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Non-interferometric Test of Collapse Models in Optomechanical Systems

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The test of modifications to quantum mechanics aimed at identifying the fundamental reasons behind the un-observability of quantum mechanical superpositions at the macro-scale is a crucial goal of modern quantum mechanics. Within the context of collapse models, current proposals based on interferometric techniques for their falsification are far from the experimental state-of-the-art. Here we discuss an alternative approach to the testing of quantum collapse models that, by bypassing the need for the preparation of quantum superposition states might help us addressing non-linear stochastic mechanisms such as the one at the basis of the continuous spontaneous localisation model.

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There is clearly a growing consensus that macroscopic tests of quantum theory are one of the most promising ways to explore the boundaries between classical and quantum framework with the scope of characterising the quantum-to-classical transition. Significant theoretical and experimental efforts have been conducted so far [1–6], and the interest in this area of investigation is increasing at a significant pace.

The (so far) lack of unquestionable observations of quantum superpositions at the macro-scale has motivated and justified the formulation of models that, by postulating an intrinsic difference between microscopic and macroscopic features, aim at pinpointing structural modifications to the Schrödinger equation that account for the explicit violation of the quantum superposition principle at the macroscopic level. The so-called Ghirardi-Rimini-Weber (GRW) [7], Continuous Spontaneous Localization (CSL) [8], and Diósi-Penrose (DP) models [10] are exemplary cases of the class of Collapse Models (CMs) [11, 12] that, generally speaking, are formulated by introducing suitable stochastic non-linear terms to the Schrödinger equation regulating the dynamics of a quantum system. Besides embodying a key test for the quantum superposition principle, and thus a fundamental exploration of the potential limitations (if any) of the quantum framework, the experimental addressing of CMs represents a tantalising experimental challenge.

The vast majority of the proposals for the test of CMs put forward so far is based on interferometric approaches in which massive systems are prepared in large spatial quantum superposition states. In order for such tests to be effective, the superposition has to be sufficiently stable in time to allow for the performance of the necessary measurements. Needless to say, these are extremely demanding requirements from a practical viewpoint. So far, matter-wave interferometry [13] and cavity quantum optomechanics [14] are generally considered as potentially winning technological platforms in this context, and considerable efforts have been made towards the development of suited experimental configurations using levitated spheres [15] or gas-phase metal cluster beams [16]. Unfortunately, the experimental state-of-the-art is still far from allowing for a conclusive test. For instance the leading matter-wave experiment is still two orders of magnitude in mass away to test a CM [17, 18], a challenging path as explained here [19]. A way forward would be the continued technical improvement of such experimental setups, aimed at reaching suited working points. Alternatively, one might adopt a radically different approach and think of non-interferometric strategies to achieve the goal of a successful test.

Here we explore one such possibility. We show that CMs (in general, any nonlinear effect on quantum systems) modify the spectrum of light interacting with a radiation pressure-driven mechanical oscillator in a cavity optomechanics setting in a way that could be revealed in a simple and effective way. More specifically, we demonstrate that the CSL-affected dynamics of the mechanical oscillator results in an additional broadening term on the noise spectrum of the light driving the oscillator. Under suitable conditions, such extra broadening can be pinpointed to gather information on the non-linear effect due, for instance, to a collapse mechanism. By bypassing the necessity of preparing, manipulating, and sustaining the quantum superposition state of a massive object, the proposed scheme would be helpful in bringing the goal of observing CM-induced effects closer to the current experimental capabilities.

The model. As anticipated, in our setting the oscillator is embodied by the moving mirror of a Fabry-Perot cavity that is driven by an external laser field. The mechanical mirror, whose oscillations are forced by its radiation-pressure coupling with the cavity field, is assumed to be in contact with a finite-temperature bath, which would
in turn be responsible for mechanical Brownian motion. In addition, we assume a non-linear mechanism to act on the oscillator, as described by a suitable CM. The setup is illustrated schematically in Fig. 1. The explicitly open-system nature of the dynamics undergone by the device is fully captured by adopting a Langevin formalism to account for the Brownian noise, the leakage of the cavity field, the input white noise to the cavity, and the effect of the CM considered in our analysis. In order to set a benchmark, we concentrate on the mass-dependent Continuous Spontaneous Localization (CSL) model, which is one of the most-studied CMs in literature. The overall dynamics is thus described by the equation

\[ \partial_t \hat{O} = \frac{i}{\hbar} [\hat{H}, \hat{O}] + \frac{i}{\hbar} [\hat{V}_t, \hat{O}] + \hat{N} \]  

(1)

with \( \hat{O} \) a generic operator of system, \( \hat{H} \) the Hamiltonian relating the coherent part of the evolution, \( \hat{N} \) the contribution due to standard environmental noise, and \( \hat{V}_t \) the intrinsic noise accounted for using many-body CSL theory.

