DOCTOR OF PHILOSOPHY

Punching Failure and Compressive Membrane Action in Reinforced Concrete Slabs

Rankin, George Ivor Barry

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PUNCHING FAILURE AND COMPRESSIVE MEMBRANE ACTION
IN REINFORCED CONCRETE SLABS

by

GEORGE IVOR BARRY RANKIN, B.Sc. (1978)

Thesis submitted to
The Queen's University of Belfast
for
The Degree of Doctor of Philosophy

Faculty of Engineering
Department of Civil Engineering

October 1982
To my parents
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SUMMARY

This study of punching failure and compressive membrane action in reinforced concrete slabs provides a basis for the development of a more realistic design approach for continuous slabs subjected to concentrated loadings.

The results of an extensive series of tests on one quarter scale models of the interior slab-column connection in a flat slab structure are reported. From an examination of the experimental evidence, fundamental concepts of slab behaviour are established and a rational method for the prediction of the punching strength of the conventional slab specimen is developed. This delineates between the various modes of punching failure, which are broadly classified as either flexural or shear. The criterion for the ultimate flexural capacity is based on elastic-plastic theory and consists of an interpolative factor which relates the applied load to the slab moment of resistance. The ultimate shear capacity is governed by the load to cause internal diagonal cracking of the slab, prior to yielding of the reinforcement or crushing of the concrete.

In order to include the effect of compressive membrane action in the method of analysis, a theory for arching action in slab strips is derived. This is combined with the rational method for the conventional slab specimen to produce the integrated procedure for the prediction of the enhanced punching strength of laterally restrained slabs. The procedure is presented as a rigorous method which is suitable for the analysis of slabs for which the degree of lateral restraint can be accurately assessed. Furthermore, a simplified approach which is applicable to the interior slab-column connection in a flat slab structure is proposed. The assumptions upon which the procedure is based are validated by the good correlation obtained from a comparison of the predicted failure loads with a wide range of test results from various sources.
NOTATION

A  effective cross-sectional area per unit width of arch leg

C₁ + C₄  real constants in elastic-plastic solution

Cₙ  compressive bending force in concrete at ultimate

C₅  force in compression steel at section with sagging moment

Cₛ  force in compression steel at section with hogging moment

D  flexural rigidity of plate

E  elastic modulus

Eₛ  elastic modulus of steel

Eₐ  elastic modulus of concrete

Eₐ  flexural rigidity parameter for grillage member

Eₐ'  flexural rigidity parameter for grillage member with increased stiffness

Eₐ(ₐ)  flexural rigidity parameter for cracked section

Eₐ'(ₐ)  flexural rigidity parameter for cracked section with concentrated reinforcement

F  lateral thrust on three-hinged arch

Fₐ  compressive arching force in concrete at ultimate

Fᵣ  non-dimensional parameters for the arching thrust

Fₙ(u)  lateral thrust at certain deformation

K  stiffness of restraint per unit width (kN/mm²)

K₁, K₂  non-dimensional parameters in Christiansen's method

L  slab span

Lₑ  half span of 'real' strip of slab with finite lateral restraint

Lᵣ  half span of 'affine' strip of slab with rigid lateral restraint

δLₑ  change in leg length of three-hinged arch

M  moment per unit width

Mₐ  arching moment of resistance of strip with finite lateral restraint

Mₐ(max)  maximum possible arching moment of resistance

Mᵣ  arching moment of resistance of strip with rigid lateral support
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\( f_c' \) cylinder compressive strength of concrete (taken as 80% of the cube compressive strength)

\( f_{cu} \) characteristic cube strength of concrete (CP110, 1972)

\( f_{pe} \) effective prestress in unbonded tendon

\( f_{sp} \) tensile strength of concrete from 'split cylinder' test

\( f_y \) yield stress of bonded reinforcement

\( h \) overall depth of section

\( h_a \) original height of three-hinged arch

\( h^* \) height of three-hinged arch in deflected position

\( k_1 \) ratio of average stress to maximum stress in concrete

\( k_2 \) ratio of depth to resultant of concrete compressive force, to depth of neutral axis

\( k_3 \) ratio of maximum concrete stress to 6" x 12" cylinder strength

\( k_a \) ratio of applied load to internal arching moment

\( k_b \) ratio of applied load to internal bending moment

\( k'_b \) elastic moment factor for slab with concentrated reinforcement

\( k_t \) ratio of applied load to ultimate moment of resistance at failure

\( k_{yl} \) moment factor for overall tangential yielding

\( k'_{yl} \) moment factor for overall tangential yielding of slab with concentrated reinforcement

\( m_u \) ultimate sagging moment in Park's method

\( \bar{m}_u \) ultimate hogging moment in Park's method

\( n \) modular ratio

\( n_u \) compressive membrane force in Park's method

\( p_i \) internal pressure on thick-walled cylinder

\( r \) undefined radius

\( r_a \) radius to supports of conventional slab specimen

\( r_c \) radius of column

\( r_f \) reduction coefficient to allow for column shape

\( r_i \) internal radius of thick-walled cylinder

\( r_o \) external radius of thick-walled cylinder
\( r_s \)  
radius of conventional slab specimen

\( r_y \)  
radius of tangential yield zone

\( s \)  
side length of conventional slab specimen

\( s_c \)  
plastic stress of concrete

\( u \)  
non-dimensional parameter for critical arching deflection

\( v_u \)  
nominal ultimate shear stress

\( w \)  
deflection

\( w_e \)  
midspan elastic deflection at yield

\( x \)  
depth of concrete compression zone

\( x' \)  
depth of concrete compression zone in section with concentrated reinforcement

\( x_1 \)  
relative depth of additional compression in Christiansen's method

\( y \)  
relative plastic deflection in Christiansen's method

\( \bar{y} \)  
distance to centroid of compressive stress distribution

\( z' \)  
ratio of bending moment of resistance to bending moment of resistance with concentrated reinforcement

\( \alpha \)  
proportion of half depth of arching section in contact with lateral support

\( \alpha_1 \)  
empirical factor which relates the maximum shear stress to the tensile strength of concrete

\( \alpha_2 \)  
empirical factor which relates the total shear force to the shear force carried by the compression zone

\( \beta \)  
proportion of total span between adjacent sagging and hogging yield sections, in Park's method

\( \beta_1 \)  
ratio of depth of equivalent rectangular stress distribution to parabolic stress distribution in concrete

\( \gamma_m \)  
partial safety factor for strength (CP110, 1972)

\( \Delta \)  
lateral displacement of restraint

\( \Delta_r \)  
radial displacement in thick-walled cylinder

\( \delta \)  
deflection at plastic hinge, in Park's method

\( \varepsilon \)  
strain
$c_{av}$ average strain
$c_b$ measured strain on bottom surface of slab
$c_c$ plastic strain of idealised elastic-plastic concrete
$c_{cy}$ compressive strain in concrete at yield
$c_m$ maximum compressive strain at outer fibres
$c_r$ resultant membrane strain
$c_t$ measured strain on top surface of slab
$c_{tm}$ strain due to tangential bending moment
$e_u$ ultimate compressive strain of concrete
$e_y$ yield strain of reinforcement
$\theta$ rotation at yield section
$\theta_u$ ultimate curvature
$\theta_y$ curvature at first yield
$\nu$ Poisson's ratio
$\xi_s$ slab depth factor (CP110, 1972)
$\rho$ reinforcement ratio at principal section
$\rho'$ reinforcement ratio at section with concentrated reinforcement
$\bar{\rho}$ reinforcement ratio at section with hogging moment
$\rho_{(bal)}$ reinforcement ratio at balanced moment of resistance
$\rho_e$ equivalent reinforcement ratio
$\rho_{ps}$ ratio of unbonded prestressed reinforcement
$\rho_s$ ratio of ordinary bonded reinforcement
$\sigma_l$ principal tensile stress in concrete at neutral plane
$\sigma_r$ radial stress in thick-walled cylinder
$\sigma_\theta$ circumferential stress in thick-walled cylinder
$\tau_m$ maximum shear stress in concrete at neutral plane
$\phi_c$ diameter of loaded area
$\omega$ reinforcement index ($\rho_{fy}/f'_c$)
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Chapter 1

INTRODUCTION

1.1 BACKGROUND TO THE RESEARCH

1.2 SCOPE OF THE THESIS
1.1 BACKGROUND TO THE RESEARCH

The problem of punching failure is of major concern to the designers of reinforced concrete slabs subjected to concentrated loadings. In particular, the punching strength of the interior slab-column connection is usually the decisive factor in the design of a flat slab structure and consequently, the viability of this most efficient form of construction is often controlled by the shear requirements of the relevant building code.

Since the introduction of ultimate strength methods into design practice, the punching strength of slabs has received considerable attention, however, the complexities involved have precluded the development of a satisfactory theoretical treatment. Thus, design provisions are based on certain empirical relationships which have been derived from the results of tests on simple laboratory specimens. Unfortunately, the extent to which the primary variables influence the behaviour of the conventional slab specimen remains unresolved and there is, at present, no generally accepted and reliable approach to the calculation of the punching strength. This unsatisfactory situation has been further exacerbated by research which has demonstrated that the slab boundary conditions have an important influence on the behaviour and ultimate capacity. In this respect, the development of compressive membrane action is considered to be an important aspect of slab behaviour which has been ignored in the formulation of the present design requirements for continuous slabs.

It is widely recognised that the effect of compressive membrane action in a laterally restrained slab is to significantly increase the load capacity above that of the equivalent conventional representation. Furthermore, it has been suggested that this enhancement in strength is likely to develop in continuous flat slabs, because of the inherent restraint. However, the possibility of utilising the compressive membrane effect in the design
provisions for punching failure has not been adequately researched. For this reason, the present study was undertaken.

1.2 SCOPE OF THE THESIS

This thesis is concerned with improving the present understanding of the principal factors which influence the punching strength of reinforced concrete slabs. The primary objective is to provide a sound basis for the development of a more realistic design approach for continuous slabs subjected to concentrated loadings. Although the study is mainly directed towards the prediction of the punching strength of the interior slab-column connection, this treatment is derived from more general concepts of punching failure and compressive membrane action in reinforced concrete slabs.

The historical background to the analysis and design of flat slab structures is first summarised and the relevant literature on punching failure and compressive membrane action is reviewed. Subsequently, the experimental investigation into the influence of the slab flexural capacity and the effect of the slab boundary conditions on the punching strength of the interior slab-column connection is described. On the basis of the test results and the observations of slab behaviour, fundamental concepts are established from which the rational method for the prediction of the punching strength of the conventional slab specimen is developed. This is successfully combined with a theory for arching action in slab strips to produce the integrated procedure for the prediction of the enhanced punching strength of laterally restrained slabs. As the interior slab-column connection represents a particular application of this method, some further assumptions are introduced which enable the use of a simplified approach for flat slab structures.
Although the research is mainly concerned with punching failure in isotropically reinforced slabs subjected to concentric loading, the effects of banding of the reinforcement, eccentricity of loading and prestress are also considered.

Throughout the thesis, a special effort is made to ensure that the assumptions upon which the individual methods of analysis are based, are consistent with the final integrated approach. Furthermore, the validity of the various procedures is individually checked by extensive comparisons of the predicted and measured failure loads for a wide variety of test specimens. Other methods of prediction, including the present code procedures, are also included for comparative purposes. The major points of importance which relate to each part of the work are concluded in detail at the end of the relevant chapters.

The practical application of the integrated procedure and the implications of utilising the effects of compressive membrane action in the design of continuous reinforced concrete slabs subjected to concentrated loading are briefly discussed. Finally, the general conclusions are drawn from an overall perspective of the study and recommendations are made for future research on this subject.
Chapter 2

HISTORICAL REVIEW

2.1 INTRODUCTION

2.2 HISTORY OF FLAT SLAB CONSTRUCTION
   2.2.1 Early development
   2.2.2 Analysis and design
   2.2.3 Present design requirements

2.3 PUNCHING FAILURE
   2.3.1 Failure under concentrated loading
   2.3.2 Previous research
   2.3.3 Design provisions

2.4 COMPRRESSIVE MEMBRANE ACTION
   2.4.1 The arching effect
   2.4.2 Previous research
   2.4.3 Utilisation in design

2.5 CONCLUSIONS
2.1 INTRODUCTION

Before giving consideration to a programme of research, it is first useful to examine the historical background to the problem and identify the important aspects involved. Therefore, in this chapter a review is made of the relevant literature on punching failure and compressive membrane action in reinforced concrete slabs.

The history of flat slab construction is only summarised, as detailed accounts have been prepared by Sozen and Seiss (1963) and Faulkes (1974). It is, however, made clear that the early methods for the analysis and design of flat slab structures were pervaded by an apparent disregard of the fundamental principle of static moment resistance - a misconception which was eventually established in the various building codes. Despite this anomaly, the excellent performance record of existing structures suggests that the enhancing influence of compressive membrane action is an important aspect of slab behaviour which has not been recognised in the formulation of the present design requirements for continuous slabs.

The major problem in the design of the modern flat slab without column capitals is in achieving adequate strength at the slab-column connections. Consequently, much research effort has been devoted to the study of punching failure in reinforced concrete slabs under concentrated loading. The previous research on this subject is surveyed, however, detailed strength expressions are omitted as a comprehensive state-of-the-art report has been compiled by the Joint ASCE-ACI Task Committee 426 (1974). Further to this, the present design provisions to guard against punching failure are described and the inconsistencies between the British (CP110, 1972) and American (ACI 318-77) code recommendations are illustrated.

In recent years, the phenomenon of compressive membrane action has been the subject of a number of research studies, owing to the possibility of producing more economical design procedures. The relevant work in this
area is therefore reviewed and important aspects concerning the utilisation of the compressive membrane effect in the design of continuous slab structures are discussed.

Finally, on the basis of the conclusions drawn from the review of the literature, the principal objectives of the research are summarised.

2.2 HISTORY OF FLAT SLAB CONSTRUCTION

2.2.1 Early development

The beginning of reinforced concrete monolithic construction dates from the pioneering work of Hennibique in the late Nineteenth Century. Initially, the structural form of buildings was a direct imitation of the ancestral steel framework with timber flooring, however, new ideas concerning the use of reinforced concrete slabs were soon to emerge.

One particularly important form of construction, which originated quite independently on opposite sides of the Atlantic at approximately the same time, was the flat or 'mushroom' slab, illustrated in Fig. 2.1. This invention, which was primarily due to the efforts of Turner (1905) in the USA and Maillart (1908) in Switzerland, quickly found much favour owing to the many advantages it presented over alternative types of building, such as the two-way beam and slab system. The unobstructed soffit of the flat slab meant greater economy in construction, with improved architectural, utility and safety characteristics.

Initially, the monolithic connection between the floor slab and supporting column comprised a distinct column capital which ensured adequate strength, however, this feature was later to disappear in favour of the more expedient flush connection. Although flat slabs without column capitals were known in earlier years (Taylor et al, 1925), the
simplification was mainly a post World War II development which accompanied the construction of high rise apartment buildings.

From many viewpoints, the flat slab presents an ideal floor system, with obvious importance in modern building construction. However, the design of such structures is usually governed by the strength criteria for one single detail - namely the slab-column connection, which is susceptible to a localised 'punching' type failure.

2.2.2 Analysis and design

In the early years, no method of slab analysis was available and consequently there were dramatic differences in the amount of reinforcement required by the various design procedures. As shown in a comparative study by McMillan (1910), the material cost of the reinforcement could vary by as much as 400% depending upon the particular design method utilised.

In Europe, the basis for design had been established from the experimental flat slab constructions of Maillart (1908-1910). From these tests, the carefully measured slab deflections were compared with those of similar beams under known bending moments and in this way, the influence of the loading was adjudged. On the basis of the knowledge gained from such experiments, it was possible to proceed with the construction of major buildings, which were themselves verified and assessed for design improvements, by the application of test loadings.

In America, due to the inconsistencies in the methods of design, proof loading tests on completed structures were often carried out for the satisfaction of the building commissioners. The pioneering work of Lord (1910) on the testing of flat slab structures, provided the first basic information on the magnitude of strains and deformations in actual buildings.
During the period 1910-1920 the results from many such tests were used to justify the most economical methods of slab design.

The first major contribution to the analysis of flat slab structures was made by Nichols (1914), who showed that the total panel moment of resistance should equal $\frac{WL}{8}$ for static equilibrium. This straightforward analysis provoked considerable controversy at the time and was clearly not supported by the vast amount of experimental evidence. For instance, it has been adequately demonstrated that flat slab buildings designed for much lower moments could pass their loading tests with ease. Consequently the early building codes adopted an arbitrary approach which required the provision of reinforcement for only a proportion (approximately $\frac{1}{3}$) of the total static moment.

The distribution of the total moment within the panel was not resolved until the publication of the treatise on the analysis and design of slabs, by Westergaard and Slater (1921). It was also shown that because of the use of long gauge extensometers and the neglect of the tensile stress in the concrete, the measured strains in previously reported slab tests had been hitherto misinterpreted. This reasoning was widely accepted to explain the apparent discrepancy between the experimental and theoretical moments in flat slabs. However, it was acknowledged that there were indications of greater strength in the panels than even the corrected test results suggested.

Although Nichol's analysis had been substantiated by elastic plate theory, the concept of static moment resistance was not entirely endorsed. Consequently, in recognition of the satisfactory performance of existing structures, the total panel moment in the empirical method of slab design was to remain less than the static requirement for a further five decades.

The equivalent frame method for the analysis of multiple panel flat slabs was introduced as a more general approach and because of the
anticipated effects of pattern loadings on the slab and column bending moments. However, in order to obtain comparable results with the empirical method of design, the maximum negative moment was, for many years, stipulated to be that at a certain distance from the centre of the column.

It is only in recent years that this anomaly has been removed and the British (CP110, 1972) and American (ACI 318-77) codes of practice now require the provision of reinforcement for the total static moment.

2.2.3 Present design requirements

With the acceptance of the principle of static moment resistance, the excellent performance record of existing structures was largely ignored in the formulation of the present design requirements for continuous slabs. For example, in the empirical method for the design of flat slab structures, the design moment adopted for CP110 (1972) was 25% greater than that of the previous British Code (CP114, 1957), although unsafe buildings had not become apparent.

There are several reasons why the less demanding requirements of earlier standards have not resulted in unsatisfactory design. One factor of particular importance is the influence of 'compressive membrane action' in continuous slabs. In a recent examination of the inconsistencies in the codified methods of slab design, Beeby (1981) concluded that, because of membrane forces, the critical flexural failure mechanisms in slab systems will be ones that extend across the whole width of a structure. Consequently, as the intensity of the characteristic floor loading decreases with increasing area, it was suggested that the use of reduced design loads would be appropriate.

Although this less restrictive approach may be justified in terms of overall flexural failure in flat slab structures, adequate shear capacity
must also be ensured at the slab-column connections. In this respect, the problem of punching failure and the influence of compressive membrane action are important aspects of slab behaviour which require careful consideration if more economical design procedures are to be introduced.

2.3 PUNCHING FAILURE

2.3.1 Failure under concentrated loading

Punching failure occurs in reinforced concrete slabs subjected to concentrated loading when a cone of concrete is suddenly pushed through the slab immediately under the load. This mode of failure is principally associated with the strength of the slab-column connections in flat slab construction, however, it is also an important consideration in the design of other concrete structures such as bridge decks under concentrated wheel loading and column footings. The characteristic form of the failure zone at an interior slab-column connection is illustrated in Fig. 2.2.

Unfortunately, little warning precedes punching failure, although the final shearing of the concrete is characterised by considerable localised destruction and the audible dissipation of energy. Furthermore, unless adequate precautions can be taken, the strength of the slab after rupture is much less than the ultimate capacity, in which case the applied load cannot be sustained after failure. Thus, the initiation of punching failure in a flat slab structure can have disastrous consequences, as the redistribution of load is likely to result in overstressing of the adjacent slab-column connections and lead to a progressive collapse of the complete building. In recent years, the catastrophic nature of progressive collapse has been spectacularly demonstrated with the destruction of several multistorey flat slab buildings during construction (eg, collapse at Cocoa Beach, Florida, 1981).
The problem of punching failure is of major concern in design and consequently, much research effort has been devoted towards the understanding of the strength and behaviour of slabs subjected to concentrated loading. In particular, the punching strength of the slab-column connection has proved to be the decisive factor in the design of flat slab structures and as such, has received most attention. The majority of tests have therefore been conducted on the simply supported conventional slab specimen, which is representative of the portion of slab circumscribed by the nominal line of contraflexure, as shown in Fig. 2.3.

2.3.2 Previous research

Since the earliest work of Talbot (1913) on the shearing strength of wall and column footings, the major portion of research on punching failure has been concerned with the generation of experimental data from which empirical strength expressions have been derived. As an indication of the importance of this initial study, Talbot's original concept of a limiting shear stress on a critical section around the loaded area, has remained the basic design approach in all the major codes of practice, to the present day.

In the early years, research on punching failure progressed with the experimental investigations of Bach and Graf (1915), Graf (1933, 1938), Richart and Kluge (1939) and Forsell and Holmberg (1946).

One of the most important investigations, which provided a considerable amount of experimental data, was that of Richart (1948) on the shearing strength of column footings. The results from this extensive series of tests were later to be utilised by other researchers for the derivation of empirical strength criteria.
In a re-evaluation of Richart's test results, Hognestad (1953) introduced the slab flexural capacity as one of the parameters in an empirical relationship which was based on the shear stress at the perimeter of the loaded area. Further to this, Elstner and Hognestad (1953) reported a series of tests in which the influence of several major variables such as the concrete strength, level of tension and compression reinforcement and size of loaded area was examined. On the basis of the earlier test results and those from an additional number of slabs, Elstner and Hognestad (1956) slightly modified the previous empirical relationship and included a term for the effect of shear reinforcement.

The local moment capacity at the critical section was first incorporated in an empirical strength criterion, by Whitney (1957). On consideration of the mechanism of failure and from an analysis of previously reported test results, Whitney realised that the punching strength was dependent upon the section moment of resistance. In addition, the span/depth ratio was included in the ultimate shear stress taken at one half of the slab effective depth from the loaded area. This work marked a significant advancement in the understanding of the influence of the localised flexural capacity on the punching strength of slabs.

The most comprehensive study of punching failure yet undertaken was that of Moe (1961), in which the basic mechanism of failure was physically examined and a statistical analysis of previously reported test results was carried out. On the basis of his observations of slab behaviour, Moe delineated the various modes of failure in terms of the shear force at which inclined cracks form and the shear force at the ultimate flexural capacity of the slab. Further to this, an empirical interaction relationship between the ultimate shear stress at the column periphery and the slab
The flexural capacity was developed for the prediction of the failure load. The work of Moe has probably had the most significant influence on current thinking regarding punching failure in reinforced concrete slabs.

The first serious attempt to establish a theoretical method of analysis was that of Kinnunen and Nylander (1960). In the derivation of this theory, a mechanical model of failure was assumed on the basis of observations from a series of tests on circular slab-column specimens. The punching strength was considered to be controlled by the attainment of a characteristic tangential strain on the surface of the slab at the periphery of the loaded area. By applying the conditions of equilibrium to the segment of slab bounded by the inclined shear crack, two radial cracks and the slab perimeter, an iterative procedure was derived for the prediction of the ultimate capacity.

The original method of Kinnunen and Nylander was applicable only to slabs with a radial and circumferential arrangement of reinforcement, however, the procedure was later extended by Kinnunen (1963) to deal with slabs having two-way reinforcement. In this case, the influence of dowel and tensile membrane action was included to allow for the increased load capacity.

A simpler theoretical approach was developed by Reimann (1963) in which the ultimate capacity was related directly to the localised slab bending moment by means of an anisotropic axisymmetric plate analysis. This was utilised to allow for the difference between the radial and tangential stiffness of the slab after cracking. The assumed criterion of failure was the attainment of the ultimate moment of resistance at the perimeter of the loaded area.

The important influence of the localised slab bending moments on the punching strength was made further apparent by Blakey (1966). From
an analysis of previously reported test results, Blakey proposed the use of the load to cause tangential yielding at the column periphery as the criterion of failure for slabs with practical reinforcement ratios. This load was determined directly from the elastic-plastic solution for an axisymmetric plate, as derived by Brotchie (1960).

In the following year, a theoretical method of analysis for the calculation of the punching load of a column and flat slab structure was presented by Long and Bond (1967). This was based on elastic thin plate theory, from which the stresses in the compression zone in the immediate vicinity of the column were derived. From an octahedral shear stress criterion of failure, the corresponding moment at the column periphery, which was directly related to the punching load, was determined. The influence of the slab support conditions and the increase in load due to dowel and tensile membrane action were recognised, and empirical correction factors were introduced to allow for these effects. The validity of the method was confirmed by comparing the calculated punching loads with the test result from various sources. This theoretical approach was also extended for the analysis of combined shear and transfer of moment type loading at interior slab-column connections (Long, 1973).

