A-level Mathematics Options – Views of Secondary-level Teachers


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ABSTRACT

The A-level Mathematics qualification is based on a compulsory set of pure maths modules and a selection of applied maths modules with the pure maths representing two thirds of the assessment. The applied maths section includes mechanics, statistics and (sometimes) decision maths. A combination of mechanics and statistics tends to be the most popular choice by far. The current study aims to understand how maths teachers in secondary education make decisions regarding the curriculum options and offers useful insight to those currently designing the new A-level specifications.

Semi-structured interviews were conducted with A-level maths teachers representing 27 grammar schools across Northern Ireland. Teachers were generally in agreement regarding the importance of pure maths and the balance between pure and applied within the A-level maths curriculum. A wide variety of opinions existed concerning the applied options. While many believe that the basic mechanics-statistics (M1-S1) combination is most accessible, it was also noted that the M1-M2 combination fits neatly alongside A-level physics. Lack of resources, timetabling constraints and competition with other subjects in the curriculum hinder uptake of A-level Further Maths.

Teachers are very conscious of the need to obtain high grades to benefit both their pupils and the school’s reputation. The move to a linear assessment system in England while Northern Ireland retains the modular system is likely to cause some schools to review their choice of exam board although there is disagreement as to whether a modular or linear system is more advantageous for pupils. The upcoming change in the specification offers an opportunity to refresh the assessment also and reduce the number of leading questions. However, teachers note that there are serious issues with GCSE maths and these have implications for A-level.

**Keywords:** mathematics, A-level, mechanics, module choice, secondary-level education, teachers.

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INTRODUCTION

In the UK, five examination boards provide A-level curricula, exams and assessment and schools are free to register with whichever exam board they believe is most suitable to their needs. A-levels were reformed in 2000 with a modular structure introduced. A-level mathematics was further reformed in 2004 in an attempt to encourage more students to study this subject (Porkess, 2003; QCA, 2007).
The exam board based in Northern Ireland is the Council for the Curriculum, Examinations and Assessment (CCEA). It provides an A-level Mathematics curriculum (CCEA, 2010) which involves four core modules of pure maths (C1 – C4) plus a selection of two out of four optional modules of applied maths (M1, M2, S1, S2). The module content is summarised below:

**C1 (pure):** indices, quadratic equations, polynomials, graphs of functions, straight line, differentiation.
**C2 (pure):** circle geometry, sequences, series, binomial expansion, solving triangles, trigonometry, logs, integration for area.
**C3 (pure):** partial fractions, parametric equations, exponential function, further differentiation (product, quotient, chain rules), solving equations by iterative methods, numerical integration.
**C4 (pure):** functions, differential equations, volume of revolution, implicit and parametric differentiation, integration by substitution and parts, vectors (algebra, geometry, scalar product).

**M1 (mechanics):** uniform acceleration, force vector, friction, equilibrium, Newton’s laws of motion, impulse, momentum.
**M2 (mechanics):** integration and differentiation of vectors, projectiles, circular motion, potential and kinetic energy, work-energy principle, power.
**S1 (statistics):** presentation of data, summary measures, probability laws and functions, discrete and continuous probability distributions, normal distribution.
**S2 (statistics):** expectation algebra, random sampling, central limit theorem, confidence intervals, hypothesis testing, bivariate distributions, linear regression.

The modules all have the same assessment weighting and therefore the A-level Mathematics qualification comprises two-thirds pure maths and one-third applied maths. The only permitted combinations of optional modules are: M1-M2, M1-S1, S1-S2. Within the mechanics and statistics streams of modules, the level of difficulty increases sequentially. The data provided by CCEA demonstrates a wide variety in popularity of the three combinations although almost all students take at least one module of mechanics (Figure 1a).

Students who follow an English exam board have even more flexibility with regard to applied maths content with extra options in decision maths (D1, D2). Topics in decision maths include linear programming, critical path analysis and flows in networks. Thus, with the Edexcel exam board, for example, there are six possible combinations of optional modules: M1-M2, M1-S1, S1-S2, D1-D2, M1-D1, S1-D1. Figure 1b indicates the relative popularity of particular module combinations for students following the Edexcel exam board in 2013, demonstrating that a mixture of mechanics and statistics was most popular, about one quarter of students took at least one decision module and 37% of students didn’t do any mechanics.

