\[ \begin{align*}
Y_{d,E} &= G^T X_{BS} + N_E \\
&= \sqrt{\rho_d \tau_d \beta_E} G^T (\alpha_p W + \alpha_n Z) \Phi^H + N_E,
\end{align*} \]  
(17)

where $N_E \sim \mathcal{CN}(0, \sigma_n^2 I_{N_E})$ is the noise at the eavesdropper, elements of $N_E$ follow $\mathcal{CN}(0, \sigma_n^2 I_{N_E})$.

Since the eavesdropper can easily have the knowledge of $\Phi$, it can process the received signal as follows:
\[ \begin{align*}
Y_{d,E} \Phi &= \sqrt{\rho_d \tau_d \beta_E} G^T (\alpha_p W + \alpha_n Z) + N_E \Phi \\
&= \frac{\sqrt{\rho_d \tau_d \beta_E} G^T (\alpha_p W + \alpha_n Z) + N_E \Phi}{N_E}
\end{align*} \]  
(18)

The eavesdropped information related to $U_k$ can be demonstrated as
\[ \begin{align*}
\hat{y}_{d,E,k} &= \sqrt{\rho_d \tau_d \beta_E} G^T (\alpha_p w_k + \alpha_n z_k) + \tilde{n}_{E,k} \\
&= \sqrt{\rho_d \tau_d \beta_E} \alpha_p c_k + \rho_d \tau_d \beta_E \alpha_n d_k + \tilde{n}_{E,k}. \]  
(19)

where $d_k = G^T z_k$.

The eavesdropper uses the MMSE to estimate $c_k$ for decoding confidential messages in the payload data transmission phase. The estimated effective channel at the eavesdropper is
\[ \hat{c}_k = \mathbb{E}\{c_k\} + \Lambda_{\hat{c}_k, \hat{y}_{d,E,k}} \Lambda_{\hat{y}_{d,E,k}, \hat{y}_{d,E,k}}^{-1} (\hat{y}_{d,E,k} - \mathbb{E}\{\hat{y}_{d,E,k}\}), \]  
(20)

where
\[ \Lambda_{\hat{c}_k, \hat{y}_{d,E,k}} = \rho_d \tau_d \beta_E (\alpha_p \mathbb{E}\{c_k\} + \alpha_n \mathbb{E}\{d_k^H c_k^H\} - \alpha_n \mathbb{E}\{c_k\} \mathbb{E}\{d_k c_k^H\}) + \sigma_n^2 I_{N_E}. \]  
(21)

D. Choosing Beamforming Matrix

In this paper, we consider two well-known precoding methods, i.e., MRT and ZF, which are widely implemented in massive MIMO systems. Being simple for implementation, MRT suffers from intra-interference, i.e., the undesired signals among users. Meanwhile, ZF precoding can effectively eliminate undesired signals among users at the price of high complexity.

1) MRT: For MRT, the precoding matrix is
\[ w_k^{MRT} = \frac{h_k^*}{\|h_k^*\|}. \]  
(23)

Following Appendix A, we have
\[ \mathbb{E}\{a_{kk}^{MRT}\} = \sigma_{u,k} \Gamma \left( \frac{2M+1}{2} \right) \Gamma(M), \]  
(24)

\[ \mathbb{E}\{|a_{kk}^{MRT}|^2\} = (M - 1) \sigma_{u,k}^2 + 1, \]  
(25)

\[ \text{var}(a_{kk}^{MRT}) = (M - 1) \sigma_{u,k}^2 + 1 - \sigma_{u,k}^2 \left( \frac{2M+1}{2} \right) \Gamma(M)^2. \]  
(26)

2) ZF: For ZF, the precoding matrix is chosen to null out the undesired signals from multi-users with the knowledge of the channel estimated during the training phase. The ZF precoding matrix is
\[ w_k = \frac{v_k}{\|v_k\|}, \]  
(27)

where $v_k$ is the $k$-th column of $V$:
\[ V = \hat{H}^* (\hat{H}^T \hat{H})^{-1}. \]  
(28)
Because $H^TV = I$, we have:

$$
\begin{align*}
\begin{cases}
\hat{h}_k^T v_k = 1, \\
\hat{h}_k^T v_l = 0, \quad l \neq k.
\end{cases}
\end{align*}
$$

Therefore,

$$
\begin{align*}
\mathbb{E}\{a_{kk}^{ZF}\} &= \frac{\Gamma(M-K+\frac{3}{2})}{\sigma_{u,k}\Gamma(M-K+1)}, \\
\mathbb{E}\{|a_{kk}^{ZF}|^2\} &= \frac{M+1-K}{\sigma_{u,k}^2} + 1 - \sigma_{u,k}^2, \quad \text{and} \\
\text{var}(a_{kk}^{ZF}) &= \frac{M+1-K}{\sigma_{u,k}^2} + 1 - \sigma_{u,k}^2 \\
&- \left(\frac{\Gamma(M-K+\frac{3}{2})}{\sigma_{u,k}\Gamma(M-K+1)}\right)^2.
\end{align*}
$$

The detailed proof of (32) is shown in Appendix A.

**Remark 1:** From (26) and (32), it is observed that

$$
\begin{align*}
\frac{\Gamma(M+\frac{1}{2})}{\Gamma(M)} &\xrightarrow{M \to \infty} \sqrt{M}, \\
\frac{\Gamma(M-K+\frac{3}{2})}{\Gamma(M-K+1)} &\xrightarrow{M \to \infty} \sqrt{M-K+1}.
\end{align*}
$$

Therefore, \(\text{var}(a_{kk}^{MRT,ZF}) \approx 1 - \sigma_{u,k}^2\).

The following conditions hold true for both MRT and ZF.

From (11), we have the condition for \(a_p\) and \(a_n\) as

$$
\begin{align*}
\alpha_p^2 \mathbb{E}\{\text{tr}(WW^H)\} + \alpha_n^2 \mathbb{E}\{\text{tr}(ZZ^H)\} &= K, \\
\alpha_p a_n \mathbb{E}\{\text{tr}(WZ^H)\} + \alpha_p a_n \mathbb{E}\{\text{tr}(ZW^H)\} &= 0.
\end{align*}
$$

Similarly, from (6), we have the condition for \(a_p^{IT}\) and \(a_n^{IT}\) as

$$
(a_p^{IT})^2 + (a_n^{IT})^2 = K^{-1}.
$$

### E. Estimated Effective Channel at End User

1) **MRT:** Following Appendix B, we obtain

$$
\mathbb{E}\{a_{kk}^{MRT}\} = 0 \quad \text{and} \quad \mathbb{E}\{b_{kk}(a_{kk}^{MRT})^*\} = 0. \quad \text{Plugging}
$$

$$
\mathbb{E}\{a_{kk}^{MRT}\} \quad \text{and} \quad \mathbb{E}\{b_{kk}(a_{kk}^{MRT})^*\} \quad \text{into (15) and (16), we have}
$$

$$
\Lambda_{a_{kk}^{MRT}} = \sqrt{\rho_d \tau_d \beta_k \alpha_p \text{var}(a_{kk}^{MRT})},
$$

and

$$
\Lambda_{b_{kk}^{MRT}} = \rho_d \tau_d \beta_k [\alpha_p^2 \text{var}(a_{kk}^{MRT}) + \alpha_n^2 \text{var}(b_{kk})] + \sigma_0^2.
$$