The applicability of the yield-line theory to punching failure was examined by Gesund and Kaushik (1970). From their analysis of test results reported in the literature, the value of an empirical parameter was determined, below which the failure load predicted by the yield-line method was less than the actual punching strength. On this basis, less conservative design recommendations were proposed.

In an attempt to develop a generalised approach to the whole subject of shear, Regan (1971) determined the position of the critical section from the column perimeter, such that the product of its length and the unit shearing resistance corresponding to the value used for beams empirically
gave the correct prediction of the punching strength. Consequently, the critical perimeter was located at an unusually large distance (1.75d) from the column face, and the nominal ultimate shear stress depended only upon the reinforcement ratio and the concrete strength. Probably for the first time, the use of a critical perimeter with the corners rounded-off was advocated for square columns.

An improved experimental procedure for determining the punching strength of flat slab structures was presented by Long and Masterson (1974). The simulation of the correct boundary conditions was accomplished by the use of a full panel specimen with floating edge moment restraint, illustrated in Fig. 2.3. The significant enhancement in the strength over that of the conventional slab specimen was attributed to the development of compressive membrane action in this type of model. Consequently, in the same year, Masterson and Long (1974) included the enhancing effect of compressive membrane action in an analytical flexural approach, based on the development of localised plasticity at the column periphery.

The relevance of the conventional test piece for the investigation of force transfer from slab to column was also questioned by Clyde and Carmichael (1974). From an examination of the redistribution of moments in slabs, it was deduced that only the specimens with floating edge moment restraint could accept the tightening of the ring of radial moment contraflexure which accompanies tangential yield, and would show an enhancement of the lower bound flexural collapse load.

The previously reported analytical procedures were later simplified by Long (1975) in a two-phase approach to the prediction of the punching strength of the conventional slab specimen. The basis of this formulation was the prediction of the punching load as the lesser value from either a lower bound flexural elastic analysis or a semianalytical shear criterion of failure. This approach was shown to give better correlation with
reported test results than the alternative empirically based procedures and various code methods.

In recent years, an approach based on the theory of plasticity has been pursued by researchers in Europe. By equating the external work done by the punching force to the internal work dissipated on the fracture surface, Braestrup (1979) has presented an upper bound solution for the prediction of the ultimate punching load. The importance of yielding of the reinforcement on the failure of lightly reinforced unrestrained slabs was recognised and consequently, only collapse modes characterised by the punching out of a concrete body from a comparatively rigid slab were considered. In this respect, the plastic analysis offered a description of punching failure, whether by actual punching of a slab, or by pull-out of an embedded disc. To predict the ultimate load, it was necessary to account for the limited ductility of the concrete in compression by means of a reduced effective strength. As this was empirically related to the square root of the cylinder strength, the plastic analysis was utilised to confirm the applicability of the nominal shear stress on a control surface as a design variable.

More recently, on completion of a Construction Industry Research and Information Association research project, Regan (1981) has proposed the use of an inclined fracture surface in conjunction with an empirically derived nominal ultimate stress. Although this change in concept presented possibilities for the treatment of a range of phenomena which could not be dealt with satisfactorily on the basis of a notional critical perimeter, the governing strength parameters remained consistent with the earlier generalised approach.

Evidently, further research is required to achieve an improved understanding of the principal factors which influence punching failure in reinforced concrete slabs. In particular, it is apparent that many
aspects of slab behaviour concern the interaction of flexural and shear effects. Consequently, the matters of major importance still relate to the influence of the primary variables and the slab boundary conditions. Until these issues are resolved, there cannot be a generally accepted and reliable approach to the prediction of the punching strength of reinforced slabs.

2.3.3 Design provisions

In all of the major codes of practice the design provisions to guard against punching failure still adhere to the original concept of a nominal ultimate shear stress on a critical section around the loaded area. The success of this approach can be attributed purely to the generality of application and is not because of any true relationship with the actual mode of failure.

The present American (ACI 318-77) code procedure makes use of a much simplified version of the empirical relationship derived by Moe (1961). The initial development of this method for the 1963 ACI Building Code has been outlined by the ACI-ASCE Committee 326 (1962). For square or circular concentrated loadings, the ultimate shear stress is equal to:

\[ v_u = 0.33 \sqrt{f'_c} \]

The punching strength is given by the product of this nominal shear stress and the area of the critical section at one half of the slab effective depth from the loaded area. This nominal capacity is multiplied by a strength reduction factor of 0.85 to give the design shear strength.

A similar approach was in existence in the United Kingdom until the introduction of the present Code of Practice, CP110 (1972). Now, instead of the nominal shear stress being dependent only upon the concrete strength,
as was the case in CP114 (1957), the influence of the reinforcement ratio is recognised and the critical section is defined at one and a half times the overall slab depth from the column perimeter. These changes were introduced as a result of the work of Regan (1971) and provide a means of utilising the same ultimate shear stresses for beams and slabs. This is given by:

\[ v_u = \frac{0.27 (100f_{cu})^{1/3}}{\gamma_m} \]

Included in the nominal ultimate shear stress is the partial factor of safety \( \gamma_m \) which allows for the difference between the strength of the material in the actual structure and that derived from test specimens. The basic shear stress is also modified by a factor which permits higher stresses in thinner slabs.

The main differences between the British and American design provisions for the punching strength of slabs are illustrated in Fig. 2.4. It can be seen that there are three basic inconsistencies which can be summarised as follows:

i) The critical perimeters for shear are located at significantly different distances from the column periphery.

ii) The relative shear strengths do not increase in the same proportion, with an equivalent increase in the concrete strength.

iii) The influence of the reinforcement ratio is only recognised in the British approach.

These markedly different design recommendations of the two building codes are indicative of the unsatisfactory situation concerning the present understanding of punching failure in reinforced concrete slabs.
2.4 COMPRESSION MEMBRANE ACTION

2.4.1 The arching effect

With the derivation of the elastic theory solutions by Westergaard and Slater (1921) and the subsequent introduction of the simplified yield-line theory for thin plates by Ingerslev (1923), it would appear that within a relatively short period since the beginning of reinforced concrete monolithic construction, the complete analysis of two-way slabs was made possible. However, until more recently, the inclusion of compressive membrane action in methods of analysis has generally been avoided, largely because of the additional complexity which would have been otherwise involved.

The arching phenomenon was evidently recognised by the pioneers of flat slab construction from the very beginning. Indeed the original American patent (Norcross, 1902) was for a flat arch which resembled the mushroom structure, but had preceded it. In reference to the flat slab, Turner (1909) wrote:

"Such a slab will at first act somewhat like a flat dome and slab combined, but as the deflection gradually increases it will gradually commence to act like a suspension system in which the concrete will merely hold the rods together and distribute the load over them."

The arching effect was also mentioned in the works of Westergaard and Slater (1921) and Taylor et al (1925).

In later years, with the more comprehensive development of the yield line theory by Johansen (1941), the arching effect was largely forgotten and research efforts were concentrated towards validating this approach for ultimate strength design. It was not until the historic tests by Ockleston (1955) on a complete building - the Old Dental Hospital in Johannesburg, that the interest in compressive membrane action was revived. These full scale destructive load tests on the interior panels of the beam and slab floors, revealed collapse loads of more than twice those predicted by
Johansen's yield-line theory. Shortly afterwards, Ockleston (1958) correctly attributed this enhancement in strength to the development of compressive membrane forces caused by the restraint against lateral expansion of the panels. The potential load carrying capacity due to this phenomenon was adequately demonstrated by means of a simple perspex model which was severed along the yield lines, but restrained within a stiff surround.

The various stages in the behaviour of a laterally restrained slab are illustrated in Fig. 2.5. It can be seen that after the onset of cracking in the slab, most of the rotational deformation is concentrated along narrow bands which represent the incipient yield-lines. Due to the migration of the neutral axis at these cracked sections, the centres of rotation are close to the compression surface of the slab, and are at different levels for the sagging and hogging moments. Consequently, vertical deflection of the panel is accompanied by a tendency for lateral expansion, which if restrained, is responsible for the development of internal arching forces. In this way, the ultimate capacity of the slab is increased beyond the normal bending strength, until the sudden collapse of the inherent arching mechanism. Subsequently, the load may again increase until fracture of the reinforcement, with the development of tensile membrane action under further deformation. This latter phenomenon is of little interest with regard to the design ultimate strength, but is still of importance in consideration of the post-failure capacity.

Although the development of arching action in a beam and slab floor is relatively easy to visualise, it is more difficult to conceive the nature of the phenomenon in a flat slab structure. In this case, compressive membrane action increases both the capacity of the panel as a whole and the localised strength of the slab-column connections. It is the enhanced punching strength which is of interest in the present investigation.
2.4.2 Previous research

One of the earliest records of enhanced collapse loads in laterally restrained slabs was in the reports of pre-war laboratory tests by Thomas (1939), although at the time, the influence of the slab boundary conditions was not fully appreciated. Later, Powell (1956) conducted a series of tests on encastre model slabs and the collapse loads were again extraordinarily large.

Probably the earliest theoretical treatment of arching action was that of McDowell et al (1956), in which a method was developed for predicting the load-deflection curves of masonry walls constrained between essentially rigid supports. The theory was based on the assumption that the principal resistance to lateral load stemmed from a crushing action at the end supports and centre of the panel. It was therefore necessary to include an idealisation of the stress-strain properties of masonry materials. Although this represented a rather radical departure from the conventional assumptions of loading resistance for this type of construction, the theory was not applied to reinforced concrete slabs.

However, since Ockleston (1958) demonstrated the importance of compressive membrane action in beam and slab construction, almost 25 years ago, the phenomenon has been the subject of a number of research studies. In general, the 'domed effect' in slabs and 'arching action' in beams have been considered as being analogous.

With the advent of CP114 (1957) in Britain, the ultimate load design of slabs was permitted and consequently, the first serious attempt was made to incorporate compressive membrane action into a method of analysis.

In a fundamental treatise on the elastic and plastic methods of slab design, Wood (1961) determined the modified yield criterion for a laterally restrained slab, based on the interaction of membrane force and bending
moment derived from rigid-plastic theory. On examination of various test results, this approach was found to overestimate the enhanced ultimate capacity, apparently because of instability and the influence of elastic deflections. It was therefore suggested that empirical reduction factors, which were dependent upon the level of reinforcement, could be used in practice. Wood's theory marked the real beginning of the search for a rational method of incorporating the compressive membrane effect in the analysis of reinforced concrete slabs.

Shortly after this, a somewhat different approach was adopted by Christiansen (1963), who derived an elastic-plastic theory for arching action in beams and one-way slabs with finite lateral restraint. In this analysis, the magnitude of the additional arching moment was determined from the deformations which corresponded to the development of plastic hinges at the supports and midspan. A graphical method for estimating the ultimate strength of interior slab panels due to the combined effect of bending and membrane stresses was also presented.

The treatment of arching action in the interior panel of a flat slab structure was first attempted by Brotchie (1963), after the derivation of a refined theory which included the effect of in-plane forces in reinforced concrete slabs. In particular, the arching within the panel as a whole, due to the restraint provided by the surrounding panels was considered. From an approximate solution, Brotchie determined that the reduction in moment resisted by bending, was of the same order of magnitude as the reduction in design moments allowed in the empirical methods of slab design. In addition, the development of localised arching action, due to the differential moments around the slab-column connections was recognised. This latter effect was described as a localised system of forces, formed by flexural cracking in the region of the column, producing radial compression in the uncracked section and tangential tension in the
surrounding plate. Unfortunately, the analysis of this particular case was not presented.

At about the same time, Park (1964) developed a yield-line theory for the determination of the ultimate strength of uniformly loaded laterally restrained slabs. In this method, a rigid-plastic strip approximation was utilised to include the effect of compressive membrane stresses on the yield moments of resistance. An empirical critical deflection was introduced to enable the theoretical ultimate load to be obtained from the virtual work equations for rectangular slabs with either all or three edges restrained against lateral movement. The theoretical predictions were compared with the results of appropriate laboratory tests.

Subsequently, Park (1964) extended the theory to include the effects of long-term behaviour and partial lateral restraint. The experimental investigation showed the influence of creep under high levels of sustained loading on thin slabs to be significant, however, under practical levels of working load, the effect was negligible.

In the following year, Park (1965) also investigated the strength and stiffness of the surrounding exterior panels in a slab and beam floor, which were required to resist the membrane action in the interior panel. It was found that tie reinforcement around the interior panel was essential and that the exterior panels should be almost square in order to avoid lateral bowing. These observations give some insight into the nature of the restraining forces which are likely to develop.

Although considerable interest in the enhanced flexural capacity of laterally restrained slabs had been shown for several years, it was not until this time that researchers gave some attention to the influence of the slab boundary conditions upon the punching strength under concentrated loading.
In an experimental study, Taylor and Hayes (1965) examined the effect of edge restraint on the punching strength of conventional slab specimens. The increase in the strength of restrained slabs was found to be more appreciable when corresponding unrestrained slabs were close to flexural failure at collapse. Significantly, this was attributed to the influence of yielding of the reinforcement on the depth of the compression zone at the periphery of the loading plate.

Further full scale tests on uniformly loaded slab and beam floors in an actual building were reported by Leibenberg (1966). In addition, a series of laboratory tests were conducted on laterally restrained slab strips and an empirical-phenomenological method of analysis was subsequently developed.

The enhanced load capacity of laterally restrained strips was also investigated by Roberts (1969), who modified Wood's original approach to include the effect of support flexibility and in-plane elasticity. A comprehensive test programme was completed and the results were analysed by this theory.

In the succeeding years, much research effort was devoted to the study of compressive membrane action, culminating in 1971 with an American Concrete Institute special publication which presented the most recent reports on the subject.

The behaviour of uniformly loaded square slabs with various types of lateral support, from which the horizontal reactions to the membrane forces were measured was examined by Brotchie and Holley (1971). In addition, simplified theoretical expressions were developed for predicting the behaviour of laterally restrained slabs and it was suggested that external restraint and internal steel were basically similar in effect, and only partially additive.
The possibility of allowing for membrane action in the design of a reinforced concrete slab and beam floor was investigated by Hopkins and Park (1971). The earlier method of analysis derived by Park was used to determine the enhanced strength of the various panels in a nine panel (three by three) model. Although the anticipated ultimate uniform loading was attained, the magnitude of crack widths and deflections under the service load gave some cause for concern.

At this time, the economic advantage to be gained from the recognition of compressive membrane action in the design of bridge deck panels became apparent. In this form of construction, the restraint against lateral expansion is provided by the slab supporting beams.

The ultimate strength of continuous two-way bridge slabs, subjected to concentrated wheel loading, was investigated by Tong and Batchelor (1971). On the basis of the results from a series of tests on small scale bridge panel models, an empirical relationship between the enhanced punching strength and the slab flexural capacity, which included a contribution due to compressive membrane action, was developed. It was suggested that substantial savings in reinforcement could be derived by the use of the yield-line theory instead of the current elastic methods of design.

The problem of the serviceability criterion for the ultimate load design of laterally restrained slabs was considered by Hung and Nawy (1971). From an examination of the deflections of slabs with various boundary conditions, a serviceability factor concept, which involved higher load factors than normally assumed, was proposed for use in limit design.

A popular approach to the prediction of the enhanced punching strength of laterally restrained slabs was to incorporate the compressive membrane effect into an existing method which was applicable to the conventional unrestrained specimen. Aoki and Seki (1971) considered the effect of
arching on the ultimate moment of resistance and proposed the substitution of an enhanced flexural capacity into Moe's empirical relationship. A slightly different method was presented by Hyttinen (1971), who assumed that the effect of compressive membrane action was to increase the shear resistance in the same proportion as it increased the depth of the compression zone at the periphery of the loaded area. Both procedures were accordingly compared with their proponents test results.

A more theoretical approach was developed by Hewitt and Batchelor (1975), in which the boundary restraining forces and moments were incorporated into Kinnunen and Nylander's idealised model of failure. This involved an iterative procedure for the prediction of the punching load of slabs with known boundary restraints. For practical situations, it was suggested that a boundary restraint factor could be estimated and this was accordingly evaluated from previously reported test results. On this basis, the restraint factors for a variety of slabs were determined and limiting values tentatively recommended for use in estimating the punching strength. Furthermore, it was confirmed that the reinforcement required for temperature and shrinkage purposes in isotropically reinforced bridge slabs, provided an adequate factor of safety against punching failure.

The enhanced punching strength of two-way bridge slabs was further investigated by Batchelor and Tissington (1976), who introduced some modifications to the earlier method of analysis proposed by Tong and Batchelor in 1971. In addition, the experimental aspects of this study were related to the influence of model scale, boundary conditions and slab reinforcement percentage. It was suggested that bridge slabs provided with minimum isotropic reinforcement could be expected to perform satisfactorily in service.

The results of the various studies on the enhanced punching strength of bridge slabs were verified by an extensive series of field tests on
existing structures by the Ontario Ministry of Transportation and Communications in Canada. Consequently, an empirical method of design was permitted in the Ontario Highway Bridge Design Code (1979), which required minimum isotropic reinforcement (0.3%) in bridge decks, provided certain boundary conditions were satisfied. The background to this successful approach has been reviewed by Csagoly (1979).

More recently, Kirkpatrick et al (1982) have reported an investigation into the strength of M-beam bridge decks, which are commonly used in the United Kingdom. As part of this study, a model bridge deck was constructed and tested under simulated concentrated wheel loading, in the Department of Civil Engineering, Queen's University of Belfast. The punching strength of the deck slab was found to be practically independent of the level of reinforcement and the beam spacing, and was considerably in excess of that predicted by both the British (BS 5400, 1978) and North American (OHBDC, 1979) bridge codes. Consequently a more appropriate method of prediction, which allowed for the enhancement in strength due to compressive membrane action, was developed. This involved the introduction of an equivalent percentage of flexural reinforcement, derived from the arching characteristics of the slab, in the simplified shear criterion proposed by Long (1975). This approach was successfully validated by correlation with a wide range of test results from various sources.

2.4.3 Utilisation in design

In view of the considerable enhancement in the strength of laterally restrained slabs, it would appear advantageous to recognise the influence of compressive membrane action in design. The provision of less reinforcement presents an attractive proposition for greater economy in construction. Evidently, the most profitable way in which allowance can be made for this
phenomenon is by exploiting the inherent capacity of continuous slab structures. Indirectly, this has been the case, for many years, with the use of the empirical method for the design of flat slab buildings.

At present, the only recognised design standard which allows for the enhancement in strength due to compressive membrane action is the Ontario Highway Bridge Design Code (1979). The adoption of an empirical approach was the direct result of careful investigation into the enhanced strength of continuous bridge slabs under concentrated wheel loading.

In flat slab construction, the consequences of punching failure are much more serious and there is less information on the influence of the slab boundary conditions. However with some realistic appraisal of the additional margin of safety provided by compressive membrane action, more economical design procedures could be introduced.

The serviceability criteria for structures designed to incorporate the compressive membrane effect may depend upon the form of construction involved. For exposed slabs, such as bridge decks, the reduced amount of reinforcement can be expected to improve the serviceability performance with regard to spalling of the concrete. In contrast, the long-term effects of creep and shrinkage in monolithic building construction, which is often subjected to high levels of sustained loading, may be cause for concern unless adequate load factors are employed.

On consideration of the preceding remarks, it is evident that the most immediate benefits to be derived from the utilisation of compressive membrane action in design, are in the construction of continuous slabs subjected to localised transient loadings. Typical examples of such structures include aircraft runways, road pavements, protective shelters, marine platforms and of course bridge decks, which are already in the forefront of this technology. Consequently, the development of a realistic
method for the prediction of the enhanced punching strength of laterally restrained slabs is of considerable practical importance.

2.5 CONCLUSIONS

On consideration of the historical background to flat slab design and from the review of the relevant literature on punching failure and compressive membrane action, the following conclusions can be drawn:

1. In the early part of this century, the principle of static moment resistance was largely neglected in the methods for the analysis and design of flat slab construction. Despite this anomaly, which has persisted throughout the years, the most economically designed structures present a record of satisfactory performance.

2. An important aspect of slab behaviour, which has been ignored in the formulation of the present design requirements for continuous slabs, is the enhancement in strength due to compressive membrane action.

3. The problem of punching failure is of particular importance in the design of flat slab structures, however, the matters of major concern still relate to the influence of the primary variables and the slab boundary conditions. The present unsatisfactory situation is reflected in the markedly different design provisions of the various building codes.

4. Considerable economies are to be derived from the utilisation of the compressive membrane effect in design. Consequently, the development of a more realistic method for the prediction of the enhanced punching strength of continuous slabs is of considerable practical importance.
Evidently, there is substantial room for improvement in the present design provisions for reinforced concrete slabs subjected to concentrated loading. The principal objectives of this research can therefore be summarised as follows:

a) To provide, by means of carefully controlled tests, fundamental information from which the development of a more rational method for the prediction of the punching strength of the conventional slab specimen could proceed.

b) To conduct subsequent tests on slabs having a range of more realistic boundary conditions which would enable the effect of compressive membrane action to be identified and quantitively assessed.

c) To produce, by the integration of the rational method for the conventional slab specimen with a theory for arching action in slab strips, a simple and reliable procedure for the prediction of the enhanced punching strength of continuous slabs.
Fig. 2.1 DEVELOPMENT OF FLAT SLAB CONSTRUCTION

(i) Turner's mushroom structure (circa 1905)

(ii) Maillart's mushroom structure (circa 1908)

(iii) modern slab-column structure (post 1945)
Fig. 2.2 PUNCHING FAILURE OF SLAB - COLUMN CONNECTION

(i) section: cone of failure

(ii) plan: crack pattern
Fig. 2.3 TYPES OF SLAB-COLUMN TEST SPECIMEN

i) conventional slab specimen

ii) full panel specimen (Long and Masterson, 1974)
Fig. 2.4 COMPARISON OF DESIGN PROVISIONS FOR PUNCHING FAILURE

\[ b_p = 4c + 3\pi h \]  
CP110 (1972)

\[ b_p = 4(c+d) \]  
ACI (318-77)

i) critical perimeters

ii) cylinder strength - \( f'_c \) (N/mm²)

iii) reinforcement ratio - 100\( \rho \) (%)
i) arching action in cracked slab

ii) characteristic load-deflection relationship

Fig. 2.5 BEHAVIOUR OF LATERALLY RESTRAINED SLAB
Chapter 3

EXPERIMENTAL PROGRAMME

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3.1 INTRODUCTION

On examination of the many punching tests which have been carried out, primarily on isolated slab-column units, it is apparent that relatively few comprehensive investigations into the effect of major variables have been pursued. In addition to this, a review of existing methods of analysis reveals a pervasion of different opinions regarding the influence of flexural behaviour, and slab boundary conditions. Consequently it was considered that the most profitable line of research would involve an extensive series of tests, covering a fairly limited number of variables. The programme derived is described in the following sections.

3.2 TEST SPECIMENS

3.2.1 Types of specimen

A realistic configuration of flat slab was selected, and representative portions simulating conditions at an interior slab-column connection subject to pure shear loading were constructed and tested.

Two types of model were considered appropriate:
a) The conventional type of specimen, simply supported around the nominal line of contraflexure.
b) Larger panel models, extending beyond contraflexure, to provide more realistic boundary conditions.

Both specimens can be considered as statically equivalent idealisations of the real situation, and are illustrated in Fig. 3.1. The simplicity of the conventional model justifies the approximation to the real problem, however localised kinematic conditions at ultimate are thought to be more closely simulated by the latter type.
To clarify the situation regarding flexural behaviour, primary variables of reinforcement percentage and slab effective depth, were chosen as a basis for examination. Span and column size remained constant throughout, and a consistent concrete strength of 40 N/mm² was intended.

For reasons of economy, and laboratory constraints, ¹⁄₈ scale models were considered appropriate. Meaningful results have been obtained by previous researchers on tests to this scale.

3.2.2 Contraflexure models

This series of tests comprised 27 isotropically reinforced square slabs, extending to the nominal line of contraflexure, taken as 0.2 L from the column centre. Tension reinforcement only was included in these models, and was varied over the range 0.4% - 2.0%, for span/depth ratios from 25 - 35. The majority of tests were however, carried out on a mid-range span/depth ratio of 31, this being more typical of practice, and enabling correlation with previously reported test results such as those of Moe (1961) and Elstner and Hognestad (1956). Model dimensions are shown in Fig. 3.2 and details of the reinforcement are presented in Figs. 3.3 - 3.5. The range of reinforcing levels can be appreciated from the meshes shown in Plate 3.1.