A separate A-level qualification in Further Mathematics involves an additional six modules. Under the CCEA specification, three modules are in pure maths while the other three involve some combination of applied maths modules depending on which of these have already been taken for A-level Mathematics. Topics in further pure maths include matrices, determinants, groups, complex numbers, proof by induction and hyperbolic functions. Further mechanics topics include centre of mass, relative velocity, simple harmonic motion, satellite motion and Newton’s law of restitution. Other exam boards have slightly different requirements in terms of which modules are compulsory and module content also varies slightly between exam boards.
The current A-level maths structure means that two students could have an A-level maths qualification, with the same grade, but have studied different topics. In an extreme example, a student could begin an engineering degree programme having the required A-level maths qualification, but without having studied mechanics during A-level maths at school.

The author has previously investigated what can be expected of first-year engineering undergraduate students in terms of mathematical knowledge (Cole, 2014a) and analysed the mathematics backgrounds of the students and whether there was any difference in first-year performance for students with different A-level maths module combinations (Cole, 2014b). The current study considers the perspective of maths teachers in secondary education, reporting their views of the Mathematics and Further Mathematics curricula and associated issues, and aims to understand how teachers make decisions regarding curriculum options including the choice of applied maths modules and the choice of exam board.

**LITERATURE REVIEW**

Ward-Penny et al. (2013) have investigated how maths teachers in a wide range of schools in England and Wales perceive each of the three streams of applied maths. Mechanics was considered the most difficult due to the modelling, problem solving and high algebraic content, but the most interesting. Statistics was described as the least engaging, for both students and teachers. Mechanics and statistics could be easily related to real-world experiences, more so than decision maths. Many teachers appreciated the values of the various applied modules to different careers and noted that certain modules would assist students’ wider education. For example, studying mechanics was beneficial to A-level Physics students. The authors reported that statistics was the most commonly offered stream of applied maths; they expressed
disappointment that this position has arisen due to strategic considerations with statistics seen as "easier for the weaker students to access" (Ward-Penny et al., 2013).

An earlier review of A-level mathematics (QCA, 2007) had already highlighted that much strategic selection of applied maths modules exists with the strategic decisions almost always made by teachers rather than pupils. There was a clear view that M1-M2 is the most challenging combination and S1 and D1 are the easiest modules. That the apparently most difficult combination (M1-M2) had the highest proportion of A grades and S1-D1 the lowest proportion (Edexcel results, Summer 2006) was attributed to the more able pupils tending to take M1-M2 and less able pupils tending to pursue S1-D1 (QCA, 2007).

Lee et al. (2007) investigated why schools did not offer both modules of an applied maths stream. For both the mechanics and statistics streams, timetable constraints was a dominant concern but the most common reason for not offering M1-M2 was that mechanics was the most difficult of the applied topics. Despite this perception, others praise the importance and value of mechanics in that, unlike statistics and decision maths, it offers opportunities to practise and develop skills in the use of algebra, calculus and trigonometry (Kitchen et al., 1997).

Minards (2013) explored teachers’ perceptions of their students’ experiences of the pure maths modules. The most common reason given by teachers for difficulties in A-level maths was novelty – unfamiliar wording or structure of questions and therefore the need to make a decision, rather than the mathematics itself, caused the difficulty. The author states that this unpreparedness for unfamiliar questions suggests a lack of relational understanding (defined by Skemp (1976) as “knowing both what to do and why”) and comments that many students with good grades in maths are not confident in problem solving.