The estimate of the effective channel at \(U_k\) is

$$
\hat{a}_{kk}^{MRT} = \frac{\rho_d \tau_d \beta_k \alpha_p^2 \text{var}(a_{kk}^{MRT})}{\rho_d \tau_d \beta_k \alpha_p \text{var}(a_{kk}^{MRT}) + \sigma_0^2} + \frac{\rho_d \tau_d \beta_k \alpha_n \text{var}(b_{kk})}{\rho_d \tau_d \beta_k \alpha_n \text{var}(b_{kk}) + \sigma_0^2} + \frac{\rho_d \tau_d \beta_k \alpha_p \text{var}(a_{kk}^{MRT})}{\rho_d \tau_d \beta_k \alpha_p \text{var}(a_{kk}^{MRT}) + \sigma_0^2} + \text{var}(\hat{a}_{kk}^{MRT})}
$$

From (39), the mean and variance of \(\hat{a}_{kk}^{MRT}\) are

$$
\mathbb{E}\{\hat{a}_{kk}^{MRT}\} = \mathbb{E}\{a_{kk}^{MRT}\},
$$

and

$$
\text{var}(\hat{a}_{kk}^{MRT}) = \frac{\gamma_d \tau_d \alpha_p^2 \text{var}(a_{kk}^{MRT})}{\gamma_d \tau_d \alpha_p \text{var}(a_{kk}^{MRT}) + \rho_d \tau_d \beta_k \alpha_n \text{var}(b_{kk}) + \sigma_0^2}.
$$

The downlink channel estimation error is defined as

$$
\hat{a}_{kk}^{MRT} = a_{kk}^{MRT} - a_{kk}^{MRT}.
$$

From (42), the mean and variance of \(a_{kk}^{MRT}\) are

$$
\mathbb{E}\{\hat{a}_{kk}^{MRT}\} = 0,
$$

and

$$
\text{var}(\hat{a}_{kk}^{MRT}) = \frac{\gamma_d \tau_d \alpha_p \text{var}(a_{kk}^{MRT})}{\gamma_d \tau_d \alpha_p \text{var}(a_{kk}^{MRT}) + \gamma_d \tau_d \alpha_n \text{var}(b_{kk}) + \sigma_0^2}.
$$

2) **ZF:** Following Appendix B, we have \(\mathbb{E}\{a_{kk}^{ZF}\} = 0\) and \(\mathbb{E}\{b_{kk}(a_{kk}^{ZF})^*\} = 0\). Similar to the MRT case, we have

$$
\mathbb{E}\{a_{kk}^{ZF}\} = \mathbb{E}\{a_{kk}\}, \quad \mathbb{E}\{b_{kk}\} = 0,
$$

$$
\text{var}(\hat{a}_{kk}^{ZF}) = \frac{\gamma_d \tau_d \alpha_p^2 \text{var}(a_{kk}^{ZF})}{\gamma_d \tau_d \alpha_p \text{var}(a_{kk}^{ZF}) + \gamma_d \tau_d \alpha_n \text{var}(b_{kk}) + \sigma_0^2}.
$$

and

$$
\text{var}(\hat{a}_{kk}^{ZF}) = \frac{\gamma_d \tau_d \alpha_p \text{var}(a_{kk}^{ZF})}{\gamma_d \tau_d \alpha_p \text{var}(a_{kk}^{ZF}) + \gamma_d \tau_d \alpha_n \text{var}(b_{kk}) + \sigma_0^2}.
$$

**Remark 2:** From (44) and (46), it is observed that \(\text{var}(\hat{a}_{kk}^{MRT,ZF}) \approx \gamma_d \tau_d (1-\sigma_0^2) + 1\). From (35), we have condition for AN noise power parameters as \(\alpha_p^2 + \alpha_n^2 = 1/K\). It means that the estimation error of the downlink at the legitimate users is independent to AN power parameters when the number of transmit antennas at the BS is large. Besides,\(\text{var}(\hat{a}_{kk}^{MRT,ZF})\) can be reduced by improving the uplink training process and decreasing the number of users.
Estimated Effective Channel at the Eavesdropper

Following Appendix C, for both MRT and ZF, we have
\[ \mathbb{E} \{ c_k \} = 0, \mathbb{E} \{ d_k \} = 0, \Lambda_{c_k, c_k} = I_N, \Lambda_{d_k, d_k} = I_N, \]
\[ \mathbb{E} \{ c_k d_k^H \} = 0, \mathbb{E} \{ d_k c_k^H \} = 0. \]
As a consequence, we have
\[ \Lambda_{c_k, \hat{g}_{d,k}} = \sqrt{\rho_d \tau_d \beta_k \alpha_k} \Lambda_{c_k, c_k} = \sqrt{\rho_d \tau_d \beta_k \alpha_k} I_N, \]
\[ \Lambda_{\hat{g}_{d,k}, \hat{g}_{d,k}} = \rho_d \tau_d \beta_k (\alpha_k^2 + \alpha_k^2) I_N + \sigma_k^2 I_N. \]

(47) estimate of the effective channel at the eavesdropper is
\[
\hat{c}_k = \mathbb{E} \{ c_k \} + \Lambda_{c_k, \hat{g}_{d,k}} \Lambda^{-1}_{\hat{g}_{d,k}, \hat{g}_{d,k}} \left( \hat{g}_{d,k} - \mathbb{E} \{ \hat{g}_{d,k} \} \right)
\]
\[
= \frac{\sqrt{\rho_d \tau_d \beta_k \alpha_k^2}}{\rho_d \tau_d \beta_k (\alpha_k^2 + \alpha_k^2) + \sigma_k^2} \left( \sqrt{\rho_d \tau_d \beta_k \alpha_k} c_k + \sqrt{\rho_d \tau_d \beta_k \alpha_k d_k} + \hat{n}_{E,k} \right),
\]
where \( \gamma_k = \frac{\alpha_k^2 \beta_k}{\sigma_k^2} \). We have \( \mathbb{E} \{ \hat{c}_k \} = 0 \) and
\[
\Lambda_{\hat{c}_k, \hat{c}_k} = \frac{\gamma_k \tau_d \alpha_k^2}{(\gamma_k \tau_d \alpha_k^2 + \alpha_k^2) + 1} = \sigma_k^2 I_N.
\]
(50) Estimation error at the eavesdropper is
\[ \hat{c}_k = c_k - \hat{c}_k, \]
with \( \mathbb{E} \{ \hat{c}_k \} = 0 \) and
\[
\Lambda_{\hat{c}_k, \hat{c}_k} = \frac{(\gamma_k \tau_d \alpha_k^2 + 1)}{(\gamma_k \tau_d \alpha_k^2 + \alpha_k^2) + 1} = \sigma_k^2 I_N.
\]
(52)

Remark 3: The estimation error of the downlink at the eavesdropper depends on the AN power parameters which means that we can adjust the AN power parameter to maximize the advantage for the legitimate side. The downlink payload data transmission phases of the two proposed AN schemes are described in details in Section III and Section IV.