3.2.3 Large panel models

Models, similar to the conventional simply supported specimens, but in which the extent of slab beyond the line of contraflexure was increased to a maximum of a full interior panel (Fig. 3.2), constituted this second
series of tests. The major variables were again the level of negative reinforcement (0.5% - 1.1%) and span/depth ratio (25 - 35). Outside the line of contraflexure, positive reinforcement equal to approximately half of the negative reinforcement percentage was provided, as would be typical in a real slab system.

In addition to the 16 short-time tests carried out in this series, one full panel model was subjected to various levels of sustained loading over a period of four months, to determine the importance of long-term effects in relation to compressive membrane action.

Reinforcement layouts for all the panels tested in this series are shown in Figs. 3.6 - 3.12, and typical top and bottom meshes can be seen from Plate 3.2.

3.3 MATERIALS AND FABRICATION OF MODELS

3.3.1 Reinforcing steel

Two types of 6 mm diameter high yield reinforcement suitable for \(\frac{1}{2}\) scale modelling work were available. Cross-ribbed Swedish steel, having a yield strength of 530 N/mm\(^2\), proved of very consistent quality, and was used as negative reinforcement in all of the models. Diagonally ribbed German steel which exhibited a slightly more variable yield strength of 513 N/mm\(^2\), was used as reinforcement in the positive moment regions of the large panel models, where yield was not expected. Both types of reinforcement had previously been annealed to give a well defined yield point, and characteristically exhibited little or no strain hardening until very high strains. Typical stress-strain properties for both Swedish and German steel are shown in Fig. 3.13, and an example of the more important Swedish reinforcement can be seen from Plate 3.3.
3.3.2 Model concrete

A model concrete mix based closely on that developed by Gilbert (1979) in the course of previous model studies, was used throughout. The main characteristics of this concrete were that it possessed a ratio of tensile to compressive strength similar to that of a full scale mix, at the same time maintaining a proper scaling relationship for maximum aggregate size. Three types of locally available aggregate were used, combined in the following proportions:

- 30% - 6 mm crushed basalt
- 48% - coarse grit
- 22% - zone 2 sand

The combined aggregate grading is given in Table 3.1, from which it can be seen that a relatively low proportion of fines (23% passing the 1.18 mm sieve) was necessary to give the hardened concrete the desired strength properties:

<table>
<thead>
<tr>
<th>sieve size</th>
<th>10 mm</th>
<th>5 mm</th>
<th>2.36 mm</th>
<th>1.18 mm</th>
<th>600 μm</th>
<th>300 μm</th>
<th>150 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>% agg. passing</td>
<td>100</td>
<td>90</td>
<td>44</td>
<td>23</td>
<td>12</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE 3.1 COMBINED AGGREGATE GRADING**

A preliminary series of trial mixes indicated the desired cube strength of 40 N/mm² would be obtained in approximately 14 days using the following mix:

- water/cement ratio : 0.58
- aggregate/cement ratio : 4.0

These proportions resulted in a medium workability mix which was used throughout the test programme.
3.3.3 Fabrication

In order to minimise the effects of possible variation of the concrete, fabrication, casting and testing of the contraflexure models were carried out on a three-model batch basis. The large panel models were cast individually, however it proved possible to maintain two sets of forms in operation to speed production. Formwork consisted simply of a flat blockboard base, stiffened underneath, around which the detachable side forms were bolted. For expedience, column shuttering was not included, however small column stubs were attached to the slabs prior to testing.

After cutting and bending of the reinforcement, the oxide coating formed during the annealing process was removed by wire brushing, and finally bars were wiped clean using a cloth dampened with inhibited trichlorethlene. Reinforcement was then assembled and positioned within the formwork by means of small L-shaped steel stools as shown in Fig. 3.14. These were bolted to the wooden base from underneath, and thus, after hardening of the concrete could be detached from the formwork, the stools remaining in the slab. Stools were used to secure the uppermost reinforcement of both top and bottom meshes, and were carefully located so as to avoid any possibility of acting as shear reinforcement in the column region. In all cases a stool was positioned directly over the column itself (Plate 3.3) to ensure accurate control of the effective depth at the critical section.

Small spring clips ('Stabil' binders) proved a most convenient method of achieving a rigid reinforcing mesh. After completion, the top reinforcement was carefully removed from the formwork, in the case of the contraflexure models by hand, and for the larger panels, by means of a specially constructed wooden jig slung from an overhead crane. This permitted oiling of the formwork without risk of smearing
the steel. Subsequently reinforcement was repositioned and firmly secured by wiring to the stools. In each model, two lifting hooks were anchored into the concrete beneath the top reinforcement.

Constituents of the concrete were batched by dry weight prior to mixing in the 0.08 m³ capacity pan mixer sited in the Concrete Laboratory. Good consistency was obtained by mixing dry for about one minute, and for approximately three minutes after addition of the water. Compaction of the contraflexure models was achieved by means of a vibrating table in the laboratory. Concrete was placed in two layers, each being vibrated for a period of 30 seconds. The same procedure was not possible for the large panel models however, and these were cast in the neighbouring Structures Laboratory, and compacted by means of a 25 mm diameter internal poker vibrator. Two batches of concrete were generally required for the larger models, in which case the first batch was placed from the column region outwards.

After compaction by vibration, the surface of the concrete was tamped in each direction before screeding to the level of the sideforms. Finally, a smooth surface finish was achieved by steel floating at approximately three and five hours after casting. The concrete was then covered in wet hessian, and allowed to cure for a period of about one week.

Subsequently, after removal from the formwork, the models were turned upside-down to allow attachment of a small column stub to the slab soffit. This consisted of a 100 mm concrete cube glued to the prepared surface using Ciba-Geigy XD 800 epoxy adhesive. A joint strength equivalent to that of monolithic connection resulted.
3.3.4 Control specimens

To ensure an accurate knowledge of material properties, careful monitoring of control specimens was required.

Tensile testing of reinforcement was frequently carried out, particularly as a consistency check between different bundles of steel. In all, approximately 90 samples were tested.

For measurement of concrete compressive and tensile strength, cubes and cylinders were used respectively. Ideally the specimens size should be scaled in the same proportion as the model to prototype, however, very small samples tend to exhibit higher strengths due to the effect of machine stiffness being relatively larger. Suitable specimen sizes for which this trend was not apparent were 100 mm cubes and 100 mm x 200 mm cylinders. For each set of contraflexure models, and for concrete around the column region of the larger panels, at least six cubes and two cylinders were made. Two cubes were considered adequate from the second batch of concrete for the large panel models. Specimens were cast in two layers, each compacted for a period of 30 seconds on the vibrating table. After one day, the cubes and cylinders were removed from the steel moulds and cured in an ambient temperature water bath until testing.

3.4 APPARATUS AND INSTRUMENTATION

3.4.1 Test rig

The basic requirements of a suitable test arrangement were as follows:

1. Ease of application of concentric loading, with reactions supplied at the nominal line of contraflexure.

2. An ability to accommodate slabs of variable extent beyond the supports, to a maximum of a full interior panel model.
3. The placement of models with 'tension face' upwards, allowing observation of crack progression and development of failure.

4. An adequate strength and stiffness for the loads likely to be encountered.

A rig based on the above criteria was designed, constructed and employed in conjunction with a new 200 kN capacity hydraulic loading system. Details of the test rig are shown in Fig. 3.15 and an overall view with a contraflexure model in position ready for testing can be seen from Plate 3.5.

3.4.2. Measurement of load

To enable a precise determination of applied load, two methods of measurement were employed. An accurately calibrated Budenberg 10000 psi hydraulic pressure gauge, connected to the hand pump outlet, provided a direct indication of load, however a more exact record was obtained by means of an electronic load cell positioned between the jack and column stub. Although great care was taken to ensure the loading apparatus was centrally positioned, a 100 mm diameter spherical seating was also included to minimise any possible eccentricity. A view of the loading arrangement beneath a contraflexure model is shown in Plate 3.6.

In the case of the full panel sustained load test, the desired load levels were temporarily maintained by means of Clockhouse Type D82 constant pressure apparatus. After an initial period of several days however, the manual hand pump was found to be adequate for this purpose.
3.4.3 Measurement of deflection

For all models, profiles of deflection along the column line between parallel slab edges were recorded. Linear variable displacement transducers (Novatech Type R101, 25 mm travel), were positioned vertically against the slab soffit, and clamped firmly to a rigid steel supporting frame. This arrangement allowed an unobstructed view of the top surface and avoided any risk of damage to the transducers at failure.

In a selected number of the larger panel models, displacements along a diagonal line, and horizontal slab expansion were also measured.

The general arrangement of transducer locations is shown in Fig. 3.16.

3.4.4 Measurement of strain

Strains of both reinforcement and concrete were measured, particularly at the critical location of the column periphery. In addition surface strains outside the line of contraflexure of the large panel models were recorded, in an attempt to identify in-plane or membrane forces.

Because of the existence of high strain gradients at the slab-column junction, small gauge lengths were essential, and thus electrical resistance strain gauges were utilised at this location. On the side of a negative reinforcing bar in each slab, a 3 mm epoxy backed foil gauge (TML/FLA-3-11) was attached, in a position approximately 10 mm inside the column periphery. Gauges were bonded to the carefully prepared steel surface using a cyanoacrylate adhesive (M Bond 200), and made waterproof with an elastomeric coating (Berger PRC). A layer of silicone rubber was added for further protection from damage during fabrication. An in-place reinforcement gauge can be seen from Plate 3.3.
Similar type gauges of length 10 mm (TML/FLA-10-11) were surface mounted on the concrete in this zone. In general these were used to measure radial strain on the slab compression surface, however in some cases tangential strain, and axial strain on the face of the column were also recorded. Preparation of the concrete surface also involved smoothing and cleaning, however, in this case, bonding was by means of a much slower setting epoxy adhesive (TML/P-2). This had the advantage of better filling properties, more suited to a porous surface such as concrete. Typical radially mounted gauges can be seen from Plate 4.6.

In the case of the large panel models, surface strains on the concrete outside the supports were measured by means of acoustic vibrating wire strain gauges (Gage Technique - Type TSR/2.5) of length 63 mm. Pairs of gauges were carefully located on both top and bottom surfaces so as to enable the separation of bending and membrane strains. The gauges were adhered to the concrete (using Isopon/P-38) along either a slab diagonal, or column line, as can be seen from Plate 3.4.

The principal positions of strain instrumentation for both reinforcement and concrete are shown in Fig. 3.17.

3.4.5 Data acquisition

An electronic data acquisition system (Intercole-Modulog) in conjunction with either a printer or punch tape recorder, proved invaluable due to the large number of measurements involved. In addition, a scan speed of one channel per second meant that creep effects between readings were negligible. This equipment was used for monitoring of transducer, load cell and electrical resistance strain gauge instrumentation. The vibration period of the acoustic
vibrating wire strain gauges was monitored manually using a miniature strain recorder (Gage Technique/Type GT 1174).

3.5 MODEL TESTING

3.5.1 Preparation for testing

After removal from the formwork and attachment of the various surface gauges, the models were painted with a thin white emulsion to facilitate crack detection during testing. Subsequent installation into the test rig involved several steps. Firstly the slab was positioned on the floor of the laboratory beneath the test frame by means of a series of rollers. In the case of the large panel models this meant that one of the threaded rod legs of the rig had to be temporarily removed. Using an overhead crane attached to the embedded lifting hooks, the slab was then raised until it contacted the supports, and carefully positioned. Underneath the column stub, the loading arrangement was subsequently assembled and centralised. This could then be used to carry the weight of the model, dispensing with the overhead crane. Following this, it was necessary to lower the slab about 50 mm from the supports in order to insert strips of masking tape coated with about 3 mm of soft Isopon. Finally, by jacking the model back against the supports, which were oiled to prevent the Isopon adhering, proper bedding at each side was ensured.

With the panel in position, the deflection measuring apparatus could then be assembled, and instrumentation connected to the data acquisition equipment, ready for testing.
3.5.2 Test procedure

The procedure adopted for testing both the contraflexure and large panel model series was basically the same; however, the full panel sustained loading test is described separately.

(i) short-time tests

Initial readings from all the instrumentation were recorded at the minimum jacking pressure required to hold the model against the supporting frame. Load was then applied in 4.45 kN increments, further measurements being recorded after a settling period of at least one minute at each stage. As failure was approached, or seemed imminent the magnitude of each load step was reduced to 1.11 kN in order to detect the punching load with reasonable precision. Throughout loading, examination of both top and bottom surfaces were frequently carried out, and photographs periodically taken.

(ii) sustained loading test

For this test, the procedure can be outlined as follows:

1. 70% of the expected short-time ultimate load (STUL) was applied in the normal way as described previously. This level of load was maintained for a period of 12 weeks.

2. Applied loading was then removed, and the model allowed to recover for a period of 25 hours.

3. Reloading to 80% of the expected STUL was then carried out, and maintained for a period of one week.
4. Subsequently a weekly addition of 5% loading was applied bringing the load level, after 14 weeks to 90% of the expected STUL. This was maintained for a further period of four weeks.

5. Loading was then increased to 95% of the expected STUL, whereupon failure suddenly occurred.

This particular test covered a total period of 18 weeks, during which measurements were recorded at regular weekly intervals.
Fig. 3.1  STATICALLY EQUIVALENT MODELS
Fig. 3-2 TEST SPECIMENS
Fig. 3:3 LAYOUT OF REINFORCEMENT: CONTRAFLEXURE MODELS
Fig. 3.4 LAYOUT OF REINFORCEMENT: CONTRAFLEXURE MODELS
Fig. 3.5  LAYOUT OF REINFORCEMENT: CONTRAFLEXURE MODELS
Fig. 3.6  LAYOUT OF REINFORCEMENT : MODELS R-08

R1-08

R2-08

R3-08
Fig. 3.7 LAYOUT OF REINFORCEMENT: MODELS R-08
Fig. 3.8 LAYOUT OF REINFORCEMENT: MODELS R-11
Fig. 3.9  LAYOUT OF REINFORCEMENT: MODELS R-05
Fig. 3.10  LAYOUT OF REINFORCEMENT : MODELS RA-08
Fig. 3.11 LAYOUT OF REINFORCEMENT: MODELS RB-08
Fig. 3-12 LAYOUT OF REINFORCEMENT : MODELS RC-08
Fig. 3.13 STRESS-STRAIN PROPERTIES OF REINFORCEMENT

Fig. 3.14 STOOLS FOR POSITIONING REINFORCEMENT
Fig. 3.15 TEST RIG
Fig. 3.16 GENERAL LOCATIONS OF DISPLACEMENT TRANSDUCERS

Fig. 3.17 GENERAL LOCATIONS OF STRAIN GAUGES

Key:
- Vibrating wire strain gauge
- ERS gauge on concrete
- ERS gauge on reinforcement
Chapter 4

PRESENTATION AND DISCUSSION OF TEST RESULTS

4.1 INTRODUCTION

4.2 CONTROL SPECIMENS
   4.2.1 Compressive strength of concrete
   4.2.2 Tensile strength of concrete
   4.2.3 Properties of reinforcing steel

4.3 CONTRAFLEXURE MODEL TESTS
   4.3.1 Model details
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   4.3.3 Slab deflections
   4.3.4 Reinforcement strains
   4.3.5 Concrete strains
   4.3.6 Ultimate strengths

4.4 LARGE PANEL MODEL TESTS
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   4.4.3 Slab deflections
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   4.4.7 Ultimate strengths

4.5 SUSTAINED LOAD TEST
   4.5.1 General behaviour
   4.5.2 Ultimate strength

4.6 CONCLUSIONS
4.1 INTRODUCTION

The correct assessment of experimental results is of paramount importance if any progress is to be made towards a more rational analytical approach. In this chapter, physical observations and experimental measurements are presented and discussed in generalised terms, to promote an understanding of the nature of punching and the characteristics associated with the various modes of failure. Fundamental differences between the types of specimen tested are highlighted and the importance of long-term effects is made apparent.

4.2 CONTROL SPECIMENS

4.2.1 Compressive strength of concrete

Average cube strengths at the time of model testing are presented in Tables 4.1 and 4.2, from which it can be seen that the variation of compressive strength was of little consequence.

4.2.2 Tensile strength of concrete

A ratio of tensile to compressive strength similar to that of a prototype concrete was found to exist, the average split cylinder strength of 3.1 N/mm² corresponding closely to that given by the relationship:

\[ f_{sp} = 0.53 \sqrt{f_c} \]

This equation has been proposed by Mirza et al (1979), on the basis of a comprehensive statistical investigation of concrete properties.
4.2.3 Properties of reinforcing steel

Variation of reinforcement properties proved to be negligible for the Swedish steel used at all critical locations, but of more significance for the German type. Strain measurements showed however, that the latter variety never reached yield and therefore a smaller degree of control was acceptable. Average stress-strain properties were as presented in section 3.3.1.

4.3 CONTRAFLEXURE MODEL TESTS

4.3.1 Model details

This series of tests can be sub-divided into four sets of slabs, according to span/depth ratio. Principal details of the models are presented in Table 4.1.

4.3.2 Observed behaviour

Initial circumferential cracking, almost directly above the column periphery, occurred at an early stage of load application. Increased loading subsequently caused an approximately symmetric outward spread of radial cracking, in general originating from the corners of the peripheral crack. This fan-like pattern tended to be more predominant along the slab diagonals, a low level of reinforcement being characterised by a small number of fairly wide cracks and a high steel percentage by many fine cracks. The sequence of cracking shown in Plates 4.1 and 4.2 illustrates this behaviour. It can also be seen that at similar proportions of ultimate load and for equal levels of reinforcement, the distribution of cracking appears to be the same regardless of slab depth, indicating a similar
flexural response. Significantly, lightly reinforced models were also
typified by very noticeable widening of the circumferential and radial
diagonal cracks prior to failure, as can be appreciated from an examination
of Plate 4.3. In contrast, crack opening in more heavily reinforced slabs
was limited to the circumferential crack only, if at all.

At failure, the zone of destruction caused by reinforcement tearing
out and breaking away concrete cover became more confined as the level of
reinforcement increased, probably due to the smaller deformations experienced
before punching. This trend is shown for a series of models in Plate 4.4.

In addition to the various modes of cracking, being related to the
level of reinforcement, important differences were apparent in the compression
zones adjacent to the column. Coincident with punching, crushing of the
concrete at the column periphery and to some extent along the diagonals of
a lightly reinforced model which exhibited considerable ductility prior to
failure, can be seen from Plate 4.5. In comparison, a more concise shearing
of the concrete, associated with higher levels of reinforcement and a more
brittle mode of failure is shown in Plate 4.6. Thus although in all cases,
failure finally occurred by punching of the column stub through the slab,
the percentage of reinforcement had a considerable influence on the eventual
mode of failure.

A column stub circumscribed by the primary flexural crack is shown in
Plate 4.7 and the familiar inclined rupture surface revealed after failure
can be seen from Plate 4.8.

4.3.3 Slab deflections

Load-central deflection curves are shown in Fig. 4.1 for a range of
reinforcement percentages and extreme span/depth ratios. Almost linear
behaviour was apparent before cracking, after which the degradation of
stiffness became pronounced, particularly for the more lightly reinforced slabs. In fact, difficulty was experienced in measuring the penultimate deformation of these models, due to the high degree of plasticity exhibited.

Deflection profiles along the column lines of models at each end of the reinforcement and span/depth ranges are shown in Fig. 4.2. It can be seen that radial bending of the section between the column periphery and edge support was negligible, and that practically rigid body rotation about the column face occurred. Cracking associated with deformations of these magnitudes, particularly for the more lightly reinforced models, must be accompanied by a considerable loss of bond to the reinforcement passing over the column.

A linear variation of displacements at the supports was recorded at all loadings and the small corrections involved have been accounted for in the above presentations.

4.3.4 Reinforcement strains

Strain measurements from the reinforcement at the column periphery for a range of models are presented in Fig. 4.3. Preceding first cracking, a small degree of linear straining occurred and subsequent to this, strain increased at a much greater rate, depending upon the level of slab reinforcement. In the case of lightly reinforced models, steel reached yield well before failure, however for the highest level of reinforcement \((\rho = 2\%)\), yield was only attained in the case of the thinnest slab, having a span/depth ratio of 35. Strain levels in the thicker slabs were close to but did not quite reach the yield value.
4.3.5 Concrete strains

Strain measurements from the surface of the compression zone around the column periphery are shown in Fig. 4.4 for a selection of models. At the centre of the column side, radial strain was found to be significantly greater than at the column corners and this is consistent with the distributions reported by previous researchers such as Moe (1961). Frequently surface strains began to decrease at between 70% and 80% of ultimate load. This phenomenon has been noted by researchers of beam shear (Scordelis et al, 1974) and has been attributed to the development of internal diagonal cracking. Swamy (1979) has also reported decreasing compressive strains in punching specimens before failure, and Moe's visual observations of early diagonal cracking would support this.

In general the magnitude of the compressive strain was not as would be expected from a cracked section analysis, considering corresponding straining of the reinforcement (Fig. 4.3). This highlights the complex state of stress which exists at a slab-column connection.

4.3.6 Ultimate strengths

Failure loads of all the contraflexure models are plotted against reinforcement percentage in Fig. 4.5. The consistency of the tests can be appreciated from the well defined trend existing within each group of results. It is apparent that the influence of reinforcement was significantly greater at low steel percentages, tending to decrease at higher levels, particularly for the deeper slabs. Evidently, the punching strength is very dependent upon the flexural capacity of the slabs and can normally be effectively increased by providing additional tension reinforcement or reducing the span/depth ratio.
4.4 LARGE PANEL MODEL TESTS

4.4.1 Model details

This series of tests comprised two sets of slabs delineated by either reinforcement percentage, or span/depth ratio. Details of these models and the coding sequence used for identification purposes are presented in Table 4.2.

4.4.2 Observed behaviour

Crack pattern development in the large panel models was basically similar to that of equivalent contraflexure specimens. Noticeably however, crack widths became smaller as panel size increased, resulting in a more brittle mode of failure. At fairly modest load levels radial cracking in the vicinity of the slab diagonals progressed beyond the line of contraflexure, meeting the panel edge at a short distance from the outer corners. Cracking at mid-side differed, in that it appeared to initiate from the outside edge, penetrating inwards, an observation which will be considered further in relation to the restraining action within a panel.

On the underside of these models, directly beneath the supports, circumferential cracking developed well before failure, indicating a somewhat different radial moment distribution than would exist in equivalent contraflexure specimens. These cracks did however remain very small relative to top surface cracking and practically closed up after failure. A view of a slab soffit and a full panel model in the test rig after punching can be seen from Plates 4.9 and 4.10 respectively.

Failure patterns for some large panel models of different depths are shown in Plate 4.11. It can be seen that for intermediate and full panel specimens, similar zones of destruction occurred and as for the contraflexure
models, the distribution of cracking appears to be independent of span/depth ratio. A complete distribution of cracking in a typical full panel model is illustrated in Plate 4.12.

4.4.3 Slab deflections

Load-central deflection curves for the full panel models are shown in Fig. 4.6. For these larger slabs, an apparent delayed occurrence of stiffness degradation was evident, being well after the first observations of cracking around the column periphery. This characteristic is associated with the deformation response of a laterally restrained slab, as opposed to that controlled by normal flexural action. Central deflections were of much smaller magnitudes than from equivalent contraflexure specimens and can be seen to depend upon the level of reinforcement and span/depth ratio.

Profiles of deflection along the column lines and slab diagonals of two full panel models close to failure are presented in Fig. 4.7. As for the contraflexure models, maximum radial rotation occurred at the column periphery, however, some reversal of curvature existed, principally over the supports, a condition conducive to the development of compressive membrane action. The general form of deformation was fairly similar to that observed by Masterson (1971) for models having floating edge moment restraint.

As for the contraflexure specimens the small support displacements have been accounted for in the presentation of deflections.
4.4.4 Reinforcement strains

Load-strain curves from reinforcement over the column periphery of two full panel models at extremes of the span/depth range are shown in Fig. 4.8. Smaller strains were exhibited from these models than in the equivalent contraflexure specimens, the discrepancy being much larger than could be accounted for by different flexural characteristics. In all cases however, reinforcement reached yield before failure, in general at between 70% and 90% of the ultimate load.

Strain measurements from the reinforcement over the supports of these models are also presented in Fig. 4.8. It can be seen that strain levels at this location increased significantly after yielding of the reinforcement over the column, but remained well below the yield strain up to ultimate.