**METHODOLOGY**

Semi-structured interviews were conducted with A-level maths teachers representing 27 grammar schools across Northern Ireland. The participants were selected using purposeful sampling – that is, according to the purpose of the study (Krueger & Casey, 2009). “Information rich” cases – people whose knowledge or experiences will be particularly helpful in the study – were desired (Krueger & Casey, 2009). Through previous work, the author had identified the schools from which the aerospace and mechanical engineering students had come and had ascertained the applied maths modules taken by the students at A-level. The schools which tend to choose M1-M2 for A-level Mathematics form a much smaller group so it was intended to target as many of them as possible. A much larger group of feeder schools seemed to use the M1-S1 combination so there was greater choice in terms of which to select for the interviews. Schools were selected to ensure that, across the whole sample, there was a variety of school types (boys’, girls’, coeducational) and exam boards represented and a geographical spread. A further motive for the study was to develop links with key teaching staff in feeder schools in the hope of promoting mutual benefits for the schools and our School of Mechanical and Aerospace Engineering regarding recruitment and sharing of knowledge, for example.

Through telephone contact with the schools, the head of the maths department was identified and an email was subsequently sent inviting this person to take part in a telephone interview as part of this research study. The author conducted the interviews which typically lasted 15 – 30 minutes. In a couple of cases, where the teacher was well known to the author, a face-to-face interview occurred. The interviews were conducted during the period January 2013 to January
2015 and therefore, in all but a few cases, the teachers were not aware of the proposed content of the new A-level specifications due to be taught first in 2016/17.

**ANALYSIS OF TEACHER INTERVIEWS**

This section reports the opinions of the maths teachers with the analysis grouped into five topics.

*Choice of Applied Modules*

In some schools, where more than one maths class runs, the pupils are given a choice between the M1-S1 and M1-M2 combinations. Module M1 tends to be taught in year 13 and, thus, performance in this can help decide whether to take M2 or S1 in year 14. Teachers have a strong influence on the choice with pupils doing physics and/or interested in proceeding to an engineering or actuarial degree advised to do M2. Pupils without physics or who are interested in geography or business studies are advised to take S1. Weaker pupils are often directed towards S1, the teachers believing this to be an easier option. It is noted that S1 is an AS-level module while M2 is at A2-level. Thus, both pupil ability and potential career are factors in choosing applied modules in these schools. One large school used to offer the relatively rare S1-S2 combination but this was dropped as the classes were becoming imbalanced. Furthermore, some pupils felt they had made the wrong choice so the decision was made easier by requiring all pupils to start with M1.

Obviously, schools want their pupils to achieve the best possible results, for both the pupils’ future and the school’s reputation. For many pupils, this means doing S1. Those doing M2 tend to be the more able pupils who are keen on mechanics and physics. Teachers indicate that these pupils score as highly as the M1-S1 pupils given their interest in mechanics.

A small number of schools require all pupils to follow M1-M2, believing this to be more useful for future career paths (maths, physics and engineering are very common) and confident that results would be as good as for M1-S1 (or even better). They contend that S1 has limited benefits – it doesn’t go as far as t-tests, for example – and potential medical/geography students can learn these topics later. While recognising that mechanics is challenging and requires more understanding of concepts compared to statistics, teachers believe that M2 links well with pure module C4 and to physics. One teacher highlighted that the majority of pupils cover mechanics as part of GCSE Further Maths – M1 doesn’t present much new material to them and M2 is therefore not much extra stress. A teacher at a girls’ school which does M1-M2 stated that the focus on mechanics was not an issue for girls – she described how the girls’ enjoyment of the subject depends on how enthusiastically the teacher delivers the subject and captures their interest. Of course, teachers have their personal preferences for certain topics and a bias towards mechanics was clear for some teachers in the above schools – this could be affecting their views as to which topics their pupils will find easy and beneficial.

Another small group of schools used to offer M1-M2 but have recently moved to M1-S1 only, even though teachers in these schools were more comfortable with mechanics. They believe S1 to be a more accessible module, noting that pupils without physics struggle with M2, while they want to support a wider career choice or other departments in the school (such as biology).

A large majority of the schools surveyed offer M1-S1 only, with two main reasons suggested. There is a perception that this combination is easier, with an expectation that extremely high marks (80 – 90%) are achievable in S1 in particular. Secondly, a broader maths curriculum is
also important in these schools to cater for a variety of potential careers. Some are concerned that the M1-M2 option appears elitist and could put pupils off studying A-level maths. Those pupils who don’t study physics would especially be concerned at the prospect of taking extra mechanics. In contrast, M1-S1 gives a greater opportunity for pupils to access A-level maths – at this level, few struggle with both M1 and S1. The M1-S1 option also has practical advantages in terms of teaching – for example, M1 can be taught in year 13 when the mechanics is fresh for the pupils just finished GCSE Further Maths and the easier S1 module can be pursued in year 14 alongside the tougher C3 and C4 modules. S1 is independent of the other modules in terms of content but M2 depends on topics in C3 and C4 and therefore there would be greater pressure to complete it in the time available.