III. ACHIEVABLE SECRECY RATE IN DOWNLINK TRAINING ARTIFICIAL NOISE AIDING SCHEME

In this section, we develop the tight approximations for the achievable secrecy rate of the considered system when the downlink training phase AN-aiding scheme is applied.

A. Achievable Legitimate Rate

1) MRT: In the payload data transmission phase the BS transmits information to the users without AN. By plugging \( u_k = \frac{h_k^*}{\| h_k \|} \) into (7) and setting \( \alpha_n^T = 0 \), the received signal at \( U_k \) is
\[ y_k = \sqrt{\frac{\rho_d}{K} \beta_k} h_k^T \hat{h}_k x_k + \sum_{l \neq k} \sqrt{\frac{\rho_d}{K} \beta_l} h_l^T \hat{h}_l x_l + n_k \]
\[ = \sqrt{\frac{\rho_d}{K} \beta_k} a_{MRT,k} x_k + \sum_{l \neq k} \sqrt{\frac{\rho_d}{K} \beta_k} a_{MRT,l} x_l + n_k, \]
(53)
where \( x_k \) and \( x_l \) are the desired confidential information for \( U_k \) and \( U_l \), respectively. Obviously, \( a_{MRT,k} \sim \mathcal{CN}(0, 1) \) for \( k \neq l \). \( U_k \) only has knowledge of \( a_{MRT,k} \). The achievable legitimate rate at \( U_k \) is formulated in (54) on the top of the next page [25, Eq. (2.46)].

2) ZF: By plugging \( u_k = \frac{v_k}{\| v_k \|} \) into (7) and setting \( \alpha_n^T = 0 \), the received signal at \( U_k \) is
\[ y_k = \sqrt{\frac{\rho_d}{K} \beta_k} h_k^T \frac{v_k}{\| v_k \|} x_k + \sum_{l \neq k} \sqrt{\frac{\rho_d}{K} \beta_l} h_l^T \frac{v_l}{\| v_l \|} x_l + n_k \]
\[ = \sqrt{\frac{\rho_d}{K} \beta_k} a_{ZF,k} x_k + \sum_{l \neq k} \sqrt{\frac{\rho_d}{K} \beta_l} a_{ZF,l} x_l + n_k. \]
(56)

Obviously, \( a_{ZF,k} \sim \mathcal{CN}(0, (1 - \sigma_k^2)) \) for \( k \neq l \). Similar to the case of using MRT, the achievable legitimate rate at \( U_k \) is expressed in (57) on the next page.

B. Achievable Eavesdropping Rate

From (7), setting \( \alpha_n^T = 0 \), the received signal at the eavesdropper in both cases of using MRT and ZF is
\[ y_E = \sqrt{\rho_d K^{-1} \beta_E} c_k x_k + \sum_{l \neq k} \sqrt{\rho_d K^{-1} \beta_E} c_l x_l + n_E. \]
(58)
The eavesdropper performs MRC
\[ y_{E, MRC} = \sqrt{\rho_d K^{-1} \beta_E} \frac{c_k}{\| c_k \|} x_k \]
\[ + \sum_{l \neq k} \sqrt{\rho_d K^{-1} \beta_E} \frac{c_l}{\| c_l \|} x_l + \frac{\hat{c}_k}{\| \hat{c}_k \|} n_E. \]
(59)
The eavesdropper only has knowledge of \( c_k \). Therefore, the achievable eavesdropping rate is described in (60) on the next page.

C. Achievable Secrecy Rate

From (54), (57), and (59), the following lemma is given.

Lemma 1: When downlink training phase AN-aiding scheme is applied, the achievable secrecy rate of the considered system is
\[ R_{k, DT}^A = [R_k^A - R_E]^+, \]
(61)
where \( A = \{ \text{MRT, ZF} \}, R_k^A \) and \( R_E \) are given in (54), (57), and (60), respectively.

IV. ACHIEVABLE SECRECY RATE IN BOTH PHASES ARTIFICIAL NOISE AIDING SCHEME

In this section, we develop a tight approximation for the achievable secrecy rate of the considered system when both phases AN-aiding scheme is applied. In this scheme, the AN is added in the downlink training phase as in Section III. In the payload data transmission phase, AN is injected into the null-space of the legitimate channels to confuse the eavesdropper. The AN matrix is chosen based on the knowledge of the estimated legitimate channels, i.e., \( H^T J = 0 \), where \( J_1 \in \mathbb{C}^{M \times K}, \) where \( \| J_k \|^2 = 1 \), is the AN matrix.

\[ R_{k, DT}^A = [R_k^A - R_E]^+. \]

\[ 1 \] This is a lower bound of the real secrecy rate \( \mathbb{E} \{ [R_k^A - R_E]^+ \} \).
\[ R_k^{\text{MRT}} = \]
\[ \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho_d \beta_k K^{-1} \mathbb{E} \left\{ a_k^{\text{MRT}} | \hat{a}_k^{\text{MRT}} \right\}^2 \right) \right\} \]
\[ \approx \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho_d \beta_k K^{-1} \mathbb{E} \left\{ |\hat{a}_k^{\text{MRT}}|^2 \right\} \right) \right\} \]
\[ \approx \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\gamma_d K^{-1} M \sigma_{a_k}^2}{\gamma_d K^{-1} \mathbb{E} \left\{ |\hat{a}_k^{\text{MRT}}|^2 \right\} + \gamma_d K^{-1}(K-1)(1 - \sigma_{a_k}^2) + 1} \right) \right\}, \]  

where step (b) is obtained by using the law of large number
\[ \frac{1}{M} \|v\|^2 \overset{M \to \infty}{\to} \mathbb{E} \left\{ \|v\|^2 \right\}, \]

where \( v \sim \mathcal{CN}(0, I_M) \). The details of step (a) is described in Appendix D.

\[ R_k^{\text{ZF}} = \]
\[ \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho_d K^{-1} \mathbb{E} \left\{ a_k^{\text{ZF}} | \hat{a}_k^{\text{ZF}} \right\}^2 \right) \right\} \]
\[ \approx \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho_d K^{-1} \mathbb{E} \left\{ |\hat{a}_k^{\text{ZF}}|^2 \right\} \right) \right\} \]
\[ \approx \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\gamma_d K^{-1} M \sigma_{a_k}^2}{\gamma_d K^{-1} \mathbb{E} \left\{ |\hat{a}_k^{\text{ZF}}|^2 \right\} + \gamma_d K^{-1}(K-1)(1 - \sigma_{a_k}^2) + 1} \right) \right\}, \]  

The details of step (a) is demonstrated in Appendix E.

**A. Achievable Legitimate Rate**

1) MRT: By plugging \( u_k = \frac{k}{\|k\|} \) into (7), the received signal at \( U_k \) is
\[ y_k = \sqrt{\rho_d \beta_k} \alpha_p^T a_{kk}^\text{MRT} x_k + \sum_{l \neq k} \sqrt{\rho_d \beta_k} \alpha_l^T a_{kl}^\text{MRT} x_l + \sigma_k^2 + n_k. \]  

The achievable rate at \( U_k \) is formulated in (63) on the next page.