By using Eq. (1) as the building block of our analysis, our goal is to show that signatures of the intrinsic collapse noise are visible in the density noise spectrum (DNS) of the mechanical oscillator. In the following, we assume the mirror to have mass \( m \), natural oscillation frequency \( \omega_m \), and energy damping rate \( \gamma_m \). The cavity of length \( L \) sustains a single mode of radiation of frequency \( \omega_c \) described by the bosonic annihilation and creation operators \( \hat{a} \) and \( \hat{a}^\dagger \). The external pump has frequency \( \omega_0 \) and input power \( P \). In a rotating frame at the frequency of the external pump, the model Hamiltonian reads

\[ \hat{H} = \hbar (\omega_c - \omega_0) \hat{a}^\dagger \hat{a} + \frac{1}{2} m \omega_m \hat{q}^2 + \frac{P^2}{2m} - \hbar \chi \hat{a}^\dagger \hat{a} \hat{q} + i \hbar \mathcal{E} (\hat{a}^\dagger - \hat{a}) , \]  

(2)

where \( \hat{q} \) is the position operator of the center-of-mass of the mechanical mirror, \( \chi = \omega_c / L \) is the optomechanical coupling rate, and \( \mathcal{E} = \sqrt{2kP / \hbar \omega_0} \) quantifies the cavity-pump coupling (\( k \) is the cavity single-photon decay rate). The interaction term \( -\hbar \chi \hat{a}^\dagger \hat{a} \hat{q} \), which puts together the mechanical mirror and the cavity field, describes the optomechanical coupling under the assumption of large free spectral range [22]. As illustrated in the Supplementary Information available at [20], the stochastic linear potential \( \hat{V}_t \) can be cast into the form

\[ \hat{V}_t = -\hbar \sqrt{\lambda} \hat{w}_t \hat{q} , \]  

(3)

where \( \hat{w}_t \) describes white noise characterized by the statistical properties \( \mathbf{E}(\hat{w}_t) = 0 \) and \( \mathbf{E}(\hat{w}_t, \hat{w}_s) = \delta(t - s) \). Here \( \mathbf{E}(\cdot) \) indicates expectation value and \( \mathbf{E}(\cdot, \cdot) \) stands for a correlation function. Moreover [20]

\[ \lambda = \frac{2}{3} \sum_{k=1}^{3} \int \frac{e^{\frac{|\mathbf{r} - \mathbf{r}'|^2}{4\sigma^2}}}{(2\sqrt{\pi} \sigma R)^3} \partial_{\mathbf{r}'} \varrho(\mathbf{r}) \partial_{\mathbf{r}''} \varrho(\mathbf{r}'') d\mathbf{r}' d\mathbf{r}'' \]  

(4)

with \( \varrho(\mathbf{r}) \) the mass density of the mechanical mirror, \( R \) is the optomechanical coupling under the assumption of large free spectral range [22]. As illustrated in the Supplementary Information available at [20], the stochastic linear potential \( \hat{V}_t \) can be cast into the form

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(4)

with \( \varrho(\mathbf{r}) \) the mass density of the mechanical mirror, \( r_C = 10^{-7} m \) a characteristic length entering the CSL model, and \( \gamma \) a coefficient that measures the strength of the coupling with collapse noise. Ghirardi, Pearle and Rimini [8] set \( \gamma_{GRW} \approx 10^{-36} m^3 s^{-1} \), while Adler [9] sets \( \gamma_{A} \approx 10^{-28} m^3 s^{-1} \). Much larger or smaller values are ruled out [8, 9]. As a benchmark for the quantification of \( \gamma \), one can consider a homogeneous spherical object of radius \( R \) and mass \( m \). Using Eq. (4), one thus gets

\[ \lambda \approx \frac{3 \gamma m^2}{8\pi^2 m_0^3 R^4} (1 - e^{-R^2/\sigma^2}) \quad [m_0 = 1 \text{ amu}] . \]  

(5)

Let us now get back to Eq. (1). We now have all the ingredients to write explicitly as a set of quantum Langevin equations reading [21]

\[ \partial_t \hat{q} = \hat{p} / m , \]

\[ \partial_t \hat{p} = -mw_m^2 \hat{q} + \hbar \chi \hat{a}^\dagger \hat{a} - \gamma_m \hat{p} + \hat{\xi} + \hbar \sqrt{\lambda} \hat{w}_t , \]

\[ \partial_t \hat{a} = i(\omega_0 - \omega_c) \hat{a} + i \hat{q} \hat{a} - \hat{a} + \sqrt{2k} \hat{a}_{in} , \]

where we have introduced the cavity input noise operator \( \hat{a}_{in} \), the Brownian-motion Langevin operator \( \hat{\xi} \) (describing the incoherent motion of the mechanical mirror arising from the coupling with the background of phononic modes due to its physical support). These sources of noise are characterized by the two-time correlators [21]

\[ \mathbf{E}(\hat{\xi}(t), \hat{\xi}(t')) = \frac{\hbar m \gamma_m}{2\pi} \int \omega e^{-i\omega(t-t')} \text{coth}(\beta \omega) + 1 \) d\omega , \]

\[ \mathbf{E}(\hat{a}_{in}(t), \hat{a}_{in}(t')) = 0 , \]

\[ \mathbf{E}(\hat{a}_{in}(t) \hat{a}_{in}^\dagger(t')) = \delta(t - t') \]  