One of the striking features of the teacher interviews in this study was the wide range of opinions even on issues where more agreement might have been expected (as illustrated by the above discussion). This is also true with regard to the order in which modules are taught – teaching S1 in year 13 and leaving M1 until year 14 was a reasonable alternative since the mechanics will be easier at this time due to pupils studying some relevant physics in the meantime.

Most schools in Northern Ireland follow the CCEA specification which doesn’t include decision maths. However, even in schools using other specifications, the decision modules are generally not offered within A-level maths. Mechanics and statistics occur in GCSE Further Maths so it is natural to take these options through into A-level maths. Choosing D1 would probably mean dropping M1 and this is deemed unrealistic. Moreover, some teachers haven’t encountered decision maths previously so wouldn’t be comfortable teaching it.

**Syllabus Content and Structure**

There was strong agreement between teachers supporting the dominant role of pure maths within the A-level curriculum and the current balance between pure and applied. They recognise that pure maths (algebra, calculus) has a critical, underpinning role encompassing the basic tools, analysis and problem solving. The current level of difficulty was considered appropriate given the current standard at GCSE level; GCSE Further Maths is very useful preparation for A-level maths but the jump to A-level is large for those without this extra preparation. Pure modules C3 and C4 (including calculus) are identified as relatively difficult for pupils. In particular, C4 is challenging and stretches the pupils, helping to differentiate those of different abilities. Thus, teachers believe that hard work and a methodical approach are definitely required to achieve grade A* overall.

Some teachers desired an increase in applied maths, recalling that the mechanics content has declined over the years, and one suggested incorporating topics in financial maths. The early drafts of the new specifications being prepared in line with the ALCAB recommendations (ALCAB, 2014) for teaching from 2016/17 show an expansion for both mechanics and statistics but little reduction in pure topics. While this proposal ensures that fundamental topics are generally retained, and some teachers will see their favoured mechanics strengthened, some will be concerned with the overall size of the syllabus given that they currently have difficulties covering the syllabus and are restricted in the time available to pursue mathematical investigations/discussion with the pupils.

Teachers note that there are serious issues with GCSE maths and these have implications for A-level. Pupils follow different routes to A-level maths (with/without GCSE Further Maths), mechanics is not included in GCSE Maths and it is possible to achieve GCSE Maths grade A without mastering algebra. Since algebra has increased significance at A-level, teachers have
to manage the expectations of pupils whose GCSE grade gives an optimistic outlook for their prospects at A-level maths.

Teachers report that statistics is the least popular topic for pupils; while the learning of methods and number-crunching can be tedious and uninteresting, S1 is recognised to be relatively elementary and straightforward and pupils realise they can boost their overall result through it. It was suggested that A-level statistics needs to be made more meaningful; analysis of a large data set, as proposed for the new A-level specification, might be a way to achieve this.

Mechanics is challenging since it involves application of learning, understanding concepts and intuition. Teachers agree that the mechanics within maths ties in well with A-level physics – pupils can benefit through seeing two approaches to the same topic – but, for some schools, the lowest results are attained in a mechanics module. Comments from teachers in girls’ schools indicate that mechanics should not pose particular problems for girls although participation in physics seems to be a key factor associated with enjoying mechanics within maths, for both girls and boys.

Only a few teachers interviewed had experience of the decision modules. D1 was described as practical, useful for programming applications and relatively easy.