2) ZF: By plugging \( u_k = \frac{w_k}{\|w_k\|} \) into (7), the received signal at \( U_k \) is
\[ y_k = \sqrt{\rho_d \beta_k} \alpha_p^T a_{kk}^\text{ZF} x_k + \sum_{l \neq k} \sqrt{\rho_d \beta_k} \alpha_l^T a_{kl}^\text{ZF} x_l + \sigma_k^2 + n_k. \]  

The achievable rate at \( U_k \) is in (65) on the next page.

**B. Achievable Eavesdropping Rate**

The eavesdropper uses MRC to process the received signal in (8). After performing MRC, the processed signal at the eavesdropper is in (66) on the next page.

The achievable eavesdropping rate at the eavesdropper is in (67) on the next page.

**C. Achievable Secrecy Rate**

From (63), (65), and (67), the following lemma is given.

*Lemma 2:* When downlink training phase AN-aiding scheme is applied, the achievable rate of the considered system is
\[ R_k^A = [R_k^A - R_E]^+, \]  

where \( [x]^+ = \max(0, x) \), \( A = \{\text{MRT, ZF}\} \), \( R_k^A \) and \( R_E \) are given in (63), (65), and (67), respectively.
\[ R_E = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho_d K^{-1} \beta_E \left\| \mathbf{e}^H_{k_k} \mathbf{c}_k \right\|^2}{\rho_d K^{-1} \beta_E \text{var} \left( \mathbf{e}^H_{k_k} \mathbf{c}_k \right) + \sum_{k \neq l} \rho_d K^{-1} \beta_E \mathbb{E} \left\{ \left| \mathbf{e}^H_{k_k} \mathbf{c}_l \right|^2 \right\} + \sigma^2_E} \right) \right\} \]

\[ = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\gamma_E K^{-1} \left\| \mathbf{c}_k \right\|^2}{\gamma_E K^{-1} \sigma^2_{\mathbf{c}_k} + \gamma_E K^{-1} (K - 1) + 1} \right) \right\} \]

\[ \approx \log_2 \left( 1 + \frac{\gamma_E K^{-1} N \sigma^2_{\mathbf{c}_k}}{\gamma_E K^{-1} \sigma^2_{\mathbf{c}_k} + \gamma_E K^{-1} (K - 1) + 1} \right), \quad (60) \]

where step (a) uses the identity (55).

\[ R_{k}^{MRT} = \log_2 \left( 1 + \frac{\gamma(\alpha_p^{(IT)})^2 M \sigma^2_{u_k}}{\gamma(\alpha_p^{(IT)})^2 \text{var}(\tilde{u}_{k_k}) + \gamma(\alpha_p^{(IT)})^2 (1 - \sigma^2_{u_k}) + \gamma(K - 1) \left( \frac{1}{K} - (\alpha_p^{(IT)})^2 \sigma^2_{u_k} \right) + 1} \right), \quad (63) \]

\[ R_{k}^{ZF} = \log_2 \left( 1 + \frac{\gamma(\alpha_p^{(IT)})^2 M - K}{\gamma(\alpha_p^{(IT)})^2 \text{var}(\tilde{u}_{k_k}) + \gamma(\alpha_p^{(IT)})^2 (1 - \sigma^2_{u_k}) + \gamma(K - 1) \left( 1 - \sigma^2_{u_k} \right) + 1} \right), \quad (65) \]

\[ y_E = \sqrt{\rho_d \beta_E \left\| \mathbf{c}_k \right\|^2} \alpha_p^{(IT)} \mathbf{c}_k \mathbf{x}_k + \sqrt{\rho_d \beta_E \left\| \mathbf{c}_k \right\|^2} \alpha_p^{(IT)} \mathbf{u}_k \mathbf{z}_k + \sum_{l \neq k} \sqrt{\rho_d \beta_E \left\| \mathbf{c}_k \right\|^2} \alpha_p^{(IT)} \mathbf{c}_l \mathbf{z}_l + \alpha_p^{(IT)} \mathbf{u}_l \mathbf{z}_l + \hat{\mathbf{c}}_k \mathbf{n}_E, \quad (66) \]

\[ R_E = \mathbb{E} \left\{ \log_2 \left( 1 + \frac{\rho_d (\alpha_p^{(IT)})^2 \beta_E \left\{ \mathbf{e}^H_{k_k} \mathbf{c}_k \right\} \left\{ \mathbf{e}^H_{k_k} \mathbf{c}_k \right\}}{\rho_d (\alpha_p^{(IT)})^2 \beta_E \text{var} \left( \mathbf{e}^H_{k_k} \mathbf{c}_k \right) + \mathbb{E} \left\{ \left( \mathbf{e}^H_{k_k} \mathbf{c}_k \right) \left( \mathbf{e}^H_{k_k} \mathbf{c}_k \right) \right\} + \sigma^2_E} \right) \right\} \]

\[ = \log_2 \left( 1 + \frac{\gamma(\alpha_p^{(IT)})^2 N \sigma^2_{\mathbf{c}_k}}{\gamma(\alpha_p^{(IT)})^2 \sigma^2_{\mathbf{c}_k} + \gamma(\alpha_p^{(IT)})^2 (K - 1) + 1} \right), \quad (67) \]

V. RESOURCE ALLOCATION

In this section, optimization algorithms for power allocation are proposed to maximize the achievable secrecy rate of the considered system.

A. AN in Training Phase

1) MRT: Plugging (44) and (35) into (54), the achievable legitimate rate of the considered system when using AN in downlink training phase with MRT can be rewritten as

\[ R_{k}^{MRT} = \log_2 \left( 1 + \frac{\gamma_d K^{-1} \sigma^2_{\mathbf{c}_k} M}{\gamma_d K^{-1} \sigma^2_{\mathbf{c}_k} M + \text{var}(\tilde{u}_{k_k}) + \gamma(K - 1)} \right) \]

\[ = \log_2 \left( 1 + \frac{A_1 \alpha_p^2 + A_2}{A_3 + A_4} \right), \quad (69) \]

where

\[ A_1 = \gamma_d^2 \tau_d K^{-1} M \sigma^2_{\mathbf{c}_k} \text{var}(\tilde{u}_{k_k}) - \text{var}(b_{k_k}), \]

\[ A_2 = \left( \gamma_d^2 M \sigma^2_{\mathbf{c}_k} \text{var}(b_{k_k}) K^{-2} + \gamma_d K^{-1} M \sigma^2_{\mathbf{c}_k} \right), \]

\[ A_3 = [\gamma_d K^{-1} (K - 1) + 1] \gamma_d \text{var}(\tilde{u}_{k_k}) - \text{var}(b_{k_k}), \] and

\[ A_4 = \left( \gamma_d \tau_d K^{-1} \text{var}(b_{k_k}) + 1 \right) \gamma_d K^{-1} (K - 1) + \gamma_d \text{var}(\tilde{u}_{k_k}) + 1]. \]
where \( A_1 = \gamma_d^2 K^{-1} \tau_d \frac{M-K}{\sigma^2_{s,k}} \) (\( \text{var}(a^{ZF}_{kk}) \) - \( \text{var}(b_{kk}) \)), \( A_2 = \gamma_d^2 K^{-1} \tau_d \frac{M-K}{\sigma^2_{s,k}} \) (\( \text{var}(b_{kk}) + 1 \)), \( A_3 = [\gamma_d(K-1)K^{-1}(1-\sigma^2_{u,k}) + 1] \gamma_d \tau_d (\text{var}(a^{ZF}_{kk}) - \text{var}(b_{kk})) - \gamma_d^2 \tau_d K^{-1} \text{var}(a^{ZF}_{kk}) \text{var}(b_{kk}) \), and \( A_4 = (\gamma_d \tau_d K^{-1} \text{var}(b_{kk}) + 1)(\gamma_d K^{-1} \text{var}(a^{ZF}_{kk}) + \gamma_d K^{-1}(K-1)(1-\sigma^2_{u,k}) + 1) \).