4.4.5 Concrete strains

Radial strain at the column periphery varied in a manner similar to that of the contraflexure specimens, as can be seen from Fig. 4.9. At similar proportions of ultimate load, strains generally decreased, indicating a delayed occurrence of internal diagonal cracking.

In several tests, tangential strains at the column periphery were also measured and a comparison of radial and tangential strain distributions is shown in Fig. 4.10. It can be seen that at the centre of the column side, tangential strain remained at a fairly low level throughout loading, however at the column corners, radial and tangential strains were approximately equal.

The distribution of axial strain on the column face was measured for one test only and is shown for different stages of loading in Fig. 4.11. As is generally recognised, a concentration of shearing force was found to
exist at the column corners, becoming more pronounced at higher load levels, as 'biting-in' of the corners increased.

Resultant in-plane surface strain profiles along the column line and slab diagonal are presented in Fig. 4.12 for various full panel models. These distributions were typical for all sizes of the larger panels and are indicative of the nature of the restraining action within these slabs.

Along the column line, tensile strains increased towards the slab edge, in contrast to the development of in-plane compression on the diagonal line as the panel corner was approached. These characteristics can be considered in terms of the combined action of in-plane bending and hoop tension within the concrete outside the supports. After cracking in the outer region, it was impossible to differentiate between tangential bending and in-plane strains, however measurements were not thought to be affected by this at load levels of less than 50% ultimate.

4.4.6 Edge rotation and slab expansion

Edge rotation and slab expansion are shown in Fig. 4.13 for two full panel models of different span/depth ratios. The considerably greater rotational deformation of the thinner slab was accompanied by a higher magnitude of lateral expansion and this can be seen to have increased rapidly after yielding of the reinforcement at the column location. The rate of expansion of the deeper slab also increased after yield, although to a lesser degree.

4.4.7 Ultimate strengths

Failure loads of the large panel models for the various levels of reinforcement and span/depth ratios, are plotted against slab size in Fig. 4.14.
It can be seen that by increasing the extent of slab beyond the nominal line of contraflexure, a significant improvement in load capacity resulted. For the full panel models, the magnitude of this enhancement was found to be in the range 30% - 50%, depending upon the percentage of reinforcement and slab effective depth. Masterson and Long (1974) have reported a similar gain in strength for models having floating edge moment restraint. The trend of results indicates that some additional increase in capacity would have resulted for even larger panels.

4.5 SUSTAINED LOADING TEST

4.5.1 General behaviour

During the initial loading to 70% of the expected ultimate load, the behaviour of this model was basically similar to that of the equivalent short-time specimen. The central deflection was, however, appreciably greater, part of this discrepancy being due perhaps to the lower concrete strength at this time (43 N/mm² as opposed to 54 N/mm²). Over the 12 week period of loading maintained at this level, deformations increased considerably, the central deflection and circumferential crack width being over 100% greater as shown in Figs. 4.15 and 4.16. In addition, reinforcement passing over the column quickly reached yield and as a result little recovery ensued after unloading. This would be a point of extreme importance regarding the long-term serviceability of slabs designed to ultimate load, as service load levels in storage buildings may well approach 70% of the ultimate capacity.

It must be remembered, however, that this test specimen was an idealised representation of an internal connection and some long-term testing of prototype floor slabs would be worthwhile before judging this to be too great a problem.
The final stages of loading over a further period of six weeks, continued to show increased deformations, the central deflection at ultimate being 60% greater than that of the equivalent short-time test.

4.5.2 Ultimate strength

Failure occurred in a familiar punching mode at 95% of the expected short-time ultimate load, indicating negligible adverse effects regarding the long-term strength of such slabs. In contrast, Park (1964) has suggested that creep effects in beam and slab systems may significantly affect the ultimate capacity. At an interior column, however, redistribution mechanisms are much more complex and may include the following beneficial effects:

1. reduced stress concentrations at the column corners
2. a deepening of the slab compression zone and lowering of concrete stress levels
3. an outward spread of plasticity from the column location

Some advantage would also be gained from the strength increase of concrete with age. However, in this case the cube strength at failure was comparable to that of the equivalent short-time test.

4.6 CONCLUSIONS

From the discussion of the test results, the following general points of importance can be concluded:

1. The ultimate capacity of both the conventional and large panel specimens was significantly influenced by the level of reinforcement and the slab span/depth ratio. Failure was characterised by the extent of yielding which occurred prior to punching.
2. The punching strength of the large panel specimens was found to increase as the slab was progressively extended beyond the nominal line of contraflexure to the panel centreline. The trend of the results suggests that a further gain in strength would have been possible for even larger panels.

3. The observations and experimental measurements indicate the development of compressive membrane action in the large panel models.

4. The failure loads of the full panel models were between 30% and 50% greater than those of the equivalent contraflexure specimens. The greatest enhancement was for the slabs of the largest span/depth ratio and the lowest level of reinforcement.

5. The application of high levels of sustained loading to the typical full panel specimen did not significantly affect the ultimate capacity. After the test period of just over four months, failure occurred at approximately 95% of the expected short-time ultimate load.

6. The deformations of the full panel specimen subjected to high levels of sustained loading increased appreciably with time. After the initial 12 week period of loading at approximately 70% of the ultimate capacity, the central deflection more than doubled. The deflection at failure was approximately 60% greater than that of the equivalent short-time test specimen.
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TABLE 4.1 CONTRAFLEXURE MODEL TESTS
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*sustained loading test

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**TABLE 4.2 LARGE PANEL MODEL TESTS**
Fig. 4.1 APPLIED LOAD - CENTRAL DEFLECTION: CONTRAFLEXURE MODELS
Fig. 4.2 DEFLECTION PROFILES: CONTRAFLEXURE MODELS
Fig. 4.3 REINFORCEMENT STRAIN AT COLUMN PERIPHERY: CONTRAFLEXURE MODELS
Fig. 4.4 CONCRETE STRAIN AT COLUMN PERIPHERY: CONTRAFLEXURE MODELS
Fig. 4.5 FAILURE LOADS: CONTRAFLEXURE MODELS
Fig. 4.6  APPLIED LOAD - CENTRAL DEFLECTION: FULL PANEL MODELS
Fig. 4.7  DEFLECTION PROFILES : FULL PANEL MODELS
Fig. 4.8 REINFORCEMENT STRAINS: FULL PANEL MODELS
Fig. 4.9 CONCRETE STRAIN AT COLUMN PERIPHERY: FULL PANEL MODELS
Fig. 4.10  STRAIN DISTRIBUTION AT COLUMN PERIPHERY

Fig. 4.11  STRAIN DISTRIBUTION ON COLUMN FACE
Fig. 4.12  IN-PLANE STRAIN PROFILES
Fig. 4.13  EDGE ROTATION AND SLAB EXPANSION
Fig. 4.14  FAILURE LOADS : LARGE PANEL MODELS
Fig. 4.15 SUSTAINED LOAD TEST: CENTRAL DEFLECTION

- Short-time ultimate load
- 70% STUL
- 90% STUL
- Failure
- Applied load (kN)
- Duration of loading (weeks)
- Deflection (mm)
- U = 43 N/mm²
- U = 52 N/mm²
- 1 day
Fig. 4.16 SUSTAINED LOAD TEST:
CRACK WIDTHS & REINFORCEMENT STRESS
Chapter 5

THE PUNCHING STRENGTH OF THE CONVENTIONAL SLAB SPECIMEN

5.1 INTRODUCTION

5.2 THE NATURE OF PUNCHING FAILURE
   5.2.1 Empirical background
   5.2.2 Analytical background
   5.2.3 Method of analysis
   5.2.4 Interaction of moment and shear

5.3 A RATIONAL METHOD OF PREDICTION
   5.3.1 Local flexural strength
   5.3.2 Moment factor at ultimate
   5.3.3 Slab ductility
   5.3.4 Ultimate flexural capacity
   5.3.5 Ultimate shear capacity
   5.3.6 Modes of failure
   5.3.7 Concentration of reinforcement
   5.3.8 Eccentric loading
   5.3.9 Direct procedure

5.4 COMPARISON WITH EXPERIMENTAL RESULTS
   5.4.1 Tests reported in the literature
   5.4.2 Predictions by other methods
   5.4.3 Comparison with the author's test results
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   5.4.5 Specimen size effect
   5.4.6 Discussion

5.5 CONCLUSIONS
5.1 INTRODUCTION

The present understanding of punching failure is based largely on the experimental studies of the behaviour and strength of the conventional isolated slab-column unit. The design provisions included in the various building codes are a direct result of the empirical procedures derived from such tests.

In recent years, the treatment of punching has been considerably augmented with the introduction of theoretical methods of analysis and conceptual models of failure. However, some form of empiricism has been necessary for the determination of suitable failure criteria. In this respect, the progression towards a more rational method of analysis is not only rewarded by complex theoretical treatments, but by the integration of suitable analytical and empirical techniques. The development of the rational procedure for the prediction of the punching strength of the conventional slab specimen is based on this latter approach.

The nature of punching failure is first examined in relation to the empirical aspects of slab behaviour. In particular, slab ductility and the interaction of moment and shear are recognised as important factors concerning the mode of failure and ultimate capacity. From the analytical background, the method of prediction, which delineates between the various modes of failure, is developed. This is formulated in a simple procedure for the prediction of the punching strength of the conventional slab specimen.

The procedure is assessed in relation to a wide range of test results and the correlation is compared with that of the principal alternative approaches. Further to this, the importance of the isolation of the various modes of failure is made apparent. In addition, the relevance of the results from the reduced scale model tests is examined.
5.2 THE NATURE OF PUNCHING FAILURE

5.2.1 Empirical background

To form a basis for the rational treatment of punching failure, it is necessary to examine the evidence concerning the nature of this phenomenon. Although considerable knowledge has been acquired from the many experimental investigations, the principal factors which influence the connection capacity and mode of failure have not been completely isolated. However, in general terms, there is wide agreement that the behaviour and strength of the conventional slab specimen can be associated with the following characteristics:

a) The influence of flexural reinforcement diminishes as the steel ratio increases. For low levels of reinforcement, the strength of the slab is governed by flexural yielding.

b) The punching strength of heavily reinforced slabs is more dependent upon the concrete strength. It is generally accepted that this relationship is primarily controlled by the tensile strength of the concrete.

c) The degree of ductility exhibited prior to failure is associated with the reinforcement ratio. Heavily reinforced slabs fail in a more brittle mode than their lightly reinforced counterparts.

d) The ultimate capacity is not significantly influenced by the presence of compression reinforcement. It does however improve the post punching behaviour.

e) The highest ultimate shear stresses occur in slabs with the lowest ratio of column size to slab effective depth. This is due to the greater dispersal of shear stress from the column perimeter into the slab compression zone.
f) The influence of column shape is relevant in that the capacity is significantly reduced by the stress concentrations at column corners.

g) The concentration of flexural reinforcement towards the column does not result in an enhanced punching capacity. However, the flexural rigidity is significantly improved.

h) The ultimate capacity is reduced by the application of eccentric loading. This is due to the combination of high flexural and shearing stresses in the slab at the critical face of the column.

i) The results of model tests can be closely correlated with the full size specimens, provided both the reinforcement and concrete are properly scaled.

It was considered essential that the method of analysis should conform with the preceding empirical background. As an additional consideration, the attractiveness of a simple procedure for the prediction of the punching strength was recognised.

5.2.2 Analytical background

It has been observed from experiment, that punching failure finally occurs by shearing of the concrete in the highly stressed compression zone adjacent to the loaded area. The penultimate deformations are however, so influenced by the flexural characteristics of the slab, that the interaction of moment and shear cannot be ignored. Thus, many researchers have considered it relevant to examine both the flexural and shearing stress in the slab at failure.

For concentric loading, the average nominal shear stress around the loaded area can be found by consideration of the static equilibrium. However, the distribution of bending moments can only be determined from
the solution of the Lagrangian equation for plate bending, with the given boundary conditions. This is made difficult by the non-linear material properties of the concrete and steel. In fact, the local behaviour in the region of interest is greatly complicated by cracking of the concrete and yielding of the reinforcement.

To overcome this problem Long (1967) proposed the use of an approximate method of analysis to determine the moments in a slab at failure. This involved the interpolation between two idealised column modelling systems, based on the rotation of the slab at the column periphery, at failure. The limits of this elastic thin plate model were zero slope for a fully built-in column to maximum rotation corresponding with ring type loading. The distribution of radial and tangential bending moments for each of these systems is shown in Fig. 5.1.

Later, Masterson (1971) developed this approach and proposed a simplified finite element model for local slab conditions at ultimate. The nature of this was similar to the ring type loading system, however much more versatility was afforded by the use of plate bending finite elements. The idealised representation of the conventional slab specimen is shown in Fig. 5.2(a). Loading was applied by means of a single point load at each column corner, simulating the stress concentration effect which has been found to exist from experiment. The resultant distribution of radial and tangential bending moments is illustrated in Fig. 5.2(b).

The idealised mechanism of failure was similar to that suggested by Reimann (1963) and was equivalent to a criterion of localised yield at the column periphery. As a state of constant moment existed over the column region, this mode corresponded to the commencement of tangential yield. The nature of this mode of failure is illustrated in Fig. 5.2(c). By making an appropriate allowance for dowel and tensile membrane effects,
the punching capacity was found to be satisfactorily predicted for the majority of practical situations.

In conjunction with the original analytical method, Long (1975) formulated a two-phase design procedure in which the punching strength was predicted as the lesser of either a flexural or shear criterion of failure. This was found to yield a significantly improved correlation with the test results than the purely empirical methods proposed by previous researchers. Therefore, it was considered that this rational approach would serve as a suitable basis for development.

5.2.3 Method of analysis

To facilitate the further examination of the stress conditions in the slab at ultimate, it was considered necessary to utilise a reliable analytical procedure for the prediction of the moment - shear ratio. The simplified relationship proposed by Long (1975) was based on the finite element analysis of the flexible column model. This relationship is only dependent upon the ratio of the slab span to column size and is adequately suited for the normal variation encountered in practice. However, the range with which the internal moment can be predicted with accuracy can be extended by the use of a slightly improved, although more complex relationship.

Further analyses of the elastic plate model have been carried out by Cleland (1979) and Franklin (1981), utilising the plate bending finite element programme developed by Bond (1969) and Lamont (1972). The ratios of internal moment to applied load which resulted from these sample analyses are presented in Fig 5.3. It can be seen that over a wide variation in the ratio of slab span to column size, the ratio of internal moment to
applied load can be expressed closely by the relationship:

$$\frac{M}{P} = 0.04 \left[ \log_{e} \left( \frac{L}{C} \right) \right]^{1.5}$$  \hspace{1cm} 5.1

In this expression the span L is the distance between the column centres of an interior panel. This is approximately equal to 2.5 times the distance to the line of contraflexure from the column centre. It is interesting to note that the preceding expression is based on a natural logarithmic function. This is inherent in the analytical solution for ring type loading of circular plates, as derived by Timoshenko and Woinowsky-Kreiger (1959) and presented in Appendix A.

In order to determine the elastic moment factor for slabs to which the preceding relationship was not applicable, the method of 'Grillage Analysis' was utilised. This provided a simple, yet accurate means of determining the ratio of internal moment to applied load, in circumstances where the use of plate bending finite elements would have been more difficult.

The grillage analogy was originally developed by Lightfoot (1964) and has since attained considerable favour owing to the many advantages it presents over the alternative solution techniques. It is particularly useful for a material having a low Poisson's ratio, such as concrete, as the anticlastic curvature effect is not accounted for. In this respect, extensive use of the method has been made in the field of bridge deck analysis (eg, West, 1973). Furthermore, the author (Rankin, 1978) has found it to be a useful means of assessing the influence of cracking on the local behaviour of flat slabs at edge columns.

In the grillage method, the slab continuum is simulated by means of an orthogonal arrangement of rigidly connected members. The distribution of internal actions can be determined from the matrix stiffness solution for the nodal displacements.
The analytical computer programme was developed by Barton (1980) and later transcribed for use on a desktop machine by Thompson (1981). From control analyses it was found that the moment–shear ratio at the column periphery could be accurately determined from relatively coarse grillage frameworks. The average moment at the column periphery was found to be within 5% of that given by the simplified relationship derived from the finite element analyses.

5.2.4 Interaction of moment and shear

It is widely recognised that the shear capacity of slabs subject to concentrated loadings is significantly greater than that of similar beam sections. This difference in behaviour has been accounted for in the various building code recommendations by different means.

The present American (ACI 318-77) approach is primarily based on the comprehensive work of Moe (1961). The nominal ultimate shear stress is taken to be on a critical section at a distance of one half of the slab effective depth from the column perimeter. The high ultimate shear stress obtained from the test results on conventional slab specimens was attributed to the influence of triaxial compression in the vicinity of the loaded area. Thus, the ACI adopted a nominal shear stress equivalent to twice that permitted for beams.

The present British (CP110, 1972) method is based on a somewhat different approach, following from the work of Regan (1971). In order to maintain consistency between the allowable shear stresses for beams and slabs, the critical section for punching is sited at a distance of one and a half times the overall slab depth from the column perimeter. The philosophy behind this is that the enhanced shear strength of slabs
is due to the increased dowel resistance of the negative reinforcement at a greater distance away from the loaded area.

However, it has been found that the shear capacity of beams is significantly dependent on the ratio of moment to shear stress at the critical section. In the experimental investigation of Leonhardt and Walther (1965), it was found that the shear capacity of simply supported beams without shear reinforcement, increased substantially for low values of the shear span to effective depth ratio. In fact, as this ratio approached unity, the ultimate capacity tended towards the full flexural strength.

From an extensive parametric study, Kani (1966) found that the full flexural capacity could be attained beyond limiting values of the steel percentage and shear span ratio. In the region defined by these variables, the strength was governed by diagonal tension, however it was suggested that the maximum bending moment at failure could be utilised as an indicator of this capacity. The 'valley of diagonal failure' defined by the experimental results of Kani's investigation is depicted in Fig. 5.4.

In this regard, the interaction of moment and shear at the critical region of a slab-column connection is worthy of examination. On this basis, the behaviour of slabs subject to concentrated loadings can more appropriately be compared to that of similar beam specimens.

The ratio of internal moment to shear force per unit width at the critical section of the simply supported beam shown in Fig. 5.4 is given by:

\[ \frac{M}{V} = a_s \]

The moment - shear ratio can be non-dimensionalised and expressed as the shear span ratio, through division by the effective depth of section to give:

\[ \frac{M}{Vd} = \frac{a_s}{d} \]
The ratio of applied load to internal moment at the critical section of the conventional concentrically loaded slab can be found from the simplified relationship derived from the finite element analyses. For the range of the slab span to column size ratios likely to be encountered in practice, the ratio of internal moment to applied load varies in the following manner:

\[
\frac{L}{C} = 8 : \frac{M}{P} = 0.12
\]

\[
\frac{L}{C} = 20 : \frac{M}{P} = 0.21
\]

In this range, the ratio of moment to shear force per unit length of column perimeter is given by:

\[
\frac{L}{C} = 8 : \frac{M}{V} = 4c \times 0.12 = 0.06L
\]

\[
\frac{L}{C} = 20 : \frac{M}{V} = 4c \times 0.21 = 0.04L
\]

The ratio of moment to shear corresponds to an equivalent shear span ratio of a beam, for which the range can be taken as:

\[
\frac{M}{Vd} = 0.06 \frac{L}{d} \text{ to } 0.04 \frac{L}{d}
\]

Thus, for the range of slab span to effective depth ratios normally found in practice \((L/d = 20-40)\), the approximate variation in the equivalent shear span ratio will be

\[
\frac{a_s}{d} = 0.8 \text{ to } 2.4
\]

This range of the equivalent shear span ratio is illustrated with respect to Kani's beam test results in Fig. 5.4. It can be seen that in the narrow band of shear span ratios applicable to slab behaviour, the ultimate capacity is governed to a large extent by the flexural strength.
The diagonal cracking resistance becomes the principal criterion for heavily reinforced or deep sections.

The experimental data relating to the shear strength of short beams is presented in Fig. 5.5. It can be seen that the nominal ultimate shear stress for beams of shear span ratios less than approximately 2.5 is considerably enhanced. In beams with \( \frac{a_s}{d} \) ratios less than 2.5, Zsutty (1971) has proposed that the strength increase can be accounted for by multiplication of the slender beam capacity by a linear factor based on the shear span ratio. The Shear Study Group (1969) recommended the use of a similar modification factor for short beams and this has been incorporated in the British code of practice (CP110, 1972). Thus, it would seem that in view of the enhanced shear capacity in beams of short shear span ratios, the high ultimate shear stress, noted in the punching failure of the conventional slab specimen, is in some way related to the moment - shear ratio. The interaction of moment and shear at the critical section is therefore of paramount importance.

The mode of failure for short shear span ratios \( (\frac{a_s}{d} < 2.5) \) is also worth consideration. The experimental work of Kani (1966) has shown that the full flexural capacity may be attained in short beams. This evidence is supported by the tests on corbels carried out by Kriz and Raths (1965). The principal types of corbel failure were classified as follows:

a) flexural tension: the concrete crushed at the base of the corbel, but only after extensive yielding of the main reinforcement

b) flexural compression: the concrete crushed before yielding of the main reinforcement had occurred

c) diagonal splitting: after the development of the flexural crack pattern, diagonal splitting occurred along a line extending from the loading plate to the base of the corbel
d) shear failure: for extremely low shear span ratios, a series of inclined cracks developed along the interface between the column and corbel; failure was by shearing along this weakened plane.

The various classifications of corbel failure defined by Kriz and Raths bear a close resemblance to the observed modes of punching in the conventional slab specimen. This is particularly relevant in view of the low moment - shear ratio which has been shown to exist in the vicinity of an interior column connection.

The modes of failure associated with low shear span ratios ($0.8 < \frac{a_s}{d} < 2.4$), can be considered in relation to the punching strength of slabs. For the normal variation in the ratio of slab span to column size, it is appropriate to examine the modes of flexural yielding, flexural compression and diagonal splitting failure. Punching by pure shearing of the concrete is only associated with extremely small shear span ratios and is not directly relevant.

5.3 A RATIONAL METHOD OF PREDICTION

5.3.1 Local flexural strength

The ultimate capacity of the conventional slab specimen can be examined in relation to the local flexural strength. The comprehensive series of tests on the contraflexure specimens, as described in the preceding Chapters 3 and 4, are ideally suited for this purpose.

In the tests, the major variables were the level of reinforcement and slab effective depth. Thus, the ultimate capacity can be related directly to the section moment of resistance as shown in Fig. 5.6. It can be seen that for relatively low flexural strengths the results are closely grouped in a narrow band. This suggests that the punching capacity
may be related directly to the moment of resistance by means of an appropriate moment factor. However, at high flexural strengths, each series of results follows an individual path in a much wider envelope. Thus, there would appear to be no direct relationship between the ultimate capacity and the moment of resistance for the complete range of flexural strength.

The test results can be further examined in terms of the local flexural capacity of the slab at the column periphery. In this case, the local flexural strength can be determined from the multiplication of the moment of resistance by the simplified moment factor derived from the finite element analyses. The ratio of the ultimate capacity to the local flexural strength for each test, is shown with the corresponding reinforcement ratio in Fig. 5.7. It can be seen that a well-defined trend exists within the results. In addition, there would appear to be no individual grouping of tests up to a relatively high reinforcement ratio \( \rho \kappa_y/f_c = 0.2 \). The ultimate capacity is greater than the local flexural strength for all but the most heavily reinforced slabs, where the ratio is approximately unity. In the case of the very lightly reinforced slabs, the ultimate capacity is up to 100% greater than the local flexural strength.

The enhancement of the localised flexural capacity as the criterion of failure, has been attributed to various phenomena. Long (1975) proposed the use of the constant proportional factor of 30% to allow for the increased resistance caused by dowel action. However, this explanation is unsatisfactory, as the enhancement has been shown to diminish with the increased reinforcement ratio.

In a novel approach, Cleland (1979) has suggested that the local yield criterion in the conventional specimen, is modified by the development of compressive membrane forces. This action was considered to be induced
by the differential deformations between strips of slab within the line of contraflexure. The local yield moment was assumed to be enhanced by the resultant in-plane compressive forces which were balanced by the restraining hoop tensile forces in the surrounding portion of slab. However, the experimental evidence concerning the nature of failure is contradictory to this argument. The enhancement is greatest in lightly reinforced slabs which exhibit considerable ductility and spread of yielding prior to punching. In this respect, compressive membrane action is associated with reduced deformations which result in a more brittle mode of failure. Thus, the enhanced flexural criterion in lightly reinforced conventional specimens cannot be attributed to the development of compressive membrane action.