(7)
with $\beta = \hbar/(2k_B T)$, $k_B$ the Boltzmann constant, and $T$ the temperature of the phononic bath with which the mechanical mirror is at equilibrium. This set of equations is in general very difficult to solve due to the non-linear nature of the optomechanical coupling (see very recent progress towards the treatment of the full non-linear process in Ref. [23]). However, under the assumption of large pumping (i.e., large input power of the driving field), we can expand the field and mirror operators in fluctuations around their respective mean values $\overline{\varphi}$ as $\hat{\varphi} = \overline{\varphi} + \delta \hat{\varphi}$ with $\overline{\varphi} = (q, p, a)$. The steady-state mean values can be easily determined and used to derive a simplified set of equations for the fluctuation operators that can be solved in frequency space [21]. Leaving the details of an otherwise straightforward calculation aside, we can focus on the form of the symmetrized two-frequency correlation function $S(\omega) \delta(\omega + \Omega) = \mathbb{E}(\hat{q}(\omega)\hat{q}(\Omega) + \hat{q}(\Omega)\hat{q}(\omega))/2$, which embodies the DNS of the mirror's position. By assuming Markovianity of the mechanical Brownian motion, which justified in the limit of moderate temperature and small mechanical damping, we get

$$S(\omega) = \frac{2\alpha^2 \hbar^2 \kappa \chi^2 (\Delta^2 + \kappa^2 + \omega^2) + \hbar m \omega [(\Delta^2 + \kappa^2 - \omega^2)^2 + 4\kappa^2 \omega^2] [\gamma_m \coth(\beta \omega) + \Lambda]}{[2\alpha^2 \Delta \hbar \chi^2 + m (\omega^2 - \omega_m^2 - i\gamma_m \omega)] [\Delta^2 + (\kappa + i\omega)^2]^2} \tag{8}\,$$

and

$$I = \frac{\int_{-\infty}^{\infty} S(\omega) d\omega}{\int_{-\infty}^{\infty} S_{\lambda \to 0}(\omega) d\omega}. \tag{10}\,$$

which gives us a quantitative estimate of the relative increase of the area under the DNS for $\Lambda \neq 0$ with respect to the case of no CSL mechanism. In Fig. 2 (b) we show the behavior of such a figure of merit against the value of $\Lambda$, calculated using the parameter $\gamma_\lambda$ (which is much larger than the estimate provided by Ghirardi, Pearle and Rimini, and thus offers more chances of being actually observed), and for some choices of the detuning: at large detunings, $I$ appears to be a linear function of $\Lambda$ and for $\Delta/\kappa \gtrsim 1$ (we notice that we work in the bad-cavity regime, which is much easier to achieve, experimentally), and the CSL mechanism could result in a sizeable increase of the DNS area. Needless to say, the actual value by which such area increases strongly depends on the set of parameters that are used to model the dynamics of the system and by no means we claim for optimality. It is worth mentioning, finally, that as shown in Ref. [21] and

$$\lim_{(\beta, \Delta) \to \infty} S(\omega) \approx h(\gamma_m + \Lambda)/\gamma_m \xrightarrow{\gamma_m \ll \Lambda} h\Lambda/\gamma_m \gamma_m^2 . \tag{9}\,$$
exponentially demonstrated in many optomechanics experiments, the light leaking out of the cavity can be used to reconstruct the spectrum of the intra-cavity mechanical mirror. The use of standard input-output relations \( \delta a_{\text{out}} + \delta a_{\text{in}} = \sqrt{2} \kappa \delta a \), linking the extra-cavity field to the input noise and the intra-cavity signal, shows that the same signatures of the CSL mechanism persist in the extra-cavity signal, which can be effectively used to infer the value of \( \Lambda \) and, from this, the characteristic parameter \( \lambda \) of the CSL model. Details of this analysis are provided in Ref. [20].

**Conclusions.**—Our analysis supports the idea that the effects of non-linear stochastic modifications to quantum mechanics, such as those that characterize the CSL model, are observable by adopting an indirect approach that does not rely on the ad hoc creation of a quantum superposition state. We have illustrated such a possibility using a cavity optomechanics setting where the noise properties of the field leaking from a Fabry-Perot cavity with a vibrating end mirror carry information on potential influences due to collapse-like mechanisms. Our proposal appears to only require low operating temperatures of the mechanical mirror, a condition that is met in most of the cutting-edge experiments in cavity optomechanics reported so far [14]. Moreover, although we have focused our discussion to the end-mirror configuration of an optomechanical setup, it is clearly perfectly suited to be adapted to both membrane-in-the-middle and levitating-nanosphere configurations, therefore embodying a general paradigm of vast appeal.

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[20] See supplementary material for more detail.