2) ZF: Plugging (46) and (35) into (57), the achievable legitimate rate of the considered system when using AN in downlink training phase with ZF can be rewritten in (70) on the next page.

3) Achievable Eavesdropping Rate: The achievable eavesdropping rate in (60) is re-arranged as in (71) on the next page.

4) Proposed Optimization Algorithm: The achievable secrecy rate of the considered system when downlink training AN scheme is applied can be re-formulated as

\[
R^A_{s,DT} = |R^A_s - R_E|^+ = \log_2 \left( 1 + \frac{A_1 a_p^2 + A_2}{A_3 a_p^2 + A_4} \right) - \log_2 \left( 1 + \frac{A_5 a_p^2}{A_6 a_p^2 + A_7} \right),
\]

(72)

where \( A = \{ \text{MRT, ZF} \} \). To maximize the secrecy rate, the transmit power ratio for pilot/AN transmission needs to be optimized. The proposed algorithm is

\[
\text{max } \alpha_p R^A_{s,DT} \quad \text{s. t. } 0 \leq \alpha_p^2 \leq \frac{1}{K}.
\]

(73)

Making the variable change \( \alpha_p^2 \rightarrow x \), problem (73) is equivalent to the following problem:

\[
\begin{align*}
\text{max } & f(x) := \ln(a_1 + a_2 x) + \ln(a_3 + a_4 x) \\
& \text{s. t. } 0 \leq x \leq \frac{1}{K},
\end{align*}
\]

(74a)

where \( a_1 = A_2 + A_4, a_2 = A_1 + A_3, a_3 = A_7, a_4 = A_5 + A_6, a_5 = A_4, a_6 = A_3, a_7 = A_7 \) and \( a_8 = A_6 \). As its objective function in (74a) is difference of two concave (d.c.) functions \( \ln(a_1 + a_2 x) + \ln(a_3 + a_4 x) \) and \( \ln(a_5 + a_6 x) + \ln(a_7 + a_8 x) \), (74) is a simple d.c. optimization problem, which can be solved by d.c. iterations [26]. We now develop a simple path-following procedure of extremely low complexity as a feasible point found in each iteration is available in closed-form. We will use the following inequalities for all \( x > 0 \) and \( \bar{x} > 0 \):

\[
\begin{align*}
- \ln(a + bx) \geq & - \ln(a + \bar{x} x) + \frac{b \bar{x}}{a + \bar{x} x} - \frac{b}{a + b \bar{x} x}, \\
\ln(a + bx) \geq & \ln(a + \bar{x} x) + \frac{b \bar{x}}{a + \bar{x} x} - \frac{b \bar{x}^2}{a + b \bar{x} x},
\end{align*}
\]

(75)

where (75) follows from the concavity of \( \ln(1 + bx) \) while (76) follows from the convexity of \( \ln(1 + 1/x) \) [27, Theorem 6].

Suppose \( x^{(k)} \) is a feasible point found from \((k-1)\)-th iteration. Then applying (75) and (76) yields

\[
f(x) \geq f^{(k)}(x) := a^{(k)} - b^{(k)} x - \frac{c^{(k)}}{x}
\]

for

\[
0 < a^{(k)} = f(x^{(k)}) + \sum_{n=1}^{4} \frac{a_{2n} x^{(k)}}{a_{2n-1} + a_{2n} x^{(k)}},
\]

\[
0 < b^{(k)} = \sum_{n=3}^{4} \frac{a_{2n} x^{(k)}}{a_{2n-1} + a_{2n} x^{(k)}},
\]

\[
0 < c^{(k)} = \sum_{n=1}^{4} a_{2n} x^{(k)},
\]

(77)

We solve the following lower bounding maximization problem to generate the next feasible point \( x^{(k+1)} \)

\[
\text{max } x \left[ a^{(k)} - b^{(k)} x - \frac{c^{(k)}}{x} \right] \quad \text{s. t. } 0 < x \leq \frac{1}{K},
\]

(78)

which admits a closed-form solution

\[
x^{(k+1)} = \begin{cases} \sqrt{c^{(k)}/b^{(k)}} & \text{if } \sqrt{c^{(k)}/b^{(k)}} < 1/K, \\ 1/K & \text{otherwise}. \end{cases}
\]

(79)

Note that \( f(x^{(k)}) = f^{(k)}(x^{(k)}) \). Since \( x^{(k+1)} \) and \( x^{(k)} \) are the optimal solution and a feasible point for (78), it is true that

\[
f(x^{(k+1)}) \geq f^{(k)}(x^{(k+1)}) > f^{(k)}(x^{(k)}) = f(x^{(k)}),
\]

i.e. \( x^{(k+1)} \) is a better feasible point than \( x^{(k)} \) for (74). Then, it can be easily shown that (see e.g. [26] and [27]) the sequence \( \{x^{(k)}\} \) of improved feasible points converges to a solution satisfying the Karush-Kuh-Tucker condition for (74). Such iterative procedure is formalized by Algorithm 1.

**Algorithm 1**: An algorithm for solving problem (74)

1: **Initialization**: Choose a feasible point \( x^{(0)} \) for (74). Set \( \kappa := 0 \).
2: **Repeat**
3: \[ x^{(k+1)} \] by (79).
4: Set \( \kappa := \kappa + 1 \).
5: **Until** convergence of the objective in (74).
where $A_5 = \gamma_{E}^{-2} \mathbf{K}_{d} N \alpha_{p}^{2}$, $A_6 = - \gamma_{E} \mathbf{K}_{d} K^{-1}$, and $A_7 = (\gamma_E + 1)(\gamma_{E} \mathbf{K}_{d} K^{-1} + 1)$. 

\begin{align}
R^\text{MRT}_k &= \log_2 \left( 1 + \frac{\gamma_{d} (\alpha_{p}^T)^2 M \sigma_{a,k}^2}{\gamma_{d} (\alpha_{p}^T)^2 \text{var}(\tilde{a}_{kk}) + \gamma_{d} (\alpha_{n}^T)^2 (1 - \sigma_{u,k}^2) + \gamma_{d} (K - 1) \left( \frac{1}{K} - (\alpha_{n}^T)^2 \sigma_{u,k}^2 \right) + 1} \right), \\
&= \log_2 \left( 1 + \frac{A_1 (\alpha_{p}^T)^2 (A_3 + A_4 \alpha_{p}^2 + A_3) + (A_2 \alpha_{p}^2 + A_3) (A_6 \alpha_{p}^T)^2 + A_2)}{A_4 (\alpha_{p}^T)^2 (A_5 \alpha_{p}^2 + A_3) + (A_2 \alpha_{p}^2 + A_3) (A_6 \alpha_{p}^T)^2 + A_2} \right), 
\end{align}

with $A_1 = \gamma_{d} M \sigma_{a,k}^2$, $A_2 = \gamma_{d} \mathbf{K}_{d} \text{var}(\alpha_{p}^T) - \text{var}(\tilde{b}_{kk})$, $A_3 = \gamma_{d} \mathbf{K}_{d} \text{var}(b_{kk}) K^{-1} + 1$, $A_4 = \gamma_{d} \text{var}(\tilde{a}_{kk})$, $A_5 = - \gamma_{d} \mathbf{K}_{d} \text{var}(b_{kk})$, $A_6 = - \gamma_{d} \mathbf{K}_{d} (1 - \sigma_{u,k}^2) + \gamma_{d} (K - 1)$, $A_7 = \gamma_{d} (1 - \sigma_{u,k}^2) + 1$.