On the basis of the preceding empirical examination, the rational explanation of the enhanced flexural criterion is considered to be the rotational capacity of the slab. In itself, the occurrence of localised yield at the column perimeter is not a sufficient criterion of failure. However, combined with the appropriate limits of flexural deformation, the ultimate capacity may be predicted.

5.3.2 Moment factor at ultimate

The behaviour of lightly reinforced slabs is characterised by a considerable degree of ductility prior to punching. In fact, the spread of plasticity can be such as to approach that of an overall flexural collapse mechanism. Indeed, it is widely recognised that in many of the punching tests which have been reported, realistic estimates of the ultimate capacity can be obtained by means of the yield line method. Gesund (1970) has defined a critical parameter below which the yield line predictions are valid.
Alternatively, slabs can be over reinforced, as has been the case for many of these tests reported in the literature. Although unrealistic, these tests have provided valuable information towards the fundamental understanding of punching failure.

The flexural mode of punching in a reinforced concrete slab can therefore be defined between the extreme cases of overall yielding and localised compression. Further to this, for a limited degree of ductility, partial yielding may occur prior to failure. Thus, the punching capacity can be determined from the distribution of moments corresponding to these various states of plasticity. The possible modes of flexurally motivated punching failure can be considered independently and are described as follows.

a) Overall tangential yielding

The kinematically admissible yield line patterns giving the minimum loads for both square and circular simply supported slabs are shown in Fig. 5.8. The flexural load required for this distribution of moments is given by:

\[ P_{\text{flex}} = k_{y1} M_b \]

From consideration of the virtual work done by the actions on the yield lines, the solution for each mechanism is as follows:

(i) square slab and column

\[ k_{y1} = 8\left[ \frac{s}{a - c} - 0.172 \right] \]

5.2a

(ii) circular slab and column

\[ k_{y1} = 2\pi \left[ \frac{r_s}{r_a - r_c} \right] \]

5.2b

In the case of a circular slab having a square column, the solution can be taken as that for an equivalent square slab, as shown in Fig. 5.9(a).
For a square slab with a circular column, an equivalent square column of the same perimeter can be utilised, as illustrated in Fig. 5.9(b).

b) Localised compression failure

Punching failure will be precipitated in over reinforced slabs when the ultimate moment is obtained at the column periphery. In a purely elastic - plastic analysis, the occurrence of radial yield at the periphery of the rigid column is a prerequisite for the development of significant local tangential moments. However, the large rotational deformations which accompany flexural cracking at the column periphery can radically change the distribution of moments well before the commencement of radial yield. In this respect, the deflection profiles of even the most heavily reinforced slabs indicate practically rigid body rotation about the column periphery. The corresponding deformations which result from the use of the rigid and flexible column models in the elastic plate analysis are illustrated in Fig. 5.10. It can be seen that in order to simulate the slab rotational deformations at ultimate, the flexible column model is appropriate.

The localised moment in an over reinforced slab at ultimate can therefore be predicted on the basis of the elastic moment factor $k_b$ which has been previously defined as:

(i) square slab and column

$$k_b = \frac{25}{\log \left( \frac{2.5a}{c} \right)^{1.5}}$$  \hspace{1cm} 5.3a

(ii) circular slab and column (see Appendix A)

$$k_b = \frac{8\pi}{2 \log \left( \frac{r_a}{r_c} + \frac{(r_a^2 - r_c^2)}{r_a^2} \right)}$$  \hspace{1cm} 5.3b

c) Partial tangential yielding

The case of failure after a limited spread of tangential yield into the body of the slab, is more difficult to analyse. Brotchie (1960) has
developed the general elastic - plastic solution for a circular plate under ring type loading. This was utilised to form the relationship between the applied load and yield moment, in terms of the radius of the tangential yield zone. The derivation of the relationship is outlined in Appendix A. In form, the load is related to the yield moment by the expression:

\[ P = k_t M_y \]

The resultant moment factor for an equivalent square slab with localised tangential yield at the column periphery is approximately equal to that given by the simplified relationship derived from the finite element analyses. In the case of tangential yielding to the outer edge of the plate, the solution corresponds to that given by the yield-line method.

The variation of the moment factor \( k_t \) with the spread of the tangential yield zone, is shown in Fig. 5.11. It can be seen that for the normal variation in the ratio of slab span to column size, the relationship can be satisfactorily approximated as being linear. Thus, the simplified moment factor for partial tangential yielding can be interpolated between the factors for overall and localised plasticity. The relationship is therefore given by:

\[ k_t = k_b + (k_{y1} - k_b) \frac{r_y}{r_s - r_c} \]

5.3.3 Slab ductility

It has been shown that the relationship of the applied load to yield moment can be approximated by the linear interpolative moment factor. Thus,
by defining the radius of the tangential yield zone at ultimate, the corresponding load can be deduced. The extent to which yielding can occur prior to failure is dependent upon the rotational capacity of the slab. Unfortunately, no simple analytical method has been devised to determine the section ductility which corresponds to the state of tangential yield. The introduction of a semi-empirical ductility parameter is therefore necessary.

The rotational capacity of plastic hinges in reinforced concrete continuous beams has received considerable attention in relation to the redistribution of elastic moments. Principally, the analytical and experimental studies of Mattock (1959) and Cohn (1965) have formed the basis of the ACI approach to moment redistribution.

In continuous beams, adequate section ductility is ensured by relating the percentage of moment reduction to the balanced reinforcement ratio. The reasoning of this, is that the depth of the neutral axis at yield as compared to the maximum depth in a fully reinforced section, is closely related to the ratio of the curvature at yield to the ultimate curvature. Thus, a convenient ductility parameter which can be utilised to express the degree of tangential yielding in the slab prior to failure, is the ratio of the yield to the balanced moment of resistance. The form of the relationship between this parameter and the section curvature ductility is shown in Fig. 5.12.

The criterion for the radius of the tangential yield zone at ultimate can therefore be expressed in terms of the ratio of the yield to the balanced moment of resistance. In the case of overall tangential yielding, the ductility of the slab must permit unlimited rotation. The ductility parameter can therefore be taken as zero for this condition.

In over reinforced slabs, the punching strength is determined by the localised rotational capacity at the critical section. The appropriate
ductility parameter is therefore equal to unity when yield cannot be attained at the column periphery. Thus, for the limiting ductility parameters, the corresponding moment factors can be defined. However, to determine the moment factor in the case of intermediate rotational capacity, the relationship between the radius of the tangential yield zone and the ductility parameter is required. In the absence of relevant experimental information, the simplest possible relationship can be tentatively assumed. This is the linear spread of the tangential yield zone with respect to the slab ductility parameter. The resultant criterion is expressed as:

$$ \frac{r_y}{r_s - r_c} = 1 - \frac{M_b}{M_{(bal)}} $$

The assumed relationship between the radius of the tangential yield zone and the slab ductility parameter is shown in Fig. 5.13.

The moment factor corresponding to the radius of the tangential yield zone can therefore be defined in terms of the slab ductility parameter. This involves the linear interpolation between the moment factors and ductility parameters for the overall yield and localised compression modes of failure. The resultant interpolative moment factor is given by the relationship:

$$ k_t = \left[ k_{yl} - (k_y - k_b) \frac{M_b}{M_{(bal)}} \right] $$

The ductility parameter and moment factor for the various modes of flexural failure have been defined. These can now be directly related to the flexural punching capacity.
5.3.4 Ultimate flexural capacity

The punching strength can be expressed in terms of the yield moment, by means of the interpolative moment factor. Thus, for the prediction of the ultimate capacity, a suitable yield criterion must be adopted.

In the case of overall tangential yielding, the normal 'stepped' yield criterion due to Johansen (1943) is appropriate. Although the yield line method theoretically provides an upper bound solution, the actual collapse loads of lightly reinforced slabs have been found to be significantly enhanced. Wood (1961) has attributed this to the phenomenon of 'kinking' of the reinforcement across the cracks at the yield lines. From tests on slabs with reinforcement at various angles to the principal moment, Kwiecinki (1965) concluded that up to 18% increase in ultimate moment strength occurred due to partial kinking of the reinforcement. In addition, Kemp (1967) has shown that the development of tensile membrane action at moderately large deflections in unrestrained slabs will increase the yield load. However, for the purpose of simplicity, the yield moment per unit width can be taken as that adopted in American practice and given by:

\[ M_b = \rho f_y d^2 (1 - 0.59 \frac{\rho f_y}{f_c}) \]  

This expression was first proposed by Whitney (1937) and has been shown to be accurate for under reinforced sections.

In the case of failure by localised flexural compression of the concrete, the ultimate resistance of the over reinforced section is required. There is only a very modest increase in strength as the level of reinforcement surpasses the balanced ratio of equal tensile and compressive flexural capacity. Thus, a convenient lower bound estimate of the flexural compressive strength is given by the balanced moment of resistance. The balanced level of reinforcement can be found from the
conditions of stress equilibrium and strain compatibility within the section. In accordance with the proposals of Mattock et al (1961), the ACI recommendations for the ultimate strength parameters are as follows:

\[ \varepsilon_u = 0.003 \]

\[ \beta_1 = 0.85 - \frac{(f'_c - 27.6)}{6.9} \times 0.05 \leq 0.85 \]

\[ E_S = 200 \text{ kN/mm}^2 \]

The balanced level of reinforcement is given by:

\[ \rho_{(bal)} = \frac{0.85f'_c \varepsilon_u E_S}{f_y(\varepsilon_u E_S + f_y)} \]

This relationship can be utilised in conjunction with the preceding expression for the moment of resistance, to give the flexural capacity of the fully reinforced section.

It has been found that the punching strength of connections having square columns is significantly reduced by the presence of stress concentrations at the column corners (eg, Vanderbilt, 1972). Regan (1981) has suggested that connections having circular columns yield approximately 15% greater capacity than square columns of equivalent perimeters. The influence of stress concentrations can be expected to be significant in the case of failure by localised compression of the concrete. Thus, in fully reinforced slabs having square columns, the punching strength can be taken as 87% of the full connection capacity. The appropriate reduction coefficients which allow for column shape are therefore:

a) square columns : \( r_f = 1.15 \)

b) circular column : \( r_f = 1.0 \)
However, for punching failure associated with increased ductility, the influence of stress concentrations is reduced. In the case of overall tangential yielding the punching strength is not diminished by the column shape. The reduction in strength due to stress concentrations can therefore be assumed to depend directly upon the slab ductility.

The preceding expressions for the interpolative moment factor, yield criterion and reduction coefficient can be combined to relate the applied load to the moment of resistance at ultimate. Thus, the ultimate capacity in the flexural mode of punching can be predicted on the basis of the following expression:

\[ P_{vf} = \left[ k_y 1 - r_f (k_y 1 - k_b) M_b \right] \frac{M_b}{M_{(bal)}} - r_f b (bal) \]

This relationship is a rational expression of the flexural behaviour and mode of failure in the conventional slab specimen. However, punching may be precipitated by an alternative failure mechanism before the flexural capacity is realised. This mode of failure is examined in the following section.

5.3.5 Ultimate shear capacity

The possibility of the concrete in the vicinity of the loaded area failing by internal diagonal tension prior to the development of significant plasticity must be considered. This mode of failure is by definition 'brittle' in that relatively small deformations and little warning precede punching.

In order to develop a realistic model of failure it is necessary to examine the stress conditions in the slab adjacent to the loaded area. With little or no yielding of the flexural reinforcement, and in the
presence of biaxial moment, the distribution of compressive bending stress can be approximated from the elastic cracked section analysis. The resulting triangular distribution of stress is shown in Fig. 5.14(a).

Elastic theory predicts a parabolic distribution of vertical shearing stress in the compression zone. The shearing stress is equal to zero at the outer fibre and is maximum at the neutral plane, as shown in Fig. 5.14(b). Therefore, the maximum shearing stress is given by:

$$\tau_m = \frac{1.5V_c}{x}$$

Thus, the critically stressed concrete will be on the neutral plane as there is no beneficial influence of direct compression at this level.

The Mohr circle of stress for an infinitesimal element on the neutral plane is also shown in Fig. 5.14(b). It can be seen that the maximum principal stress is of equal magnitude to the maximum shear stress. Therefore, internal diagonal cracking will occur at an angle of 45° to the neutral plane when the maximum shear stress equals the tensile strength of the concrete. As the principal compressive stress is directed at 90° to the principal tensile stress, this failure mode can be likened to the transverse splitting test performed on standard concrete cylinders. In this respect, the maximum shear stress in the compression zone at the diagonal cracking load can be empirically related to the split cylinder strength by:

$$\tau_m = \alpha_1 f'_c$$

(\(\alpha_1 = 0.5 \text{ to } 0.6\))

The latest research on the shear resisting mechanisms in a flexurally cracked beam section at ultimate suggests that only a proportion of the total shear force is transmitted by the concrete compression zone. In particular, Acharya and Kemp (1965) have estimated that less than 40% of the total ultimate shear force is carried by the compressed concrete.
From tests on beams without web reinforcement, Fenwick and Pauley (1968) have found that the major proportion of the total shear force is transmitted by the action of aggregate interlock across the flexural crack. Further to this, on the basis of fundamental experimental research, Taylor (1974) has proposed that the total shear force is transmitted by three mechanisms in the following approximate proportions:

- compression zone: \( V_c = 20\% - 40\% \)
- aggregate interlock: \( V_a = 33\% - 50\% \)
- dowel action: \( V_d = 15\% - 25\% \)

Therefore, the total shear force carried by the flexurally cracked section at the internal diagonal cracking load is given by:

\[
V_u = \alpha_2 V_c
\]

\( \alpha_2 = 2.5 \text{ to } 5.0 \)

Combining this expression with the previously defined criterion for maximum shear stress in the compression zone gives the relationship for the shear force at diagonal cracking as:

\[
V_u = \frac{1}{1.5 \alpha_2} \frac{\rho}{\rho_2} \frac{F_T}{C} X
\]

It can be seen that the depth of the compression zone \( X \) has a direct influence on the magnitude of the shear force which can be sustained before internal cracking. From the conditions of strain compatibility and equilibrium of forces, the depth of compressed concrete in the elastic analysis of a cracked section is given by:

\[
\frac{X}{d} = -\rho n + \sqrt{(\rho n)^2 + 2\rho n}
\]

The modular ratio \( n \) can be found by assuming the normal modulus of elasticity relationships for steel and concrete to be:

\[
E_s = 200 \text{ kN/mm}^2 \quad E_c = 4.73 \sqrt{F_T} \text{ kN/mm}^2
\]
The relationship is shown in Fig. 5.15 for the range of steel percentage and concrete strength normally used in practice. It can be seen that a simplified approximation for the neutral axis factor $\frac{X}{d}$ is given by:

$$\frac{X}{d} = 0.35(100\rho)^{0.25}$$

Thus, the shear force at diagonal cracking becomes:

$$V_u = \frac{0.35}{1.5\sigma_1\sigma_2\sqrt{f_c'}} d(100\rho)^{0.25}$$

The coefficients $\sigma_1$ and $\sigma_2$ remain to be established on an empirical basis, however from the approximate ranges stated, the ultimate shear force per unit width can be taken as:

$$V_u = 0.48\sqrt{f_c'} d(100\rho)^{0.25}$$  \hspace{1cm} 5.9

In the case of concentrated loading on a slab, the vertical shear stress is dispersed before reaching the neutral plane. The critical section is therefore on the neutral plane at some distance from the loaded area. It has been suggested by many researchers, that the influence of stress dispersal may be adequately recognised by defining the critical perimeter at a distance of one half of the slab effective depth from the loaded area. This would seem reasonable in view of the fact that this mode of failure is more likely in heavily reinforced slabs, in which the depth of the compression zone may well approach one half of the slab effective depth. The critical section so defined has been adopted in successive ACI building codes and has only recently, with the introduction CP110 (1972), been removed from the British recommendations.

The influence of the stress concentrations at the corners of square columns must also be accounted for. As for the localised flexural compressive mode of failure, the shear capacity may be reduced by the empirical factor of 15% which has been suggested by Regan (1981).
Thus, the punching strength in the 'shear' mode of failure is given by:

\[ P_{VS} = 1.66\sqrt{\frac{F_T}{F_C}}(c + d)d(100\rho)^{0.25} \]  
\[ \text{5.10a} \]

b) circular column

\[ P_{VS} = 1.50\sqrt{\frac{F_T}{F_C}}(2r_c + d)d(100\rho)^{0.25} \]  
\[ \text{5.10b} \]

The nominal ultimate shear stress in the above expressions is similar to that proposed by Long (1975) on the basis of the simplified analytical procedure. This latter method includes an additional parameter to account for the influence of the ratio of the slab span to column size. In general, however, the effect of this is small and is therefore excluded for the purpose of simplification.

The punching strength of the conventional isotropically reinforced slab specimen may therefore be predicted on the basis of the preceding flexural and shear criteria of failure. The ultimate capacity is given by the lesser of the two predictions.

5.3.6 Modes of failure

The various modes of failure, which are considered by the proposed method for the prediction of the punching strength, are illustrated in Fig. 5.16. The range of flexural behaviour is characterised by diminishing ductility as the level of reinforcement approaches the balanced ratio. In the localised flexural compression and shear modes of failure, the capacity is primarily determined by the strength of the concrete. Failures of this type occur in heavily reinforced slabs and are therefore brittle in nature. Thus, the broad distinction between the ductile and brittle modes of punching can be made on the basis of the degree of tangential yielding prior to failure.
The relationship between the flexural capacity and the mode of failure is further illustrated in Fig. 5.17. It can be seen that the interaction of flexure and shear is important with respect to the principal mode of failure. In thin slabs, the punching strength is governed by the flexural mode of failure throughout the reinforcement range. Therefore, for the normal levels of reinforcement the 'yield' criterion is appropriate. The primary mode of punching in thick slabs is also flexural, however, for moderate levels of reinforcement, failure can be in the intermediate 'shear' mode, before the full flexural resistance is realised. In all cases, the punching strength is limited by the localised flexural 'compression' mode which corresponds to the fully reinforced capacity.

The general interaction of flexure and shear, with respect to the principal modes of punching failure, is in accordance with the behaviour of beams having short shear span ratios.

5.3.7 Concentration of reinforcement

In practice, if the British or American building code recommendations are followed, the negative reinforcement will usually be concentrated towards the column. This provides increased local flexural rigidity, however, the tests of Elstner and Hognestad (1956) and those of Moe (1961) have shown that no significant improvement in punching capacity results from such an arrangement. To understand the reason for this, it is necessary to examine the distribution of moments caused by banding of the reinforcement.

The concentration of reinforcement in a narrow strip, passing directly over the column, substantially increases the local flexural rigidity. In so doing, a higher proportion of the total slab internal moment is attracted
towards the critical section. Thus, increased strength is partially
offset by increased stress and reduced local ductility. The increased
flexural rigidity due to the concentration of reinforcement may be
estimated in the following manner.

In the context of a flexural mode of punching, the relevant flexural
stiffness is that corresponding to first yield of the reinforcement.
From the conditions of strain compatibility and the elastic theory of
bending, this is given by:

\[ E I_{(cr)} = \frac{M_b(d - x)}{\varepsilon_y} \]

In this expression, the depth \( x \) to the neutral axis may be approximated
as the depth of the equivalent rectangular stress block at ultimate. For
an over reinforced section the moment at yield can be taken as the balanced
moment of resistance. This is simple to derive and ensures consistency
with the analysis for the localised flexural compression mode of failure.
Thus, the stiffness of the strip of slab having concentrated reinforcement,
in relation to the stiffness of the surrounding slab, is given by:

\[ \frac{E I'_{(cr)}}{E I_{(cr)}} = \frac{M_b'(d - x')}{M_b(d - x)} \]

It is necessary to assess the influence of the increased flexural
rigidity on the internal distribution of slab bending moments. In this
respect, the modified elastic moment factor is required. This is dependent
upon the ratio of the slab span to column size, hence it is appropriate
to determine the influence of the increased stiffness for the variation
likely to be encountered in practice. The method of grillage analysis is
ideally suited for this purpose.

The flexible column modelling system in conjunction with the grillage
mesh shown in Fig. 5.18(a) was utilised to determine the relationship for
the modified elastic moment factor. Thus, the analyses of the extreme ratios of slab span to column size were possible by simply applying the concentrated corner load at different nodes of the same mesh. The effect of concentrated reinforcement was simulated by apportioning increased stiffness to the grillage members in direct line with the column. It was found that the torsional rigidity of these members did not influence the ratio of applied load to internal moment. Thus, the simplest means of modifying the member stiffness was adopted. This was effected by assigning an increased breadth of section to the appropriate members.

The resultant moment factors from the various analyses, are shown as a proportion of the original ratio of applied load to internal moment, in Fig. 5.18(b). It can be seen that a reasonable approximation for this ratio is given by:

\[
\frac{k'_b}{k_b} = 1 - \frac{1}{3} \log_3 \frac{EI'(cr)}{EI(cr)}
\]

Thus, by estimating the ratio of increased stiffness due to the concentration of reinforcement, the modified elastic moment factor can be determined.

The redistribution capacity of a slab is also affected by the concentration of reinforcement towards the column. In this regard, the overall flexural yield load is changed. The increased flexural resistance in the vicinity of the column can be denoted by the ratio:

\[
\frac{M'_b}{M_b} = z'
\]

From the consideration of the virtual work solution for overall yielding, the moment factors for the previously described yield line mechanisms become:

(i) square slab

\[
k'_y = 8\left[\frac{s + c(z' - 1)}{a - c} - 0.172\right]
\]

5.12a
(ii) circular slab

\[ k'_y1 = 2\pi \frac{r_s + r_c(z' - 1)}{r_a - r_c} \]  

5.12b

However, the localised slab ductility is reduced by the presence of the additional flexural reinforcement in the column region. In this case, the spread of the tangential yield zone in the slab will be governed by the reduced rotational capacity at the critical section. Therefore the ductility parameter can be taken as the ratio of the increased local yield moment to the balanced moment of resistance.

Hence, for concentrated reinforcement towards the column, the ultimate capacity resulting from a flexurally precipitated punching failure can be predicted on the basis of the following expression:

\[ P_{vf} = \left[ k'_y1 - r_f(k'_y1 - k'_b) \frac{M'_b}{M(bal)} \right] M'_b \frac{1}{r_f} k'_b \frac{M}{(bal)} \]  

5.13

This modified flexural criterion adequately recognises the influence of the increased localised moment, and the reduction in local ductility on the punching capacity of a slab having concentrated reinforcement.

The shear mode of failure, which is precipitated by internal diagonal cracking on the neutral plane, is also affected by the concentration of reinforcement towards the column. The additional reinforcement contributes towards a greater shear capacity by the increase of dowel action and the deepening of the slab compression zone. These components of shear resistance have been adequately accounted for in the method for the prediction of the ultimate shear capacity. Thus, in the case of concentrated reinforcement towards the column, the shear strength can be predicted on the basis of the concentrated level of reinforcement at the critical section. The modified shear force per unit length of the critical perimeter is given by:

\[ V_u = 0.48\sqrt{f'_c d(100\rho)^{0.25}} \]  

5.14
The ultimate capacity in the shear mode of failure can be found from the modified shear criterion in conjunction with the appropriate critical perimeter and stress reduction coefficient for the particular column shape.

It can be seen that, as the reinforcement parameter is to the power of the fourth root, a significant increase in the steel percentage will have only a modest influence upon the ultimate capacity. However, if the total amount of reinforcement in the slab is kept constant, the concentration of reinforcement over the column will have a detrimental effect upon the ultimate flexural strength. In the limiting case, if all of the reinforcement in each direction passes directly over the column, the strength is no longer controlled by slab action, but by the capacity of the perpendicular beam sections. The moment factor for this system is considerably less than for the two way slab of equivalent span. Thus, the ultimate capacity is determined by the flexural criterion of failure.

The test results of Elstner and Hognestad (1956) and those of Moe (1961) would support this. In similar specimens, the concentration of reinforcement was indeed found to be slightly detrimental towards the ultimate capacity.

The preceding examination further justifies the use of the proposed flexural approach and highlights the misleading consequences of adopting a purely shear criterion for the full range of slab behaviour.

5.3.8 Eccentric loading

The application of eccentric loading induces the more concentrated actions of shear and moment at the critical section perpendicular to the direction of moment transfer. Therefore, on consideration of the flexural mode of failure, it is apparent that eccentric loading will influence the
spread of the tangential yield zone. In the case of tangential yielding to the outer edge, the yield line pattern may be significantly changed. Failure by localised compression of the concrete or internal diagonal tension will be governed by the flexural and shearing stresses in the slab at the critical column face. However, to determine the punching strength in the case of eccentric loading, it is desirable to adopt a simple procedure which is applicable to all the modes of failure. Therefore, the use of a suitable eccentricity factor, with which the capacity of the equivalent concentrically loaded specimen can be reduced, is appropriate.