**Pupil Attitudes**

Teachers had different opinions of pupil attitudes in recent times. Some remarked that pupils worked very hard and really cared about their work, being under greater pressure to achieve certain grades. Another observed a less conscientious approach recently due to the pressure of part-time jobs and trying to maintain a work/study/social balance. Having so many exams and opportunities to resit has reduced the impact of those exams. Some commented that pupils were increasingly dependent on teachers to provide extras (such as revision lists, etc) and reluctant to play around with a problem to solve it by themselves, although some noticed an increasing openness to online learning. A strategic approach to exams has been observed with pupils focusing on C3 and C4 at the expense of S1 in order to target an A* grade and score highly overall.

**Further Maths**

In 2014 in Northern Ireland, entries in A-level Further Maths represented 6.2% of entries in A-level Maths and this proportion has been relatively constant over the last 10 years. In contrast, Further Maths entries in England accounted for 16.3% of Maths entries in 2014 and this fraction has grown steadily from 11.3% in 2004 (JCQ, 2014). The situation in Northern Ireland does not simply reflect a lack of interest by teachers or pupils; while demand from pupils in some schools is low, resourcing and timetabling issues and competition with other subjects in the curriculum are major factors.

Many good mathematicians aim for a career in medicine and therefore select maths and three sciences for A-level. Medicine is strongly promoted and it can be difficult convincing careers teachers of the value of Further Maths. Pupils often want a breadth of subjects – this was encouraged when AS-level was introduced – so they don’t want a second maths A-level unless they are likely to proceed to a maths/engineering degree.

Schools often need a minimum of six to eight pupils to make an A-level class financially viable and this can be difficult to achieve in the smaller schools in Northern Ireland. Some schools
collaborate with a neighbouring school but this can require a big commitment from pupils in terms of travel and getting accustomed to an unfamiliar learning environment. When schools have sufficient pupils to run a class in Further Maths, they typically have about five or six pupils participating. The schools often cover A-level Maths in year 13 and therefore complete the necessary C3 and C4 modules before embarking on Further Maths in year 14.

The interviews suggest that building a tradition of Further Maths in a school will help maintain pupil numbers in future years. One teacher commented that pupils were aware of the high standard of maths in his school and they respond. In another, the subject is heavily promoted by careers teachers emphasising the transferable and problem-solving skills. Another said that senior management values having Further Maths as an option at A-level, offering a challenge for the most intelligent pupils – maths is unique in having this extended option available.

Given that many prospective students have three A-levels and one AS-level, it might have been hoped that many potential engineering students would take Further Maths to AS-level, while not expecting a large number to proceed to the full A-level. Teachers generally agree that this scenario is unrealistic. With Further Maths often being taught during year 14 rather than over two years, it would be rare for a pupil to stop halfway through the year – these pupils have completed A-level Maths in year 13, they are very capable mathematically, so they will want to gain the full A-level Further Maths.

The difficulty of teaching Maths and Further Maths in parallel in year 13, due to the latter requiring knowledge of some more advanced pure topics, is often cited although at least one school in the survey has managed to surmount this problem by delaying module FP2 until year 14. A solution to this problem should be investigated by those currently designing the new maths specifications. Some schools in which A-level Further Maths has not yet occurred offer AS-level Further Maths in the first term of year 14 and they have had two to six pupils taking this each year – however, many pupils will want to focus on their three main A-levels at this stage.

**Exam Board**

Schools generally have a tradition of using a particular exam board and need a good reason to make a change. They are content with the results achieved by their pupils and they become familiar with the structure and style of the syllabus and exam questions; it would be a major upheaval to have to get used to new materials.

The majority of Northern Ireland schools use the local exam board, CCEA. When the syllabus is similar across exam boards, an important issue is the level of customer service and support and, for Northern Ireland schools, communication with CCEA is simple and the schools should be well known to CCEA. A common theme in favour of CCEA was that their exam questions are presented in a style which is more easily understood by pupils as compared to the Edexcel questions which are described as having complicated wording. However, concerns were raised that the CCEA assessment and specification needs refreshed – exam papers have become repetitive and there should be fewer leading questions. These issues could be promoting some complacency and the current drafting of a new specification is a good opportunity to deal with them.