### B. AN in Both Downlink Training Phase and Information Transmission Phase

1) **MRT**: Plugging (44) into (63), the achievable legitimate rate of the considered system when using AN in both downlink training phase and payload data transmission phase with MRT is re-formulated as in (80) on the next page.

2) **ZF**: Plugging (46) into (65), the achievable legitimate rate of the considered system when using AN in both downlink training phase and payload data transmission phase with ZF is re-formulated as in (81).

3) **Achievable Eavesdropping Rate**: 

\begin{align}
R_{E} &= \log_2 \left( 1 + \frac{\gamma_{E} \mathbf{K}_{d} N \alpha_{p}^2}{\gamma_{E} (\alpha_{p}^T)^2 \sigma_{a,k}^2 + \gamma_{E} (\alpha_{n}^T)^2 + \gamma_{E} (K - 1) \left( \frac{1}{K} \right) + 1} \right), \\
&= \log_2 \left( 1 + \frac{A_8 \alpha_{p}^2 (\alpha_{p}^T)^2}{A_9 \alpha_{p}^2 (\alpha_{p}^T)^2 + A_1} \right), 
\end{align}

where $A_8 = \gamma_{E} \mathbf{K}_{d} N \alpha_{p}^2$, $A_9 = - \gamma_{E} \mathbf{K}_{d}$, and $A_1 = (\gamma_{E} + 1)(\gamma_{E} \mathbf{K}_{d} K^{-1} + 1)$.

4) **Proposed Optimization Algorithm**: The achievable secrecy rate is re-arranged as (83) on the next page.

We seek the values for pilot transmit power ratio in downlink phase, $\alpha_{p}$, and data transmit power ratio in payload data transmission phase, $(\alpha_{p}^T)^2$, to maximize the achievable rate of the considered system. The proposed optimization problem is

\[
\max_{\alpha_{p}^T, \alpha_{p}} \quad R_{\text{BP}} \quad \text{s.t.} \quad 0 \leq (\alpha_{p}^T)^2 \leq \frac{1}{K}, \quad 0 \leq \alpha_{p}^2 \leq \frac{1}{K}. 
\]

Make the variable changes $\alpha_{p}^2 \rightarrow x$ and $(\alpha_{p}^T)^2 \rightarrow y$ to transform (84) to

\[
\max_{x, y} \quad F(x, y) := \ln (\bar{a}_1 + a_{11} x y + \bar{a}_2 + b_{2 y}) \\
+ \ln (\bar{a}_3 + a_{12} x y) \\
- \ln (\bar{a}_5 + a_{13} x y + \bar{a}_6 x + b_{6 y}) \\
- \ln (\bar{a}_7 + a_{14} x y) 
\]

s.t. \quad $0 \leq x \leq \frac{1}{K}, \quad 0 \leq y \leq \frac{1}{K}$.

where $\bar{a}_1 = A_5 A_7$, $a_{11} = A_4 A_5 + A_2 A_6 + A_1 A_2$, $\bar{a}_2 = A_2 A_7$, $b_2 = A_4 A_3 + A_3 A_6 + A_1 A_3$, $\bar{a}_3 = A_10$, $a_{12} = A_9$, $\bar{a}_5 = A_3 A_7$, $a_{13} = A_4 A_5 + A_2 A_6$, $a_{14} = A_4 A_3 + A_3 A_6$, $\bar{a}_7 = A_10$, $a_{14} = A_9 + A_8$.

Suppose $(x^{(k)}, y^{(k)})$ is a feasible point found from the $(k - 1)$th iteration. In iterating $x^{(k+1)}$ consider (74) for

\[
a_1 = \bar{a}_1 + b_{2 y}, \quad a_2 = a_{11} y^{(k)} + \bar{a}_2, \quad a_3 = \bar{a}_3, \quad a_4 = a_{12} y^{(k)}, \quad a_5 = a_{13} y^{(k)} + \bar{a}_6 y^{(k)}, \quad a_6 = \bar{a}_6 + a_{13} y^{(k)}, \quad a_7 = \bar{a}_7, \quad a_{14} = a_{14} y^{(k)}. 
\]

Recalling definition (77), $x^{(k+1)}$ defined by (79) is a better feasible point than $x^{(k)}$:

\[
F(x^{(k+1)}, y^{(k)}) > F(x^{(k)}, y^{(k)}),
\]

Similarly, in iterating $y^{(k+1)}$ consider (74) for

\[
a_1 = \bar{a}_1 + a_{2 x}^{(k+1)}, \quad a_2 = a_{11} x^{(k+1)} + b_2, \quad a_3 = \bar{a}_3, \quad a_4 = a_{12} x^{(k+1)}, \quad a_5 = \bar{a}_5 + a_{13} x^{(k+1)}, \quad a_6 = b_6 + a_{13} x^{(k+1)}, \quad a_7 = \bar{a}_7, \quad a_{14} = a_{14} x^{(k+1)}.
\]

With definition (77), according to (79),

\[
y^{(k+1)} = \begin{cases} 
\sqrt{c^{(k)} / b^{(k)}} & \text{if } \sqrt{c^{(k)} / b^{(k)}} < 1 / K, \\
1 / K & \text{otherwise}
\end{cases}
\]

is a better feasible point than $y^{(k)}$:

\[
F(x^{(k+1)}, y^{(k+1)}) > F(x^{(k+1)}, y^{(k)}).
\]

Thus, in Algorithm 2 we propose another path-following computational procedure for solving (85).
is applied, i.e., \( \text{strated. In this setup, an equal resource allocation scheme} \)
\[
M = 50
\]

Algorithm 2: An algorithm for solving problem (85)

1: **Initialization:** Take a feasible point \( x^{(0)} \) and \( y^{(0)} \) for (85).
   Set \( \kappa := 0 \).
2: **Repeat**
3:   Iterate \( x^{(\kappa+1)} \) according to (79).
4:   Iterate \( y^{(\kappa+1)} \) according to (89).
5:   Set \( \kappa := \kappa + 1 \).
6: **Until** convergence of the objectives in (85).