The present British (CP110, 1972) and American (ACI 318—77) treatments of the combined loading case, are based primarily on the work of Regan (1971) and Di Stasio and van Buren (1960) respectively. The methods differ in detail, however, in both cases, a proportion of the out of balance moment is considered to be transferred by the uneven distribution of shear on some perimeter around the loaded area. For the normal variation in the ratio of eccentricity to span, Regan (1974) has shown that little difference will result from the use of either of the two Code approaches.

The method proposed by Long (1975) was based on the elastic plate analysis for combined loading. The ratio of the applied load to internal moment at the critical section was determined for various ratios of the slab span to column size. For the range encountered in practice, the capacity reduction due to eccentric loading was expressed by means of the following simplified relationship:

\[
P_{ve} = \frac{P}{V_0} \left(1 + 15 \frac{e}{L} \right)
\]

This expression is similar to, although slightly more conservative than, the relationship adopted by CP110 (1972).
In general, no significant difference will result from the use of any of the prementioned methods to determine the punching strength of the eccentrically loaded conventional specimen. However, the capacity reduction factor suggested by Long (1975) is appropriate in the present context as it is related to both the flexural and shearing action at the critical section.

5.3.9 Direct procedure

The procedure for the prediction of the punching strength of the conventional slab specimen is outlined by the following flow chart:

```
<table>
<thead>
<tr>
<th>SLAB DUCTILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>\downarrow</td>
</tr>
<tr>
<td>MOMENT FACTOR</td>
</tr>
<tr>
<td>\downarrow</td>
</tr>
<tr>
<td>ULTIMATE FLEXURAL CAPACITY</td>
</tr>
<tr>
<td>\downarrow</td>
</tr>
<tr>
<td>ULTIMATE SHEAR CAPACITY</td>
</tr>
<tr>
<td>\downarrow</td>
</tr>
<tr>
<td>MODE OF FAILURE</td>
</tr>
</tbody>
</table>
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In the case of concentrated reinforcement towards the column, the procedure can be utilised in conjunction with the modified moment factor and ductility parameter, as described in section 5.3.7. The punching strength of an eccentrically loaded specimen can be predicted on the basis of the reduced concentric loading capacity, as shown in section 5.3.8.

An example of the direct procedure applied to a typical slab tested by Moe (1961) is presented in Appendix A. The method for the prediction of the punching strength of the conventional isotropically reinforced slab and column, is described in detail by the following steps.
1. SLAB DUCTILITY

\[ M_b = \rho f_y d^2 (1 - 0.59 \frac{\rho f'_c}{f'_c}) \]  
\[ \beta_1 = 0.85 - (\frac{f'_c}{6.9} - 27.6) \times 0.05 \} 0.85 \]  
\[ \rho_{(bal)} = \frac{0.85f'_c\beta_1(0.003 \times 200000)}{f_y \times (0.003 \times 200000 + f_y)} \]  
\[ M_{(bal)} = \rho_{(bal)} f_y d^2 (1 - 0.59 \frac{\rho_{(bal)} f'_y}{f'_c}) \]  

Ductility parameter \( \frac{M_b}{M_{(bal)}} \)

2. MOMENT FACTOR

(i) square slab and column \( (r_f = 1.15) \)

\[ k_{y1} = 8 \left[ \frac{s}{a - c} - 0.172 \right] \]  
\[ k_b = \frac{25}{\log_e \left( \frac{2.5a}{c} \right)^{1.5}} \]  

(ii) circular slab and column \( (r_f = 1.0) \)

\[ k_{y1} = 2\pi \left[ \frac{r_s}{r_a - r_c} \right] \]  
\[ k_b = \frac{8\pi}{2\log_e \frac{r_a}{r_c} + \left( \frac{r_a^2 - r_c^2}{r_a^2} \right)} \]
interpolative moment factor:

\[ k_t = \left[ k_y 1 - r_f (k_y 1 - k_b) \frac{M_b}{M(bal)} \right] \] (5.4)

3. ULTIMATE FLEXURAL CAPACITY

\[ P_{vf} = k_t b M_b \frac{1}{r_f} k_b M(bal) \] (5.8)

4. ULTIMATE SHEAR CAPACITY

(i) square column

\[ P_{vs} = 1.66 \sqrt{c + d}(100 \rho)^{0.25} \] (5.10a)

(ii) circular column

\[ P_{vs} = 1.5 \sqrt{r_c + d}(100 \rho)^{0.25} \] (5.10b)

5. MODE OF FAILURE

\[ P_{vf} < P_{vs} : P = P_{vf} \text{ (flexural mode)} \]

\[ P_{vf} > P_{vs} : P = P_{vs} \text{ (shear mode)} \]
5.4 COMPARISON WITH EXPERIMENTAL RESULTS

In this section, the proposed method of predicting the punching strength of the conventional slab specimen is examined in relation to the test results of the author and many of those reported in the literature. The relevant details of over one hundred concentrically loaded, isotropically reinforced slabs are presented. For each specimen, the predicted ultimate capacity and mode of failure is given. In addition, the approach for slabs having concentrated reinforcement and eccentric loading is compared with several of the test results. The proposed method of predicting the ultimate strength is also assessed in respect of the correlation resulting from other methods of analysis.

5.4.1 Tests reported in the literature

a) Tests by Elstner and Hognestad

The results of the full scale specimens tested by Elstner and Hognestad (1956) have been widely used as a basis for many studies of punching failure. In these tests, the main variables were the level of reinforcement, concrete strength and column size. In fact, several of the slabs had a high percentage of steel in conjunction with a low concrete strength, which resulted in over reinforced sections. The principal details of the slabs and the predicted ultimate capacities are presented in Table 5.1.

b) Tests by Base

As part of a pilot investigation into punching failure, Base (1959) reported tests on nine conventional isotropically reinforced specimens. These slabs were approximately $\frac{1}{3}$ scale models, in which the main variable was the percentage of reinforcement. The results of these tests are compared with the predicted punching strengths in Table 5.2.
c) Tests by Kinnunen and Nylander

The tests of Kinnunen and Nylander (1960) comprised circular slabs having circular columns. Unfortunately, the majority of specimens were reinforced by means of a radial and circumferential arrangement of steel. Thus, only four uniformly reinforced slabs were amenable to this analysis.

The elastic moment factor for these specimens was calculated on the basis of the ring type loading system and the absence of stress concentrations was accounted for. The principal details of these tests and the predicted ultimate capacities are presented in Table 5.3.

d) Tests by Moe

The full scale specimen tests by Moe (1961) were similar to the earlier tests carried out by Elstner and Hognestad (1956) at the Portland Cement Association. Details of the isotropically reinforced slabs and the predicted ultimate capacities are presented in Table 5.4. The results of the specimens having concentrated reinforcement and eccentric loading are compared with the predicted strengths in Tables 5.5 and 5.6 respectively.

e) Tests by Taylor and Hayes

The approximately $\frac{1}{3}$ scale conventional models tested by Taylor and Hayes (1965) served as control specimens for an investigation of the influence of lateral restraint on the punching strength. These slabs are of particular interest in that a wide range of column size was utilised. In addition, the level of reinforcement was varied. The test results and the predicted failure loads are presented in Table 5.7.

f) Tests by Dragosavić and van den Beukel

The slabs tested by Dragosavić and van den Beukel (1974) were approximately $\frac{1}{5}$ scale circular models, loaded through square column
stubs. The principal variable was the percentage of reinforcement, however, the column size and slab effective depth were changed in individual instances.

In this comparison, square slabs having equivalent overall yield capacities were analysed. The resultant elastic moment factors were approximately equal to those given by the analysis of the circular slab under ring type loading.

The results of these tests and the predicted ultimate capacities are presented in Table 5.8.

g) Tests by Criswell

In an investigation of the static and dynamic response of slab-column connections, Criswell (1974) has reported the results of several full scale conventional specimens. The main variables in these slabs were the level of reinforcement and the size of column. Details of these tests and the predicted punching strengths are presented in Table 5.9.

h) Tests by Regan

As part of the recent CIRIA investigation into flat slab behaviour, Regan (1978) has reported several tests on conventional interior column connection specimens. The slab size corresponded to approximately \( \frac{2}{3} \) of the full scale, however, the concrete mix was not proportioned accordingly. Only the uniformly reinforced concentrically loaded slabs have been analysed. Details of these specimens and the predicted punching loads are presented in Table 5.10.
5.4.2 Predictions by other methods

As a means of assessing the proposed procedure for the prediction of the punching strength of the conventional slab specimen, it is relevant to compare the correlation with test results, of the principal alternative methods. For this study, the present British (CP110, 1972) and American (ACI 318-77) code procedures, the simplified approach of Long (1975) and the more recent method of Regan (1981) are examined.

The code methods involve the limitation of the nominal shear stress on the critical perimeter at some distance from the loaded area. In the case of the ACI formula, the ultimate capacity is directly proportional to the square root of the concrete strength and the critical section is at one half of the slab effective depth from the column perimeter. The British code recognises the influence of negative reinforcement and permits a maximum shear stress in proportion to the cube root of the steel percentage and concrete strength. This must not be exceeded on a critical section at one and a half times the overall slab depth from the column perimeter.

The ACI recommendations include a strength reduction factor for safety in design. However, to compare the method with laboratory tests, this is omitted.

The CP110 formulation contains a partial safety factor to allow for the possible difference between the strength of material in the actual structure and that derived from test specimens. However, there is some confusion as to the actual magnitude of the factor which is included. According to Regan (1974), the full value of the partial factor of safety for concrete ($\gamma_m = 1.5$) is contained within the maximum allowable shear stress. This approach would seem to be somewhat illogical in view of the dependence of the shear strength on the cube root of the concrete strength. In the proposed revisions to the Code (draft for public comment, May 1982),
the maximum allowable shear stress remains basically unchanged. However, it is stated that the partial factor of safety is only 1.25. Therefore, for comparison of the present method with the results from laboratory controlled tests, it is assumed necessary to increase the ultimate shear stress by 25%.

In addition, the absolute depth of slab must be accounted for. The reason for this is that the basic method has been found to underestimate the strength of thin slabs, apparently because of concrete scale effects (Regan and Yu, 1973). Thus, the strength of slabs less than 300 mm in depth can be enhanced linearly by up to 30% for 150 mm deep slabs. It is recognised that the scale effect is reduced when the aggregate is scaled, and thus for properly scaled models it would seem reasonable to use the depth factor which corresponds to the equivalent prototype slab.

The simplified two-phase approach of Long (1975) has formed the basis of the author's method of analysis. It is however, worth separate consideration. The flexural punching capacity is the load necessary to cause localised tangential yielding at the column perimeter, with an appropriate allowance for the enhancement due to dowel action. The shear capacity is given by the nominal ultimate shear stress on a critical perimeter at one half of the slab effective depth from the loaded area. As in the proposed method, the predicted punching strength is the lesser of the flexural and shear capacities.

In the recent approach of Regan (1981), the resistance to punching failure is related to the nominal ultimate shear stress on an inclined fracture surface. The method has been derived on an empirical basis by the adoption of a standard angle of inclination for the failure plane. The parameters which influence the ultimate capacity are basically the same as in the present CP110 method.
The various procedures for calculating the punching strength of concentrically loaded interior slab-column connections are outlined in Appendix A.

5.4.3 Comparison with the author's test results

The full details of the ¼ scale conventional slab specimens tested by the author have been presented in Chapters 3 and 4. In respect of the major variables, these tests provide a comprehensive base from which to assess the interaction of flexure and shear on the punching strength.

The failure loads of the contraflexure models are shown in comparison with the predicted ultimate capacities in Table 5.11. In the case of the empirical procedures, the predicted strengths are given as the lesser of the shear and yield line capacities. The correlation coefficients obtained for each method are presented in Table 5.12.

The empirically based methods of the British (CP110, 1972) and American (ACI 318-77) codes are shown to give significantly higher average ratios of experimental to predicted punching strength, with greater variation about the mean. This is particularly true for the ACI method which takes no account of the level of reinforcement on the shear capacity.

The method recently proposed by Regan (1981), yields a correlation similar to the present CP110 (1972) approach. Indeed, although it has been presented in a form which may provide the designer with a better picture of what is involved, it has been purposefully devised to assimilate the existing Code method. In this respect, the influence of the major variables is not adequately recognised and the various modes of punching failure are not defined.
The method of Long (1975) is shown to give a significantly better correlation with the test results, than any of the purely empirical approaches. This can be expected, as the empirical methods have been calibrated with reference to particular sources of data. In this respect, by generalising the criteria for all modes of failure, the real nature of punching is not adequately reflected. Thus, the overall applicability of the empirically based methods, is to some extent diminished. The flexural criterion of Long's two—plase approach, does however, more realistically assess the punching strength of the more lightly reinforced slabs. This is particularly significant, in that the majority of slabs designed in practice cannot be heavily reinforced because of the practical limitations upon the steel arrangement.

It can be seen that the proposed procedure yields a consistent, yet conservative estimate of the ultimate capacity. The mean ratio of the test to predicted capacity is 1.29, with a coefficient of variation equal to 7.4%. This is a significantly better correlation than that given by any of the alternative methods. The variation about the mean is improved by approximately 19% over that given by Long's procedure. This is as a result of the more rational treatment of the flexural mode of failure. Although the method is more complex, the true nature of punching is more realistically modelled throughout the range of flexural behaviour. In addition, the qualitative assessment of the degree of yielding at failure is possible by means of the slab ductility parameter. For the particular configuration of the author's slab specimens, the punching capacity predicted by the proposed nominal shear stress criterion is the same as for the method of Long. This supports the view that the inclusion of the slab span to column size parameter is not fully warranted for this mode of failure.
The methods are further compared with the author's test results, in Fig. 5.19. As there was little variation of the concrete strength, the average cube strength of 40 N/mm² was utilised for the calculations.

It can be seen that in the absence of the flexural collapse loads derived by means of the yield line theory, the empirical methods are dangerously unsafe for the lightly reinforced slabs. This highlights the advantage of the modal procedures, in which the flexural criterion of failure is inherent. In this respect, the empirical methods do not properly account for the percentage of reinforcement, yield strength and slab effective depth, in the important practical range of reinforcing levels. In addition, the possibility of the localised compressive destruction of the concrete is not recognised. This can be particularly significant when the reinforcement is concentrated towards the column.

5.4.4 Overall correlation

The correlation coefficients for the various methods of prediction with the results of the tests on isotropically reinforced slabs having square columns are presented in Table 5.13. For the sake of brevity, the predictions for the individual tests are not given. The mean ratio of the experimental to the predicted failure load, by the proposed method, is 1.178, with a coefficient of variation equal to 11.8%. This correlation is significantly better than that from the alternative procedures, with the variation about the mean being improved by approximately 35% over that given by Long's method. In respect of the preceding limited comparison with the author's test results, this much wider correlation with tests from various sources is more relevant.
The overall correlation of the proposed method of prediction with the reported test results and those of the author can be assessed from Fig. 5.20(a). The mean ratio of the experimental to the predicted ultimate capacity is 1.174 with a coefficient of variation equal to 11.4%. This is very satisfactory in view of the wide range of variables involved and the probabilistic nature of failure. The best fit linear relationship for the overall correlation with the proposed procedure is also shown in Fig. 5.20(a). There is no apparent trend with the results which indicates that the principal material variables are adequately accounted for.

An examination of the predicted capacities revealed that of the 120 tests analysed, only 10% were predicted to have failed by localised flexural compression. The slabs having concentrated reinforcement accounted for approximately one third of these results. Of the remaining tests, approximately one half were predicted as flexural yielding type failures and one half as shear failures. The various modes of failure are delineated in Fig. 5.20(b). It can be seen that although not defined precisely by the reinforcement index \( \frac{f_y}{f_c} \), the general classification is as follows:

a) \( \frac{f_y}{f_c} \leq 0.1 \) : all slabs failed in a flexural mode of punching

b) \( 0.1 < \frac{f_y}{f_c} \leq 0.3 \) : an approximately equal number of slabs failed in both the flexural and shear modes of punching

c) \( 0.3 < \frac{f_y}{f_c} \leq 0.6 \) : slabs failed by shear or localised flexural compression

d) \( \frac{f_y}{f_c} > 0.6 \) : all slabs failed by localised flexural compression

In practice, even with some concentration of the reinforcement towards the column, the local reinforcement index is not likely to exceed a value of approximately 0.2. This consideration further highlights the importance
of recognising the flexural nature of punching failure for the normal range of reinforcing levels.

The correlation of the method with the tests having concentrated reinforcement is, in itself, very satisfactory. In addition, the author is aware of no other method which actually predicts the slightly detrimental effect on the ultimate capacity, which the concentration of reinforcement has been found to produce. This is caused by the attraction of an increased proportion of the flexural stress to the critical section and the reduction in ductility at this location.

The method appears to be more conservative for the very high concentrations of reinforcement. This can be attributed to the lower bound nature of the balanced failure criterion. The punching strength of the over reinforced slabs can be predicted with greater accuracy by the use of the full resistance moment for compression failure. This is however, more difficult to calculate and is not warranted for normal practice.

The influence of eccentric loading is adequately recognised by means of the suggested capacity reduction factor. Therefore, it would appear to be unnecessary to compute the alternative yield line solution for these slabs.

5.4.5 Specimen size effect

The predictions for the series of model tests are shown in comparison with those for the full scale specimens in Fig. 5.20(c). The models have been considered as all of the slabs having scaled aggregate concrete. It can be seen that although there is considerable overlap in the correlation the average model strength is slightly higher than the mean of the full scale specimens. The apparent trends within each group of results are not
significant and tend to be produced by the variation in correlation between particular sources of data. The average discrepancy is only of the order of 10% which is not too serious in view of the mean value of the experimental to predicted capacity for the full scale slabs being greater than unity. However, the reason for this apparent difference is worth some consideration.

In a recent study of the shear strength of model beams, Chana (1981) found that the ultimate shear stress increased substantially as the depth of section was reduced. The principal source of the enhancement was attributed to the presence of high strain gradients in the models, which resulted in the delayed cracking of the concrete. For a ratio of beam depths similar to that of the full scale and model slabs which have been analysed, the average ultimate shear stress was approximately 33% higher in the shallow beams. This is much greater than the 10% discrepancy which resulted from the slab tests and perhaps the reason for this can again be attributed to the interaction of flexure and shear. As shown previously, the ratio of moment to shear in a slab is equivalent to that found in beams of short shear spans \((0.8 < \frac{a}{d} < 2.4)\), such as corbels. The beams tested by Chana had shear span ratios of 3 which corresponds to a different mode of shear failure governed by 'beam action' in the shear span. In this case, the strength is limited by the bond transmission capacity of the concrete cantilevers formed between the flexural cracks in the shear span. Therefore, without due consideration to the interaction of moment and shear, the findings concerning size effects in beams, are not directly applicable to the punching strength of slabs.

An important factor in reduced scale model tests, is the correct assessment of the strength of the concrete in the structure. In general, the compressive strength of the concrete in the models which have been analysed, was determined from crushing tests on 100 mm cubes. This size
of specimen is generally selected in order to avoid problems of the machine calibration and relative stiffness effects. However, it has been suggested by Sabnis and Mirza (1979) that the strength of model concrete should be based on the strength of test cylinders of diameter equal to the minimum dimension of the structure in the region of failure. It is widely accepted that the compressive strength of concrete increases as the specimen size is reduced. Thus, it may be more appropriate to assess the model capacity on the basis of a factored 100 mm cube strength. The relationship presented by Neville (1973) would suggest that the strength of the concrete in specimens of similar scale as the models may be of the order of 5% - 10% greater than the 100 mm cube strength. In itself, this may not be of sufficient magnitude to account for the full 10% difference in the mean predictions for the models and full scale slabs. In addition, Dragosavić and van den Beukel utilised small scale cubes (40 mm) as control specimens, yet the average ratio of the experimental to the predicted capacity for these tests is also greater than that of the full scale slabs.

At present therefore, no fully satisfactory explanation can be postulated as to why the punching capacity of the model slabs would appear to be slightly greater than that of the corresponding full scale specimens. However, it must be borne in mind that the overall discrepancy is small (10%) in comparison with the degree of accuracy with which the ultimate capacity can be predicted. Thus, an awareness of the possible existence of size effects in models is important, but the seriousness of the problem should not be overestimated.
5.4.6 Discussion

It has been shown that the proposed method for the prediction of the punching strength of the conventional slab specimen, constitutes a progressive step towards the more rational treatment of punching failure. The method is, to a certain extent, more cumbersome to use than the empirically based code procedures, however, the general validity is much more acceptable. In this respect, the interaction of flexure and shear, which results in complex slab behaviour, cannot be satisfactorily dealt with by a single failure criterion. However, the procedure is not overly complicated by the treatment of various modes of punching and is adequately simple for hand calculation. In addition, it can provide a realistic assessment of the slab ductility and mode of failure.

The comparison of the predictions given by the various methods, with a wide range of test results, has shown that the proposed procedure yields a significantly better correlation than the alternative approaches. Although the method is not quite a lower bound, it can be expected that within the limits of experimental scatter, some conservatism will be inherent.

5.5 CONCLUSIONS

The following points of importance concerning the punching strength of the conventional slab specimen can be concluded:

1. The moment-shear ratio at the slab critical section is similar to that in beams of short shear span (eg, corbels). In such specimens, the mode of failure and ultimate capacity is significantly influenced by the interaction of moment and shear.

2. The slab ductility can be quantitatively expressed as the ratio of the moment at yield to the balanced moment of resistance. This parameter can be utilised to define the extent of the tangential yield zone at failure.
3. The various modes of punching failure can be broadly classified as either flexural or shear. As the flexural capacity of the slab is increased, the ductile 'yield' mode is superceded by the more brittle localised flexural 'compression' failure. The 'shear' mode of punching distinguishes the slabs which fail due to internal diagonal cracking prior to yielding of the reinforcement or crushing of the concrete.

4. The proposed method for the prediction of the punching strength of the conventional slab specimen has been shown to give better correlation with a wide range of test results than the principal alternative procedures. This is primarily due to the more rational treatment of the flexural mode of punching failure.