Another factor in CCEA’s favour has been its uniqueness among exam boards in providing GCSE Further Maths. This subject is very useful in bridging the gap to A-level and it makes sense to have a uniform style of questions from GCSE through to A-level. While reducing the content of a syllabus often attracts criticism, a sensible balance in accord with the overall
curriculum is necessary. For example, the GCSE Further Maths introduced in 2011 by AQA has less content than the CCEA course but one teacher seemed pleased at the opportunity for more enjoyable learning and problem solving as opposed to continually moving on to new topics.

One school which recently changed to CCEA indicated that their staff marked CCEA exam papers and therefore had a useful insight into what was expected of pupils.

Staff using Edexcel, probably the second most popular exam board for maths in Northern Ireland, strongly praised its teaching resources including its dedicated textbooks, website and online material, feedback on exam results and contact people. Having decision modules available for Further Maths was another reason for choosing Edexcel; there was more flexibility in the timing of modules within Further Maths and the teacher could focus more on her preferred applied maths topics.

The upcoming change to a linear A-level structure in England, with all assessment at the end of the two years, provokes serious questions for schools in Northern Ireland since the modular structure is being retained here, and a wide variety of views is apparent. Two schools in the survey have already decided to move from an English exam board to CCEA. Others are monitoring the situation but believe the change to a linear structure will make a move to CCEA more likely. Some teachers are keen to avoid the more pressurised situation of a final exam only, there is a desire that pupils should have equal opportunities across the country at their A-levels and there is concern as to whether universities will distinguish between exam boards in their requirements. Interestingly, there is disagreement as to whether a modular or linear system benefits pupils more. Clearly, more material will need revised in preparation for one final exam. However, one teacher is determined to stay with an English exam board, convinced that the linear structure is advantageous – he contends that maths is different from other subjects in that the pupils are continually building and maturing in ability and so will be in a stronger position to tackle the assessment of the year 13 topics at the end of year 14 rather than at the end of year 13.

CONCLUSIONS

This study has examined teachers’ opinions of the A-level mathematics curriculum and how they make decisions regarding curriculum options. The wide variety of views, even on issues where more agreement might have been expected, was remarkable.

The M1-S1 combination was favoured because it is perceived to be easier and it provides a broader curriculum, catering for a variety of potential careers. While statistics is the least popular topic for pupils and can be uninteresting, S1 is said to be relatively straightforward. Mechanics, which involves more understanding of concepts, is challenging but it links well with A-level physics and pure module C4. Some teachers are confident that equally good results can be achieved in M1-M2, especially when the pupils are keen on physics. Mechanics should not pose particular problems for girls although an enthusiastic teacher is a vital factor influencing enjoyment of the subject. Teachers in Northern Ireland have little experience of decision modules and they are generally not offered within A-level Mathematics. The dominant role of pure maths within the curriculum and the current balance between pure and applied were supported. The current level of difficulty was considered appropriate with C4, in particular, enabling differentiation between pupils of different abilities.
Teachers report that they currently have difficulties covering the syllabus so the designers of the new specification should consider what time is available or desirable for pursuing mathematical investigations in the classroom. The upcoming change in the specification offers an opportunity to refresh the assessment also and reduce the number of leading questions. However, teachers note that there are serious issues with GCSE maths and these have implications for A-level. Mechanics is not included in GCSE Maths and it is possible to achieve a high GCSE Maths grade without mastering algebra.

Lack of resources, timetabling constraints and competition with other subjects in the curriculum are major causes for the low uptake of Further Maths. Having an extended, challenging subject like Further Maths available at A-level is valuable for a school and it was suggested that developing a tradition of Further Maths in a school will help maintain pupil numbers in future years. The difficulty of teaching Maths and Further Maths in parallel in year 13 is a major problem which, if alleviated under the A-level specifications currently being designed, should help encourage uptake of Further Maths even if only to AS-level.

Schools need a good reason to change exam board since they become familiar with the structure and style of the syllabus. Factors influencing the choice of exam board include the style of exam questions, quality of teaching resources, availability of feedback on exam results and level of customer service. CCEA has been attractive given its uniqueness among exam boards in providing GCSE Further Maths. The upcoming divergence within the UK between linear and modular A-level structures means serious questions for schools in Northern Ireland but, interestingly, there is disagreement as to whether a modular or linear system is more advantageous for pupils.

REFERENCES