\[
R_{s, BP} = [R_k^A - R_E]^+ \\
= \log_2 \left( 1 + \frac{A_1 (\alpha_p^T)^2 (A_2 \alpha_p^2 + A_3)}{A_4 (\alpha_p^T)^2 (A_5 \alpha_p^2 + A_3) + (A_2 \alpha_p^2 + A_3) (A_6 (\alpha_p^T)^2 + A_7)} \right) - \log_2 \left( 1 + \frac{A_8 \alpha_p^2 (\alpha_p^T)^2}{A_9 \alpha_p^2 (\alpha_p^T)^2 + A_2} \right)^+. \tag{83}
\]

\[
R_k^{ZF} = \log_2 \left( 1 + \frac{\gamma_d (\alpha_p^T)^2 M - K}{\sigma^2_x} \right) \\
= \log_2 \left( 1 + \frac{A_1 (\alpha_p^T)^2 (A_2 \alpha_p^2 + A_3)}{A_4 (\alpha_p^T)^2 (A_5 \alpha_p^2 + A_3) + (A_2 \alpha_p^2 + A_3) (A_6 (\alpha_p^T)^2 + A_7)} \right), \tag{81}
\]

where \( A_1 = \gamma_d \frac{M - K}{\sigma^2_x}, A_2 = \gamma_d \sigma_d (\var{a_{kk}^P} - \var{b_{kk}}), A_3 = \gamma_d \sigma_d \var{b_{kk}} K^{-1} + 1, A_4 = \gamma_d \var{a_{kk}^F}, A_5 = -\gamma_d \sigma_d \var{b_{kk}}, A_6 = -\gamma_d (1 - \sigma^2_{u,k}), A_7 = \gamma_d (1 - \sigma^2_{u,k}) K^{-1} + \gamma_d (K - 1) K^{-1} (1 - \sigma^2_{u,k}) + 1. \]

It is because the undesired signals in ZF scheme is much smaller than that in the MRT scheme. In the MRT case, using AN in both phases has better performance than that of using AN in downlink training. The reason is that in the MRT case, the undesired signals from other users is the dominant part of interference. By using AN in payload data transmission phase, a part of transmit power that causes the interference turns into AN for confusing the eavesdropper. In the ZF case, the secrecy performance of using AN in downlink training scheme is slightly better than that of using AN in both phases scheme. It is because the user of using AN in both phases scheme suffers from the leakage of AN. Meanwhile, in using AN in downlink training scheme, the user is free from that interference.

Fig. 3 shows the effect of the number of antennas at the BS, \( M \), the number of antennas at the eavesdropper, \( N \), and the number of users, \( K \), on the secrecy performance of the two proposed AN schemes. In this setup, an equal power allocation is deployed, i.e., \( \alpha_p = \alpha_n = \sqrt{\frac{1}{2K}} \). As increasing the number of antennas at the BS and/or reducing the number of users, the secrecy performance in the two proposed AN schemes enhances. It is because each user will receive less undesired signals of the other users and the transmit power of the desired signals is also increased. Besides, increasing the number of antennas at the eavesdropper will enhance the performance of the illegitimate side. As a result, the achievable secrecy rate of the considered system decreases.

Fig. 4 and Fig. 5 demonstrate the convergence of Algorithm 1 and Algorithm 2, respectively. The parameters are set as \( x^{(0)} = \frac{1}{10K} \) and \( y^{(0)} = \frac{1}{10K} \). After a few iterations the object function of achievable secrecy rate converges to a maximum value.

Fig. 6 provides the performance comparison between the optimal AN transmit power scheme and the equal power allocation scheme. In this setup, \( M = 30 \) and \( N = 50 \) By applying the optimal power allocation, the secrecy performance of the two proposed AN schemes is better than that by the equal power allocation. Particularly, the performance of the two proposed AN schemes is similar when the optimal power
Fig. 3: AN schemes with different value sets of \( \{M,N,K\} \).

(a) Training phase

(b) Both phases

Fig. 4: Convergence of optimization algorithm 1.

Fig. 5: Convergence of optimization algorithm 2.

Fig. 6: Comparison between the optimal AN transmit power scheme and the equal power allocation scheme.

Fig. 7: Comparison among AN schemes.

Fig. 8: Comparison among AN schemes.

Therefore, the optimization scheme shows a bigger gain. Moreover, the legitimate users suffer from a larger amount of interference than in the ZF scheme. Therefore, the optimization scheme shows a bigger gain.

In Fig. 7, a comparison among AN-aiding schemes is presented. In this simulation, we compare the secrecy perfor-
mance of the two proposed AN schemes with the conventional payload data transmission phase AN-aiding scheme in which AN is placed in the null space of the estimated down-link channel. The results show that the two proposed schemes outperform the conventional AN in payload data transmission phase. The reason is that in the conventional payload data transmission phase AN-aiding scheme, both users and eavesdropper has their best estimated CSI in the training phase. In addition, in payload data transmission phase, a significant part of transmit power of the BS will be used as jamming power to degrade the performance of the eavesdropper. Consequently, the achievable rate at the user is reduced. Meanwhile, in training phase AN-aiding scheme, the outcome of power allocation process is the smaller channel estimation error at the users. As a consequence, using more power to transmit information in the payload data transmission phase leads to a better achievable rate at the users while the achievable rate at the eavesdropper is kept sufficiently low. Using AN in both phases offers the considered system a flexible solution to maximize its secrecy rate at the price of a higher complexity in the power allocation process.

VII. CONCLUSION

In this paper, two AN-aiding schemes have been proposed to enhance the secrecy performance of a massive MIMO network in the presence of a multiple-antenna eavesdropper. Analytical expressions and tight approximations for the achievable secrecy rate of the considered system have been developed to investigate the performance of the two proposed AN-aiding schemes in the presence of imperfect channel estimation. The results have shown that using AN in the downlink training phase of the massive MIMO networks can effectively suppress the eavesdropping side. Besides, deploying AN in both downlink training phase and payload data transmission phase facilitates the system a flexible solution to enhance the secrecy performance at the price of higher complexity.

APPENDIX A

The term $E\{a_{kk}^{\text{MRT}}\}$ can be computed as

$$E\{a_{kk}^{\text{MRT}}\} = E\left\{\left\|\tilde{h}_k \hat{h}_k^*\right\|_2\right\}$$

$$= \frac{\sigma_{u,k}}{\sqrt{2}} E\left\{\sqrt{X}\right\} = \frac{\sigma_{u,k}}{\sqrt{2}} \frac{\Gamma(M+\frac{1}{2})}{\Gamma(M)}, \quad (A.1)$$

where $X$ is Chi-squared distributed with $2M$ degrees of freedom.

The expression of $E\{|a_{kk}^{\text{MRT}}|^2\}$ is

$$E\{|a_{kk}^{\text{MRT}}|^2\} = E\left\{\left\|\tilde{h}_k \hat{h}_k^*\right\|^2_2\right\} = (M-1)\sigma_{u,k}^2 + 1. \quad (A.2)$$

The term $E\{a_{kk}^{ZF}\}$ can be computed as follows:

$$E\{a_{kk}^{ZF}\} = E\left\{\left(\tilde{h}_k^T + \tilde{h}_k^T \frac{v_k}{\|v_k\|}\right)\right\}$$

$$= E\left\{\left\|\frac{v_k}{\|v_k\|}\right\|_2\right\} + E\left\{\left\|\frac{\tilde{h}_k^T v_k}{\|v_k\|}\right\|_2\right\} = E\left\{1\right\}$$

$$= \frac{1}{\sqrt{2\sigma_{u,k}}} E\left\{\sqrt{X}\right\}$$

$$= \int_0^{\infty} x^{M-K+\frac{1}{2}} \exp\left(-\frac{x}{2}\right) \frac{dx}{2^{M+\frac{3}{2}}\sigma_{u,k}\Gamma(M+1-K)}$$

$$= \frac{\Gamma(M-K+\frac{1}{2})}{\sigma_{u,k}\Gamma(M-K+1)}, \quad (A.3)$$

where $x$ follows Chi-squared distribution with $2(M+1-K)$ degrees of freedom.