5. The principal effect of concentrating the reinforcement in a narrow band passing directly over the column region is to reduce the ultimate flexural capacity of the slab. Furthermore, the ultimate shear capacity is increased and hence, the mode of failure is more likely to be flexural. Thus, for normal slabs, the concentration of reinforcement can have a slightly detrimental effect on the punching strength.

6. The reduction in strength due to eccentric loading can be satisfactorily allowed for by means of the suggested capacity reduction factor. This is applicable to all the modes of punching failure.

7. The general delineation between the various modes of punching failure can be made on the basis of the reinforcement index. The majority of the specimens in the practical range of reinforcement were predicted to fail in the flexural 'yield' mode.

8. There is some evidence of 'size effects' in the results of the reduced scale models. However, the overall increase in strength is estimated to be only 10% which is a small discrepancy in comparison with the variability of the results.
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<th>d (mm)</th>
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<td>3.700</td>
<td>322</td>
<td>446</td>
<td>452</td>
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<td>26.6</td>
<td>3.700</td>
<td>322</td>
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<td>500</td>
<td>1.070</td>
<td>s</td>
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<td>A3a</td>
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<td>114.3</td>
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<td>3.700</td>
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<td>549</td>
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<td>117.6</td>
<td>26.2</td>
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<td>333</td>
<td>401</td>
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<td>14.2</td>
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<td>325</td>
<td>179</td>
<td>141</td>
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<td>B2</td>
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<td>47.7</td>
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<td>47.8</td>
<td>0.990</td>
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<td>580</td>
<td>655</td>
<td>0.885</td>
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slab size = 1829 mm
span = 1778 mm

**TABLE 5.1 SLABS TESTED BY ELSTNER AND HOGNENSTAD**
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<thead>
<tr>
<th>SLAB NO.</th>
<th>c mm</th>
<th>d mm</th>
<th>U N/mm²</th>
<th>ρ %</th>
<th>f_y N/mm²</th>
<th>P_T kN</th>
<th>P_p kN</th>
<th>P_T/P_p</th>
<th>MODE OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>57.3</td>
<td>33.1</td>
<td>1.083</td>
<td>345</td>
<td>93.9</td>
<td>79.6</td>
<td>1.180</td>
<td>s</td>
</tr>
<tr>
<td>B</td>
<td>102</td>
<td>57.3</td>
<td>35.7</td>
<td>1.083</td>
<td>345</td>
<td>103.9</td>
<td>82.7</td>
<td>1.256</td>
<td>s</td>
</tr>
<tr>
<td>C</td>
<td>102</td>
<td>57.3</td>
<td>32.8</td>
<td>1.083</td>
<td>345</td>
<td>97.9</td>
<td>79.3</td>
<td>1.235</td>
<td>s</td>
</tr>
<tr>
<td>D</td>
<td>102</td>
<td>57.3</td>
<td>34.2</td>
<td>1.083</td>
<td>345</td>
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<td>E</td>
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<td>57.3</td>
<td>37.3</td>
<td>0.725</td>
<td>345</td>
<td>81.9</td>
<td>65.3</td>
<td>1.254</td>
<td>y</td>
</tr>
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<td>F</td>
<td>102</td>
<td>57.3</td>
<td>34.7</td>
<td>0.725</td>
<td>345</td>
<td>81.9</td>
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<td>1.268</td>
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<td>G</td>
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<td>57.3</td>
<td>36.4</td>
<td>1.635</td>
<td>345</td>
<td>112.9</td>
<td>92.4</td>
<td>1.222</td>
<td>s</td>
</tr>
<tr>
<td>H</td>
<td>102</td>
<td>57.3</td>
<td>33.0</td>
<td>1.635</td>
<td>345</td>
<td>99.9</td>
<td>88.0</td>
<td>1.135</td>
<td>s</td>
</tr>
<tr>
<td>J</td>
<td>102</td>
<td>57.3</td>
<td>35.1</td>
<td>3.270</td>
<td>345</td>
<td>117.9</td>
<td>108.0</td>
<td>1.092</td>
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</tbody>
</table>

slab size = 610 mm
span = 559 mm

TABLE 5.2 SLABS TESTED BY BASE
### TABLE 5.3 SLABS TESTED BY KINNUNEN AND NYLANDER

<table>
<thead>
<tr>
<th>SLAB NO.</th>
<th>$\phi_c$ mm</th>
<th>$d$ mm</th>
<th>$U$ N/mm$^2$</th>
<th>$\rho$</th>
<th>$f_y$ N/mm$^2$</th>
<th>$P_T$ kN</th>
<th>$P_p$ kN</th>
<th>$\frac{P_T}{P_p}$</th>
<th>MODE OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A30(a)24</td>
<td>300</td>
<td>128</td>
<td>32.4</td>
<td>1.010</td>
<td>456</td>
<td>430</td>
<td>418</td>
<td>1.029</td>
<td>s</td>
</tr>
<tr>
<td>1A30(a)25</td>
<td>300</td>
<td>124</td>
<td>30.8</td>
<td>1.040</td>
<td>452</td>
<td>408</td>
<td>394</td>
<td>1.040</td>
<td>s</td>
</tr>
<tr>
<td>1A15(a)5</td>
<td>150</td>
<td>117</td>
<td>32.9</td>
<td>0.800</td>
<td>442</td>
<td>255</td>
<td>227</td>
<td>1.123</td>
<td>s</td>
</tr>
<tr>
<td>1A15(a)6</td>
<td>150</td>
<td>118</td>
<td>32.1</td>
<td>0.790</td>
<td>455</td>
<td>275</td>
<td>226</td>
<td>1.217</td>
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slab diameter = 1829 mm  
span = 1710 mm

### TABLE 5.4 SLABS TESTED BY MOE

<table>
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<tr>
<th>SLAB NO.</th>
<th>$c$ mm</th>
<th>$d$ mm</th>
<th>$f'_c$ N/mm$^2$</th>
<th>$\rho$</th>
<th>$f_y$ N/mm$^2$</th>
<th>$P_T$ kN</th>
<th>$P_p$ kN</th>
<th>$\frac{P_T}{P_p}$</th>
<th>MODE OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1-60</td>
<td>254</td>
<td>114.3</td>
<td>23.4</td>
<td>1.060</td>
<td>400</td>
<td>390</td>
<td>320</td>
<td>1.219</td>
<td>y</td>
</tr>
<tr>
<td>S5-60</td>
<td>203</td>
<td>114.3</td>
<td>22.2</td>
<td>1.060</td>
<td>400</td>
<td>343</td>
<td>288</td>
<td>1.191</td>
<td>s</td>
</tr>
<tr>
<td>S1-70</td>
<td>254</td>
<td>114.3</td>
<td>24.5</td>
<td>1.060</td>
<td>484</td>
<td>393</td>
<td>351</td>
<td>1.120</td>
<td>s</td>
</tr>
<tr>
<td>S5-70</td>
<td>203</td>
<td>114.3</td>
<td>23.1</td>
<td>1.060</td>
<td>484</td>
<td>379</td>
<td>294</td>
<td>1.289</td>
<td>s</td>
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<td>H1</td>
<td>254</td>
<td>114.3</td>
<td>26.1</td>
<td>1.150</td>
<td>328</td>
<td>372</td>
<td>309</td>
<td>1.204</td>
<td>y</td>
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<tr>
<td>R2</td>
<td>152</td>
<td>114.3</td>
<td>26.6</td>
<td>1.150</td>
<td>328</td>
<td>312</td>
<td>270</td>
<td>1.156</td>
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<td>305</td>
<td>114.3</td>
<td>20.9</td>
<td>1.500</td>
<td>482</td>
<td>434</td>
<td>403</td>
<td>1.077</td>
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slab size = 1829 mm  
span = 1778 mm

Table 5.3 and 5.4 show the test results for slabs tested by different researchers using specific parameters such as slab diameter and span.
### TABLE 5.5 SLABS TESTED BY MOE - CONCENTRATED REINFORCEMENT

<table>
<thead>
<tr>
<th>SLAB NO.</th>
<th>c (mm)</th>
<th>d (mm)</th>
<th>$f'_c$ (N/mm²)</th>
<th>$\rho$ (%)</th>
<th>$f_y$ (N/mm²)</th>
<th>$P_T$ (kN)</th>
<th>$P_p$ (kN)</th>
<th>$P_{T/p}$</th>
<th>MODE OF FAILURE</th>
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<td>254</td>
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<td>22.1</td>
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<td>400</td>
<td>357</td>
<td>361</td>
<td>0.989</td>
<td>y</td>
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<tr>
<td>S3-60</td>
<td>254</td>
<td>114.3</td>
<td>22.7</td>
<td>2.300</td>
<td>400</td>
<td>365</td>
<td>314</td>
<td>1.162</td>
<td>c</td>
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<tr>
<td>S4-60</td>
<td>254</td>
<td>114.3</td>
<td>23.9</td>
<td>3.450</td>
<td>400</td>
<td>334</td>
<td>250</td>
<td>1.336</td>
<td>c</td>
</tr>
<tr>
<td>S3-70</td>
<td>254</td>
<td>114.3</td>
<td>25.4</td>
<td>2.300</td>
<td>484</td>
<td>379</td>
<td>362</td>
<td>1.047</td>
<td>c</td>
</tr>
<tr>
<td>S4-70</td>
<td>254</td>
<td>114.3</td>
<td>35.2</td>
<td>3.450</td>
<td>484</td>
<td>375</td>
<td>290</td>
<td>1.293</td>
<td>c</td>
</tr>
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<td>254</td>
<td>114.3</td>
<td>20.5</td>
<td>3.450</td>
<td>484</td>
<td>312</td>
<td>238</td>
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slab size = 1829 mm
span = 1778 mm

### TABLE 5.6 SLABS TESTED BY MOE - ECCENTRIC LOADING

<table>
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<tr>
<th>SLAB NO.</th>
<th>c (mm)</th>
<th>$e/L$</th>
<th>$f'_c$ (N/mm²)</th>
<th>$\rho$ (%)</th>
<th>$f_y$ (N/mm²)</th>
<th>$P_T$ (kN)</th>
<th>$P_p$ (kN)</th>
<th>$P_{T/p}$</th>
<th>MODE OF FAILURE</th>
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<td>15.5</td>
<td>1.500</td>
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<td>185</td>
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<td>305</td>
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<td>482</td>
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<td>139</td>
<td>1.036</td>
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<td>25.8</td>
<td>1.500</td>
<td>482</td>
<td>293</td>
<td>269</td>
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<td>482</td>
<td>208</td>
<td>196</td>
<td>1.061</td>
<td>s</td>
</tr>
<tr>
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<td>26.5</td>
<td>1.340</td>
<td>328</td>
<td>240</td>
<td>205</td>
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<td>M7</td>
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<td>1.340</td>
<td>328</td>
<td>312</td>
<td>272</td>
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<td>M8</td>
<td>254</td>
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<td>328</td>
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<td>218</td>
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<td>178</td>
<td>142</td>
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slab size = 1829 mm
span = 1778 mm
d = 114.3
<table>
<thead>
<tr>
<th>SLAB NO.</th>
<th>c (mm)</th>
<th>d (mm)</th>
<th>U (N/mm²)</th>
<th>ρ (%)</th>
<th>f_y (N/mm²)</th>
<th>P_T (kN)</th>
<th>P_P (kN)</th>
<th>P_I/P_p</th>
<th>MODE OF FAILURE</th>
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<td>s</td>
</tr>
<tr>
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<td>76</td>
<td>63.5</td>
<td>30.8</td>
<td>1.570</td>
<td>377</td>
<td>92.9</td>
<td>81.6</td>
<td>1.138</td>
<td>s</td>
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<tr>
<td>2S4</td>
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<td>29.0</td>
<td>1.570</td>
<td>377</td>
<td>87.4</td>
<td>94.1</td>
<td>0.929</td>
<td>s</td>
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<tr>
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<td>63.5</td>
<td>27.6</td>
<td>1.570</td>
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<tr>
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<td>23.0</td>
<td>1.570</td>
<td>377</td>
<td>98.4</td>
<td>109.1</td>
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<td>s</td>
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<td>3S2</td>
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<td>63.5</td>
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<td>377</td>
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<td>76.7</td>
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<td>s</td>
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<td>28.3</td>
<td>3.140</td>
<td>377</td>
<td>117.4</td>
<td>110.6</td>
<td>1.061</td>
<td>s</td>
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<tr>
<td>3S6</td>
<td>152</td>
<td>63.5</td>
<td>27.1</td>
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<td>377</td>
<td>152.8</td>
<td>140.9</td>
<td>1.084</td>
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slab size = 889 mm
span = 864 mm

TABLE 5.7 SLABS TESTED BY TAYLOR AND HAYES
<table>
<thead>
<tr>
<th>SLAB NO.</th>
<th>c mm</th>
<th>d mm</th>
<th>$U$ N/mm²</th>
<th>$\rho$ %</th>
<th>$f_y$ N/mm²</th>
<th>$P_T$ kN</th>
<th>$\frac{P}{P_T}$ kN</th>
<th>$\frac{P}{P_T}$</th>
<th>MODE OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>30.0</td>
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<td>1.200</td>
<td>425</td>
<td>32.0</td>
<td>26.0</td>
<td>1.231</td>
<td>s</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>30.0</td>
<td>38.4</td>
<td>1.200</td>
<td>425</td>
<td>33.0</td>
<td>26.0</td>
<td>1.269</td>
<td>s</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>60.0</td>
<td>34.1</td>
<td>1.200</td>
<td>425</td>
<td>78.0</td>
<td>65.4</td>
<td>1.193</td>
<td>s</td>
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<td>4</td>
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slab diameter = 475 mm
span = 425 mm

**TABLE 5.8** SLABS TESTED BY DRAGOSAVIĆ AND VAN DEN BEJKEL
### Table 5.9 Slabs Tested by Criswell

<table>
<thead>
<tr>
<th>SLAB NO.</th>
<th>c (mm)</th>
<th>d (mm)</th>
<th>$f'_{c}$ (N/mm²)</th>
<th>$p$ (%)</th>
<th>$f_{y}$ (N/mm²)</th>
<th>$P_{T}$ (kN)</th>
<th>$P_{p}$ (kN)</th>
<th>$P_{T}/P_{p}$</th>
<th>MODE OF FAILURE</th>
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<td>291</td>
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Slab size: S2075 - S2150 = 2134 mm
Span = 2032 mm

Slab size: S4075 - S4150 = 2388 mm
Span = 2286 mm

### Table 5.10 Slabs Tested by Regan

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<th>$U$ (N/mm²)</th>
<th>$p$ (%)</th>
<th>$f_{y}$ (N/mm²)</th>
<th>$P_{T}$ (kN)</th>
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Slab size = 2000 mm
Span = 1829 mm
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* yield line prediction

TABLE 5.11 SLABS TESTED BY THE AUTHOR
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<th>COEFFICIENT OF VARIATION</th>
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**TABLE 5.12** CORRELATION OF VARIOUS METHODS WITH AUTHOR'S TEST RESULTS

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**TABLE 5.13** CORRELATION OF VARIOUS METHODS WITH RESULTS OF ISOTROPICALLY REINFORCED SLABS HAVING SQUARE COLUMNS
Fig. 5.1 PLATE MODELS USED BY LONG (1967)
Fig. 5.2 ANALYTICAL MODEL USED BY MASTERSO (1971)
Fig. 53  RELATIONSHIP OF APPLIED LOAD TO INTERNAL MOMENT

\[ M/P = 0.04 \log_{10}(L/c)^{1.5} \]

finite element analyses:
Cleland (1979)
Franklin (1981)

\[ M/P = 0.255 - 1.17 \frac{c}{L} \]
(Long, 1975)
Fig. 5-4 'VALLEY OF DIAGONAL FAILURE' (KANI, 1966)
Data of Shear Study Group (1969)

- ■ corbel tests
- ● beam tests

$\frac{v_u}{(p_u)^{1/3}} = \frac{2d}{a_s}$

Fig. 5.5 RELATIONSHIP OF ULTIMATE SHEAR STRESS TO SHEAR SPAN RATIO

Shear span / effective depth - $\frac{a_s}{d}$
Fig. 5.6 PUNCHING LOAD RELATED TO MOMENT OF RESISTANCE

Fig. 5.7 PUNCHING LOAD RELATED TO LOCAL FLEXURAL STRENGTH
Fig. 5-8 YIELD-LINE PATTERNS FOR CONVENTIONAL SPECIMENS
\[ t = 0.293 (s - c) \]

\[ r_s = \sqrt{\frac{s^2}{2} + t (t-s)} \]

\[ c = \frac{1}{2} \pi r_c \]

\[ k_{y1} = 8 \left[ \frac{s}{a-c} - 0.172 \right] \]

a) Equivalent square slab  
b) Equivalent square column

Fig. 5.9 EQUIVALENT YIELD-LINE SOLUTIONS

Fig. 5.10 DEFLECTION PROFILES OF ELASTIC PLATE MODELS
Fig. 5.11 ELASTIC - PLASTIC PLATE ANALYSIS
Fig. 5.12 GENERAL FORM OF DUCTILITY RELATIONSHIP

\[ \frac{\theta_y}{\theta_u} \approx \frac{\varepsilon_{cy}}{a_1} \frac{a_b}{\varepsilon_u} \]

Fig. 5.13 ASSUMED CRITERION FOR PARTIAL YIELDING

\[ \frac{r_y}{r_s - r_c} = 1 - \frac{M_b}{M_{(bal)}} \]
Reinforcement for neutral axis factor

\[ \tau_m = \frac{1.5V_c}{x} \]

parabolic distribution

triançulordistribution

Mohr circle

a) Compressive stress  
b) Shear stress

Fig. 5.14 ELASTIC DISTRIBUTION OF STRESS IN COMPRESSION ZONE

Neutral axis factor - \( x/d \)

- \[ f_c = 20 \text{ N/mm}^2 \]
- \[ x/d = 0.35 (100\rho)^{0.25} \]
- \[ f_c = 50 \text{ N/mm}^2 \]

Fig. 5.15 APPROXIMATE RELATIONSHIP FOR NEUTRAL AXIS FACTOR
MODES OF FAILURE IN CONVENTIONAL SPECIMENS

Fig. 5.16

- **ductile modes**
  - (i) overall yield
  - (ii) partial yield

- **brittle modes**
  - (iii) localised compression
  - (iv) shear

Fig. 5.17 PREDICTED INTERACTION OF PUNCHING MODES

![Diagram showing modes of failure in conventional specimens and predicted interaction of punching modes](image-url)
Fig. 5.18 MODIFIED MOMENT FACTOR FOR CONCENTRATED REINFORCEMENT

a) Grillage mesh for analysis of slab quadrant

b) Reduction of moment factor

1 - 0.333 \log_e (\frac{E1'}{E1})
Load kN

authors tests 
models 1-15 ($L/h = 31$)

---

Fig. 5.19 COMPARISON OF VARIOUS METHODS OF PREDICTION
Fig. 5.20 CORRELATION OF TEST AND PREDICTED FAILURE LOADS: CONVENTIONAL SLAB SPECIMENS
Chapter 6

A THEORY FOR ARCHING ACTION IN SLAB STRIPS

6.1 INTRODUCTION

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6.5 CONCLUSIONS
6.1 INTRODUCTION

The behaviour and ultimate capacity of continuous slab systems is considerably enhanced by the development of compressive membrane action. In order to promote an understanding of this phenomenon, an examination of the nature of 'arching action' in one-way strips is worthwhile.

A method of analysis is developed for the prediction of the ultimate load capacity of laterally restrained slab strips with a variety of boundary conditions and loading arrangements. For this, an existing deformation theory is utilised in conjunction with an idealised criterion for material behaviour. The simple analogy proposed takes account of the degree of lateral restraint and the interaction of bending and arching action is examined.

The proposed procedure is compatible with the present rationale of flexural analysis, and is direct in that the ultimate load can be found without recourse to the prediction of deflections.

Although based on several simplifying assumptions, the method of predicting the ultimate capacity has been validated by a comparison with a wide range of test results from various sources. It has also been found to yield significantly better correlation than other more complex methods of analysis.

6.2 TREATMENT OF THE ARCHING PHENOMENON

6.2.1 Analytical background

Although first recognition of the phenomenon of arching in reinforced concrete was in the early beginnings of monolithic construction, the subject has remained esoteric to the majority of practising engineers. This lack of acknowledgement is perhaps due to an unwillingness to depart from the
general principles of elastic analysis, which are after all, widely applicable. In addition, with the return to ultimate load design procedures, much of the experimental research has in fact purposefully eliminated any possibility of arching effects, in order to validate the plastic methods of design. In a recent review, Braestrup (1980) has traced the historical significance and treatment of the arching phenomenon.

The theoretical treatment of arching action in strips has generally been based on rigid-plastic or elastic-plastic idealisations, and the consideration of compressive membrane action in slabs has stemmed from this two-dimensional approach. The complexity of the problem is such however, that much research is necessary before one particular method will be generally accepted as a rational addition to flexural analysis. It is apparent in fact, that many treatments concentrate on the 'exact' mathematical aspect to the problem, as opposed to producing a simple model of the physical reality. In this respect, empirical corrections have frequently been introduced, particularly in the case of rigid-plastic analysis, in order to produce realistic results. Furthermore, many researchers have attempted to predict the deformation behaviour of laterally restrained strips. This approach leads to difficulties of ensuring compatibility in the case of two-way slab analysis. For the purposes of ultimate load analysis however, an accurate knowledge of deflection is not of necessity.

In view of the present 'state-of-the-art' it was considered desirable to develop a simple method of analysis for laterally restrained reinforced concrete strips. Of primary importance was the satisfactory prediction of the ultimate capacity, deformation being of less significance. In the following sections, such a method is presented and is shown to give good correlation with a wide range of test results.
6.2.2 The nature of arching action

The phenomenon of arching action in reinforced concrete occurs as a result of the great difference between the tensile and compressive strength of the material. In principle, cracking of the concrete causes a migration of the neutral axis and this is accompanied by increased rotational and lateral deformations. It is this lateral expansion, which if restrained, will significantly enhance the load carrying capacity of a member. In effect, an internal flat arch is generated, which transfers loading to the supports by direct compression. An idealised illustration of this behaviour is shown in Fig. 6.1.

This action is somewhat analogous to that of a three-hinged arch. The non-linear load-deformation response of such an arch occurs as a result of the changing geometry of the system, as illustrated in Fig. 6.2. Up to a certain critical deflection, any increment in load is accompanied by a reduction of the internal lever arm, however, this is offset by greater horizontal reactions and hence stable equilibrium is possible. Beyond a deflection of approximately 40% of the original height, however, the system becomes unstable and a sudden collapse will occur. This is termed 'snap-through' and is similar in nature to the mode of failure observed from tests on laterally restrained concrete strips. In this case however, the complexity of the problem is greatly increased by the non-linear material properties and the presence of flexural reinforcement at the articulations.

Fortunately, the deformation behaviour of reinforced concrete can be simplified to a certain extent, principally because of the large reduction of stiffness which accompanies flexural cracking. In this respect, rigid body motion is assumed for the purposes of plastic analysis and even for an elastic-plastic material, a similar approach is possible.
6.3 METHOD OF ANALYSIS

6.3.1 Idealised mode of action

The geometry of deformation of a laterally restrained, unreinforced strip, has been considered in detail by McDowell, McKee and Sevin (1956), in relation to the arching behaviour of masonry walls. This idealised two dimensional mode of action is illustrated in Fig. 6.3. By considering the distribution of material shortening at each contact zone, and assuming a linear reduction in strain along parallel fibres to the tensile cracks, an expression for strain along the contact areas was derived. Thus, from a knowledge of the material stress-strain relationship, which was assumed to be of the classical elastic-plastic form, the arching moment of resistance was found in terms of the midspan deflection. This resulted in curves of the form shown in Fig. 6.4 which are unique for any given set of geometric and material parameters.

The physical significance of this theory, and the constitutive similarity of masonry and concrete, provide a sound basis for application of the method to reinforced concrete strips. For the purpose of calculating the ultimate strength however, the maximum arching capacity is of primary interest. Thus, a direct method of predicting this was developed from the general load-deflection theory of McDowell et al.

Firstly, however, some validation of McDowell’s principal assumption of a linear variation of compressive strain was necessary. This was examined by means of a plane stress finite element analysis of a typically proportioned strip. A Highway Engineering Computer Branch programme (HECB/B/3) was utilised for the analysis of the element mesh shown in Fig. 6.5(a). The load application was such as to produce end displacements of a similar nature to those of the deformed strip, as shown in Fig. 6.5(b). It can be seen from Figs. 6.5(c) and (d) that the stress distributions along various fibres are indeed approximately linear. In addition, in Figs. 6.5(e) and (f)
the theoretical strain distributions at various sections are shown in comparison with the assumed distributions. It can be seen that the average strains are equal and that the centroids of the distributions are at the same position, giving the same line of thrust. The decrease of strain along the outer fibres is however more rapid in the simple theoretical model of Fig. 6.5(e). This means the assumption of a linear variation is conservative in that higher strains will occur at smaller deformations and thus, due to an increased lever arm, the arching component will be greater than that predicted.

6.3.2 Arching capacity

In terms of deformation, the stress distribution along the contact face takes several distinct forms, depending upon whether strain is increasing or decreasing, and if the material is elastic or has reached a plastic state. These distributions have been considered by McDowell et al over the full range of deformation and material behaviour, however only two possibilities are of consequence for the maximum arching resistance. The analytical forms for these have been described in terms of the non-dimensional parameters $R$ and $u$ defined by:

$$R = \frac{e}{c} \left( \frac{r}{L} \right)$$  \hspace{1cm} 6.1

$$u = \frac{w}{2d_1}$$  \hspace{1cm} 6.2

The parameter $R$ incorporates the material plastic strain and the geometry of the strip. It can therefore be considered to be a measure of the influence of elastic deformation on the system. The midspan deflection is simply expressed as the dimensionless proportion $u$ of the depth of section.
The lateral thrust and arching moment were defined in terms of these parameters by the analytical expressions presented in Table 6.1.

<table>
<thead>
<tr>
<th>range of $R$</th>
<th>range of $u$</th>
<th>stress distribution</th>
<th>$F_r = \frac{4F(u)}{s_c d_f}$</th>
<th>$M_r = \frac{4M(u)}{s_c d_f^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R \geq 0.5$</td>
<td>$u \geq 0$</td>
<td><img src="image" alt="Triangle" /></td>
<td>$2u(1-u)^2$</td>
<td>$8u(1-5u)(1-u)^2$</td>
</tr>
<tr>
<td>$R &lt; 0.5$</td>
<td>$0 \leq u \leq 1-\sqrt{1-2R}$</td>
<td><img src="image" alt="Square" /></td>
<td>$4\left(1-\frac{u}{2} - \frac{R}{2u}\right)$</td>
<td>$4\left(1+\frac{R}{2} + \frac{3u^2}{4} - 2u - \frac{R^2}{3u^2}\right)$</td>
</tr>
<tr>
<td>$R &lt; 0.5$</td>
<td>$1-\sqrt{1-2R} \leq u &lt; \sqrt{2R}$</td>
<td><img src="image" alt="Rectangle" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 6.1 ANALYTICAL FORMS FOR LATERAL THRUST AND ARCHING MOMENT**
(REF. McDOWELL et al, 1956)

The depth of section in contact with the support was given by consideration of the geometry of deformation and expressed as a proportion of the half depth by:

$$\text{contact depth} = \alpha d_f$$

where $\alpha = 1 - \frac{u}{2}$ \hspace{40mm} (6.3)

To formulate a direct method for predicting the maximum arching moment of resistance, the expressions for moment ratio $M_r$ were differentiated with respect to the deformation $u$. The derivative of the first expression forms the quadratic:

$$20u^2 - 32u + 8 = 0$$

with a root of:

$$u = 0.31$$
The second equation for moment ratio becomes:

\[ 6u - 8 + \frac{8R^2}{3u^3} = 0 \]

which is dependant upon \( R \). This equation was therefore solved numerically for \( u \), at discrete values of \( R \) in the range 0 to 0.26, above which the preceding solution is applicable.