APPENDIX B

The term $E\{a_{kk}^{MRT}b_{kk}^*\}$ is formulated as

$$E\{a_{kk}^{MRT}b_{kk}^*\} = E\left\{\left(\tilde{h}_k^T + \tilde{h}_k^T \frac{\hat{h}_k^*}{\|\hat{h}_k\|}\right) z_k^H (\hat{h}_k^* + \tilde{h}_k)\right\}$$

$$= E\left\{\left(\tilde{h}_k^T + \tilde{h}_k^T \frac{\hat{h}_k^*}{\|\hat{h}_k\|}\right) \right\} + E\left\{\frac{\hat{h}_k^*}{\|\hat{h}_k\|} z_k^H \tilde{h}_k\right\}$$

$$+ E\left\{\frac{z_k^H \tilde{h}_k^*}{\|\hat{h}_k\|} \right\} E\left\{\tilde{h}_k\right\} + E\left\{\frac{z_k^H}{\|\hat{h}_k\|} (1 - \sigma_{u,k}^2)\right\}$$

$$= 0. \quad (B.1)$$

The term $E\{a_{kk}^{ZF}b_{kk}^*\}$ is expressed as

$$E\{a_{kk}^{ZF}b_{kk}^*\} = E\left\{\left(\tilde{h}_k^T + \tilde{h}_k^T \frac{v_k}{\|v_k\|}\right) z_k^H (\hat{h}_k^* + \tilde{h}_k)\right\}$$

$$= E\left\{\left(\tilde{h}_k^T + \tilde{h}_k^T \frac{v_k}{\|v_k\|}\right) \right\} + E\left\{\frac{v_k}{\|v_k\|} \hat{h}_k^* z_k^H \tilde{h}_k\right\}$$

$$+ E\left\{\frac{z_k^H \tilde{h}_k^*}{\|\hat{h}_k\|} \right\} E\left\{\hat{h}_k^*\right\} + E\left\{\frac{z_k^H}{\|\hat{h}_k\|} (1 - \sigma_{u,k}^2)\right\}$$

$$= 0, \quad (B.2)$$

where step (a) is obtained by choosing $z_k$ satisfy $z_k^H v_k = 0$. Similarly, $E\{b_{kk}(a_{kk}^{MRT})^*\} = 0$ and $E\{b_{kk}(a_{kk}^{ZF})^*\} = 0$. 
Consider the following equation:

\[
\mathbb{E}\left\{ g_m^H \hat{\mathbf{h}}_k^* z^H g_n \right\} = \mathbb{E}\left\{ z^H g_m^* g_m^T \hat{\mathbf{h}}_k \right\} \quad (C.1)
\]

If \( m = n \),

\[
\mathbb{E}\left\{ z^H g_m^* g_m^T \hat{\mathbf{h}}_k \right\} = \mathbb{E}\left\{ \hat{\mathbf{h}}_k^* \right\} = 0 \quad (C.2)
\]

If \( m \neq n \),

\[
\mathbb{E}\left\{ z^H g_m^* g_m^T \hat{\mathbf{h}}_k \right\} = 0 \quad (C.2)
\]

where \( z \) follows a Chi distribution with 2\( M \) degrees of freedom. Mean value and variance of \( X \) are \( \mu = \sqrt{2} \frac{\Gamma(\frac{2M+1}{2})}{\Gamma(M)} \) and \( \sigma^2 = 2M - \mu^2 \). Therefore, \( \hat{a}_{kk}^\text{MRT} \) and \( \hat{a}_{kk}^\text{MRT} \) are mutually independent. As a consequence, \( \hat{a}_{kk}^\text{MRT} \) and \( \hat{a}_{kk}^\text{MRT} \) are independent. \( \hat{a}_{kk}^\text{MRT} \) can be approximated as a Gaussian RV, i.e., \( \hat{a}_{kk}^\text{MRT} \sim \mathcal{N}\left(\mathbb{E}\left\{ \hat{a}_{kk}^\text{MRT} \right\}, \text{var}(\hat{a}_{kk}^\text{MRT})\right) \).

**APPENDIX D**

**PROOF OF \( \hat{a}_{kk}^\text{MRT} \) APPROXIMATION**

The approximation of \( \hat{a}_{kk}^\text{MRT} \) can be derived as follows:

\[
\hat{a}_{kk}^\text{MRT} = \frac{1}{\sqrt{\sigma_{a,k}^2}} \mathbf{X}^T \mathbf{H} \approx \mathbb{E}\left\{ \mathbf{X} \right\} = \mathbb{E}\left\{ \hat{\mathbf{h}}_k \right\}
\]

where \( \mathbf{X} \) follows a Chi distribution with 2\( M \) degrees of freedom. Mean value and variance of \( \mathbf{X} \) are \( \mu = \sqrt{2} \frac{\Gamma(\frac{2M+1}{2})}{\Gamma(M)} \) and \( \sigma^2 = 2M - \mu^2 \). Therefore, \( \hat{a}_{kk}^\text{MRT} \) and \( \hat{a}_{kk}^\text{MRT} \) are mutually independent. As a consequence, \( \hat{a}_{kk}^\text{MRT} \) and \( \hat{a}_{kk}^\text{MRT} \) are independent.

**APPENDIX E**

**PROOF OF \( \hat{a}_{kk}^\text{ZF} \) APPROXIMATION**

Formulation of \( \hat{a}_{kk}^\text{ZF} \) is

\[
\hat{a}_{kk}^\text{ZF} = \frac{1}{\sqrt{\sigma_{a,k}}} \mathbf{v}_k^T \hat{\mathbf{h}}_k \approx \frac{1}{\sqrt{\sigma_{a,k}}} \mathbf{v}_k^T \mathbf{H} \mathbf{v}_k = \frac{1}{\sqrt{\sigma_{a,k}}} \mathbf{v}_k^T \mathbf{H} \mathbf{v}_k = \frac{1}{\sqrt{\sigma_{a,k}}} \mathbf{v}_k^T \mathbf{H} \mathbf{v}_k
\]

where \( \mathbf{H} \) is the channel matrix, \( \mathbf{v}_k \) is the eigenvector of \( \mathbf{H} \), and \( \sigma_{a,k} \) is the corresponding eigenvalue. Therefore, \( \hat{a}_{kk}^\text{ZF} \sim \mathcal{CN}\left(\sqrt{\frac{\sigma_{a,k}}{\sigma_{a,k}}}, 1 - \sigma_{a,k}^2\right) \). As a consequence, \( \hat{a}_{kk}^\text{ZF} \) and \( \hat{a}_{kk}^\text{ZF} \) are mutually independent.


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