The resultant \( u \) values were then back substituted into the original expressions for moment ratio, giving the maximum \( M_r \) for each value of \( R \). A relationship between these coefficients was established by means of a desktop computer and general library programme (Hewlett Packard 9845/REGD) which utilised a 'least squares' method to fit a polynomial curve. Although not required for the direct application of the arching resistance equations, a relationship for the critical deflection parameter \( u \) was also derived by the same means. This can be used to determine the magnitude of lateral thrust by substitution into the expressions for thrust ratio \( F_r \) from Table 6.1.

The form of these relationships is shown in Figs. 6.6 and 6.7, and are expressed analytically as:

(i) \( R > 0.26 \)

\[
M_r = \frac{0.3615}{R} \quad 6.4a \\
u = 0.31 \quad 6.4b
\]

(ii) \( 0 < R < 0.26 \)

\[
M_r = 4.3 - 16.1\sqrt{(3.3 \times 10^{-4} + 0.1243R)} \quad 6.5a \\
u = -0.15 + 0.36\sqrt{0.18 + 5.6R} \quad 6.5b
\]

From the first solution it can be seen that for high values of \( R \), \((R > 0.26)\), the maximum arching resistance occurs at a critical deflection of 0.31 times the overall depth, and this corresponds to a purely snap-
through effect with an elastic stress distribution. That is, failure occurs at a deflection of 0.423 times the height of arch, where this height is given by the vertical distance between the centroids of the triangular stress distributions. Thus, the height of arch is given by:

\[ h_a = 2d_1 - \frac{2}{3} ad_1 \]

but \( u = 0.31 \) and therefore \( a = 0.845 \)

hence the critical deflection is equal to:

\[ 0.423h_a = u \times 2d_1 \]

For lower values of \( R \), \( (R < 0.26) \), plasticity of the material at the contact area is predicted, and failure will occur more suddenly by instability. In the extreme case of \( R = 0 \), that is for a rigid-plastic material, the maximum arching capacity is for zero midspan deflection and thus any deformation is accompanied by a considerable reduction in resistance.

The maximum arching capacity, critical deflection and lateral thrust can thus be found directly from the analytical expressions derived.

6.3.3 Material properties

In the analysis by McDowell et al, elastic-plastic behaviour of the material was assumed, and thus for application of the maximum arching equations to concrete strips, it was necessary to formulate such an idealised relationship.

The most notable work on the stress distribution in unconfined concrete of a flexural member is that of Hognestad et al (1955), at the Portland Cement Association. In this, the properties of the ultimate stress distribution in a rectangular section were defined in terms of three
parameters, \( k_1, k_2 \) and \( k_3 \). These determined the magnitude of the compressive force and the centroid of the distribution, as illustrated in Fig. 6.8(a). In accordance with the proposal of Whitney (1937) for the use of an equivalent rectangular stress distribution of magnitude equal to 85% of the cylinder crushing strength, these coefficients were further examined by Mattock et al (1961), and a general design theory was developed. The coefficients of this equivalent rectangular stress block are shown in Fig. 6.8(b). This constitutes the present ACI (318-77) method for ultimate flexural strength and has become accepted virtually worldwide. As this approach has been found to be perfectly satisfactory for predicting the normal flexural capacity, a compatible procedure was considered desirable for the analysis of arching.

The magnitude of the compressive force resulting from the ultimate stress distribution shown in Fig. 6.8(a) is given by:

\[
C_b = k_1 k_2 f' c_1
\]

An equivalent force exerted by the rectangular stress block is thus:

\[
C_b = 0.85 f' a_1
\]

where the ratio of base lengths is equal to

\[
\frac{a_1}{c_1} = \beta_1 = \frac{k_1 k_3}{0.85}
\]

The ratio adopted by the ACI was taken to be equal to 0.85 for concrete cylinder strength up to 27.6 N/mm\(^2\) (4000 psi) and thereafter reduced by 0.05 for each 6.9 N/mm\(^2\) (1000 psi) in excess of this. The corresponding relationship for \( k_1 k_3 \) is shown in comparison with the test results from PCA (1955) and those of Rüsch (1955) in Fig. 6.10. It would seem in fact, that a linear relationship would be satisfactory over the full range of concrete strengths, and this is given by:

\[
k_1 k_3 = 0.89 - 0.0062 f'_c
\]
The ultimate stress distribution caused by a linear variation of compressive strain was considered in terms the stress-strain relationship, as shown in Fig. 6.9. Thus, the base length of the curve is given by the magnitude of the ultimate compressive strain, which after Hognestad et al (1955) is:

\[ \varepsilon_u = (4000 - 22.3f'_C) \times 10^{-6} \]

An equivalent elastic-plastic stress-strain relationship can be defined by three coefficients, namely:

(i) the maximum compressive stress \( s_C \)

(ii) the maximum compressive strain \( \varepsilon_u \)

(iii) the magnitude of plastic strain \( \varepsilon_c \)

Thus, for equivalent ultimate stress distributions, the magnitude of plastic strain is given by:

\[ \varepsilon_c = 2\varepsilon_u(1 - \beta_1) \]

From this, the relationship between plastic strain and concrete cylinder strength up to 70 N/mm\(^2\) can be derived as:

\[ \varepsilon_c = (-400 + 60f'_C - 0.33f'_C^2) \times 10^{-6} \]

The form of this relationship is shown in Fig. 6.11 and it can be seen that rigid-plastic behaviour is predicted at a concrete cylinder strength of 6.9 N/mm\(^2\) (1000 psi). The position of the centroid of the trapezoidal stress distribution defines the line of action of the compressive force. This has been evaluated over the range of concrete strengths and is shown in respect of the test results in Fig. 6.10. It can be seen that a good correlation results.

The expression for plastic strain is essential for the application of the preceding arching resistance equations.
6.3.4 Degree of lateral restraint

In the event of less than rigid restraint against lateral expansion, the arching capacity of a section will be reduced. The complexity of analysis of a strip having a finite degree of restraint is, however, greatly increased. Many researchers have ignored the issue and yet others have recourse to complicated theoretical treatments. Thus a convenient method of assessing the influence of the degree of restraint was desirable.

A simple analogy to the problem is that of a three-hinged arch with linear elastic 'spring' restraints, as illustrated in Fig. 6.12(a). The general equilibrium equation for this system has been derived following the method outlined by Lind and Puranik (1966). This solution is presented in Appendix B and yields:

\[ P = \frac{h^*EA(h_a^2 - h_a^2)}{L^3\left(\frac{EA}{KL_e} + 1\right)} \]

In the case of a rigidly restrained arch, the spring stiffness \( K \) is infinite and this equation reduces to:

\[ P = \frac{h^*EA(h_a^2 - h_a^2)}{L^3} \]

It can therefore be shown that by utilising an increased length of rigidly restrained arch, the same load-deformation response will result. This length is given by the relationship:

\[ L_r = L_e \sqrt[4]{\frac{EA}{h^*KL_e} + 1} \]

From the conditions of static equilibrium of the rigidly restrained arch shown in Fig. 6.12(b) the relationship between applied load and internal moment is given by:

\[ P = \frac{2M}{L_r} \]
Thus, under equal loading, the ratio of internal moments in the equivalent systems is given by:

\[
\frac{M_{es}}{M_{rs}} = \frac{L_e}{L_r}
\]

6.8

The load can therefore be expressed in terms of the internal moment of the rigidly restrained arch as:

\[
P = \frac{2}{L_e} M_{rs} \frac{L_e}{L_r}
\]

or \[P = k_a M_{es}\]

6.9

In this, the coefficient \(k_a\) relates the applied load to internal moment in the 'real' system.

The load capacity of a strip with any degree of lateral restraint may therefore be calculated on the basis of an 'affine' strip having rigid restraints. To determine the length of this 'affine' strip, the relative magnitude of the arch leg axial stiffness \(\frac{E_c A}{L_e}\), to that of the restraint \(K\), is required. This is somewhat difficult to define precisely, however an approximate method which is consistent with the idealised mode of action may be used.

In determining the strain distribution along the contact areas, it was assumed that a linear variation of strain existed along parallel fibres, as illustrated in Fig. 6.13(a). At any section therefore, the average strain above or below the neutral axis can be determined from the appropriate triangular distribution. This average strain also decreases linearly towards the tensile crack, however the total average strain remains constant, and is effective over a section equal to that of the contact area as shown in Fig. 6.13(b). Thus, the arch leg can be considered to be of this cross-sectional area for the purpose of calculating the axial stiffness.
The length of the 'affine' strip and the contact area are however, both dependant upon the degree of restraint. Thus, an iterative procedure is necessary for the solution. This involves the initial assumption of an effective area of arch leg, from which the length $L_r$ can be computed. The central deflection of this rigidly restrained strip can be found, and thus a new estimate of the contact area will result. This cycle can be repeated until a satisfactory degree of convergence is attained, and generally results in a difference in area of less than 1% after two or three iterations.

Alternatively an approximation to the effective area of arch leg can be taken as one half of the depth of section available for arching ($d_1$). The maximum error which could arise from this assumption is for the minimum possible contact area ($\alpha = 0.845$). This would result in an underestimation of the arching capacity by only 18% for very low degrees of restraint, and the magnitude of this inaccuracy decreases as the level of restraint increases. The variation of arching resistance with the degree of lateral restraint is shown in Fig. 6.14 for both the theoretical and approximate effective areas. The error bounds between the theoretical and approximate methods are shown in Fig. 6.15, from which it can be seen that for modest levels of restraint (nominal relative stiffness $> 2$), the difference will be insignificant.

An appropriate value of the elastic modulus of concrete ($E_c$) is also required to determine the relative stiffness coefficient $E_c A / KL$. As the three-hinged arch analogy is concerned with relative elastic strains, the initial elastic modulus may be utilised. The behaviour of members subject to bending and axial load has been studied by Hognestad (1951) and an expression for the initial elastic modulus given as

$$E_c = 4730\sqrt{f_c} \text{ (N/mm}^2)$$ 6.10

This widely used relationship is applicable in the present context.
6.3.5 Bending and arching action

To determine the ultimate strength of a laterally restrained reinforced concrete strip, an interaction criterion for bending and arching action is necessary. This should satisfy stress equilibrium and strain compatibility within the section and with the boundary conditions. To facilitate practical application, a relatively simple approach has been adopted in which the stress components due to bending and arching are found separately and superimposed. In reality however, the compressed area of concrete consists of a single compression zone in which stresses due to bending and arching cannot be segregated.

In the absence of a rigorous mathematical treatment, the following simplifying assumptions have been introduced:

a) the maximum arching component is induced as a result of the deformations associated with yielding of the reinforcement.

b) strain compatibility within the section can be neglected, however, stress equilibrium is not violated.

The justification of assuming yield occurs is based on the principal that arch action is a secondary phenomenon which enhances the normal flexural behaviour of slabs. This is a common supposition by researchers and is valid providing premature shear or crushing failure does not occur. The second simplification is reasonable in view of the fact that provided the stress conditions are satisfied, the compatibility of strains will have little influence on the prediction of ultimate strength.

The idealised behaviour of a laterally restrained strip is illustrated in Fig. 6.16. It can be seen that by relating the bending and arching resistances to the applied loading, the ultimate capacity can be predicted. This is done by means of the internal moment coefficients as shown in Fig. 6.17.
To determine the magnitude of the arching component, the following approach has been suggested by Christiansen (1963). From a knowledge of the depth of compressed concrete required to resist the tensile force in the reinforcement and the elastic deformation necessary to cause yield, the depth of section remaining available for arching can be found. The stress conditions corresponding to this idealisation are illustrated in Fig. 6.18.

On consideration of this method of analysis, it is apparent that if the deformation and load necessary to cause yielding of the reinforcement can be found accurately, then a lower bound solution will result. That is, the minimum arching component will be included in the total strength.

In view of the difficulties involved with the accurate prediction of bending deformations, and the fact that the coupling of bending and geometric stiffness has been neglected, an alternative approach is to ignore this deflection entirely. This can be considered as an upper bound method involving the maximum arching component. The corresponding stress conditions are shown in Fig. 6.19.

These idealised solutions are illustrated in respect of the true behaviour in Fig. 6.20. In fact, because of the simplifying assumptions upon which the proposed arching theory is based, most of which tend towards a conservative method, the upper bound approach has been found to give satisfactory correlation with experimental results. In addition, this method stands to be of much more convenience for the incorporation of compressive membrane action into the two-way behaviour of slabs, as in this instance, bending deformations would be exceedingly difficult to estimate with any degree of accuracy.

An important point of interest in this method is that for normal ratios of effective to total depth \(d/h = 0.8\), the maximum arching
resistance will not exceed that caused by over reinforcing. This is illustrated by considering an approximate value of the maximum flexural strength of a fully reinforced section to be given by the empirical expression proposed by Whitney (1937) as:

\[ M_{u(\text{max})} = 0.333 f'_c d^2 \]

From Fig. 6.21, the maximum possible arching moment of resistance is for a rigid-plastic material and is given by:

\[ M_{a(\text{max})} = 0.21 f'_c h^2 \]

for \( d = 0.8h \)

\[ M_{a(\text{max})} = 0.33 f'_c d^2 \]

Thus, for concrete obeying an elastic-plastic stress-strain criterion, the arching moment of resistance will be reduced by deformation and thus:

\[ M_a < M_{u(\text{max})} \]

This is consistent with the assumption that the maximum arching capacity is associated with yielding of the reinforcement, that is, for a fully reinforced section, no arching will be induced. Thus, as strain compatibility has been ignored, the ultimate strength in flexure is limited to the resistance at the balanced failure load. This ensures that the combination of bending and arching components will not cause unrealistic strain conditions in the section at ultimate. The enhanced strength of a laterally restrained strip will therefore comply with the limitations of flexural capacity as illustrated in Fig. 6.22.
6.3.6 Direct procedure

The procedure for calculating the theoretical ultimate capacity of a laterally restrained strip is based on the upperbound approach and is outlined by the following flow chart:

An example of the direct procedure applied to a strip tested by Leibenberg (1966) is presented in Appendix B. The method of analysis is described in detail by the following steps, which refer to a unit width of strip.
1. **BENDING CAPACITY**

\[ M_b = \rho f_y d^2 (1 - 0.59 \frac{\rho f_y}{f_c}) \]  
\[ P_b = k_b M_b \]  

2. **ARCHING SECTION**

\[ 2d_1 = h - (\rho + \emptyset) \frac{f_y d}{0.85 f_c} \]  

3. **AFFINE STRIP**

assume \( A = d_1 \)

\[ E_c = 4.73 \sqrt{f_c} \text{ kN/mm}^2 \]  
\[ L_r = L_c \sqrt{\frac{E_c}{K L_0} + 1} \]  

4. **ARCHING PARAMETERS**

\[ \varepsilon_c = (-400 + 60 f' - 0.33 f'^2) \times 10^{-6} \]  
\[ R = \frac{\varepsilon_c L_r^2}{4d_1^2} \]  

5. **DEFORMATION**

\[ R > 0.26 : \ u = 0.31 \]  
\[ 0 < R < 0.26 : \ u = -0.15 + 0.36 \sqrt{0.18 + 5.6R} \]  

6. **CONTACT DEPTH**

\[ \alpha = 1 - \frac{u}{c} \]  
contact depth = \( \alpha d_1 \)

For theoretical refinement, assume \( A = \alpha d_1 \) and repeat from (3)

7. **ARCHING CAPACITY**

\[ R > 0.26 : \ M_r = \frac{0.3615}{R} \]  
\[ 0 < R < 0.26 : \ M_r = 4.3 - 16.1 \sqrt{(3.3 \times 10^{-4} + 0.1243R)} \]  
\[ M_a = 0.21 f' c_1 2M_r \frac{L_e}{L_r} \]  
\[ P_a = k_a M_a \]  

8. **ULTIMATE CAPACITY**

\[ P_p = P_a + P_b + \frac{1}{4} k_b M (ba 1) \]
6.4 COMPARISON WITH EXPERIMENTAL RESULTS

6.4.1 Tests reported in the literature

In the following sections, details of tests conducted on laterally restrained slab strips from various sources are presented. A compendium of the test arrangements is shown in Fig. 6.23. The ultimate strength of each specimen has been compared with that predicted by the simplified arching theory neglecting bending deformations. A comparison with the predictions resulting from the inclusion of bending deformations is later shown. In addition, an assessment of the merits of the proposed procedure is made in respect of other methods of analysis. The ultimate deflections have not been compared as the method is not primarily concerned with predicting these.

a) Tests by Christiansen

Some of the earliest tests on artificially restrained slab strips were carried out by Christiansen (1963). Four beams were tested, in which the boundary conditions were such as to induce 'fixed end' moments, with a concentrated loading at midspan. Details of these tests and the predicted ultimate strengths are presented in Table 6.2.

b) Tests by Leibenberg

To study the behaviour of laterally restrained slabs, Leibenberg (1966) carried out many such tests on one-way strips. Unfortunately however, only details of 18 of these tests have been reported.

A midspan loading system in conjunction with a 'fixed end' moment arrangement was utilised, and the lateral expansion was carefully controlled by means of an hydraulic jack. In fact, relatively low degrees of restraint were applied and these results are therefore a particularly good test of the three-hinged arch analogy. In addition the strength of concrete was as low as 10 N/mm², which is the most extreme case of all the test
results analysed. The specimen details and predicted ultimate strengths are presented in Table 6.3.

c) Tests by Roberts

One of the most comprehensive series of tests on laterally restrained strips was that of Roberts (1969). In all, 36 such strips were tested, and for each variable at least two identical tests were carried out.

The loading was applied by means of a four point system giving a midspan moment equivalent to that from a uniformly distributed load. The end conditions were such as to simulate a simply supported restrained strip and thus the ratio of arching to bending resistance was relatively high. Although the conditions of equilibrium can be defined, the method of loading could have considerably influenced the mode of deformation, as the maximum flexural rotation would have been induced directly beneath the two innermost load points. In the analysis therefore, a reduced arching span equal to \( \frac{4}{3} \) of the clear span was utilised for the purpose of calculating the arching moment of resistance.

Details of these tests are presented in Table 6.4, in which the average results are compared with the predicted failure loads.

d) Tests by Birke

Two series of tests on small scale laterally restrained strips have been reported by Birke (1975). In this investigation, a wide range of variables was examined, but also of importance was the use of a model concrete.

The results from the first series of tests are presented in Table 6.5. These are the average from at least two similar specimens. In the case of the beam only 10 mm in depth, the result is the average from 10 individual tests. All the strips of this series were unreinforced.
The second series of tests utilised a larger specimen size and included both reinforced and unreinforced strips. An additional parameter in these tests was the influence of the degree of lateral restraint. The results of these tests are presented in Table 6.6 in which the details are given for individual specimens.

As a result of the large scale factor involved with these tests, Birke also examined the strength of small scale concentrically loaded prisms. This was found to be, on average, 20% greater than the corresponding standard (150 mm) cube strength, and on this basis, Birke assumed a higher prism strength for his theoretical predictions. In view of this, the cylinder compressive strength was taken to be 1.2 times the cube strength for the purpose of this analysis. A comparison of the predicted and experimental failure loads is shown with the specimen details.

e) Tests by Chattopadhyay

In a recent series of tests, Chattopadhyay (1981) has examined the influence of the level of reinforcement and span/depth ratio.

Concentrated loading was applied at midspan and the end conditions simulated those of a continuous structure with limited rotational and translational stiffness. In fact, the rotation of the end restraint proved to be negligible, and on this basis the degree of lateral restraint has been calculated as the axial stiffness of the four steel rods utilised.

Of particular interest in these results are the tests on unreinforced strips, and those having large span/depth ratios. Details of these tests and the predicted ultimate strengths are presented in Table 6.7.
f) Tests by McCann and Close

During the course of this research, a supplementary series of tests on laterally restrained strips was carried out by McCann (1981) and Close (1982). These strips were of a similar nature to the two-way slabs tested by the author.

A concentrated midspan loading system, in conjunction with a 'fixed end' moment arrangement was utilised, however in some cases the negative reinforcement was deliberately omitted to increase the proportion of load carried by arching. In addition to the material variables, the influence of the degree of restraint was also examined.

The results of these tests and the ultimate load predictions are given in Tables 6.8 and 6.9.

6.4.2 Predictions by other methods

Various methods have been proposed for calculating the enhanced strength of laterally restrained strips. Many of these are however, exceedingly complex, or are not direct in that the load-deflection response must be computed in order to extract the failure load. In addition, the effect of finite lateral restraint has often been neglected. The methods presented are those of Christiansen (1963) and a more recent version of that due to Park (1964). These are considered to be useful approaches and have formed the basis of many other studies. Details of the procedures are given in Appendix B.

The test results of Leibenberg (1966) and those of Roberts (1969) were considered to be the most comprehensive and appropriate for comparative purposes. These tests covered a wide range of variables and in addition the type of loading and end conditions differed, as described previously.
In the method proposed by Christiansen, the bending and arching capacities are combined give the total ultimate strength. The maximum arching capacity was obtained from the solution of the fourth order polynomial expression for plastic deformation as opposed to the less accurate graphical method presented by Christiansen. This was achieved by means of the general library computer programme (Hewlett-Packard-9845/S1LJAK).

In the case of Roberts' tests, the arching mechanism was assumed to flatten beneath the two innermost point loads to ensure a fair comparison with the proposed method. The tests of Leibenberg were unambiguous in that the loading was concentrated at midspan.

The original rigid-plastic strip theory proposed by Park (1964) required the assumption of an empirical critical deflection (taken as 0.5 h) in order to determine the ultimate capacity of a laterally restrained strip. More recently however, Park and Gamble (1980) have modified this approach to include the effect of axial straining and the degree of lateral restraint. This latter method can be used to compute the load-deflection response of the strip, and from this the ultimate load can be extracted.

A computer programme was written to evaluate the load for distinct increments of deflection (\( \frac{W}{h} = 0.01 \)), and the test results of Leibenberg and Roberts were analysed in this way. For Roberts' tests, the assumed mechanism of failure was that of two end 'hogging' moment hinges and two 'sagging' moment hinges beneath the innermost point loads. This resulted in the lowest plastic collapse load and is compatible with the previous assumption of the mode of arching in these tests. The mechanism for Leibenberg's strips, simply consisted of a 'hogging' moment hinge at each end, with a 'sagging' moment hinge at midspan.

A comparison of the predicted ultimate strengths by the methods of Christiansen and Park, with the test results of Leibenberg and Roberts is
shown in Table 6.10. In addition, the predictions resulting from the proposed approach including the elastic bending deformations are given. The deflections at yield were estimated by means of an effective cracked stiffness in conjunction with the standard formulae of elastic bending theory. The cracked stiffness was calculated from the conditions of strain compatibility at yield. It can be seen that in all cases the correlation is less satisfactory than that of the proposed procedure which neglects bending deformations. In addition, the alternative methods were found to be cumbersome to apply, in that the accurate solution of Christiansen's principal equation is difficult, and the computational effort required by the iterative procedure of Park is considerable.

The proposed method is adequately simple for hand calculation and yields a direct prediction of the ultimate capacity. Utilising the approximate effective area of arch leg and neglecting the elastic bending deformations will result in a good estimate of the ultimate strength with a considerable saving in effort.

6.4.3 Discussion

The overall correlation between the experimental and predicted ultimate loads by the proposed method, neglecting elastic bending deformations is shown in Fig. 6.24. For the 97 tests analysed, the mean value of experimental to predicted ultimate load is 1.225, with a coefficient of variation equal to 11.7%.

As the applied loadings were not uniformly distributed, the results reported in the literature would not appear to include the self weight of the strips. In fact, when an appropriate allowance was made for this on
the internal moments, the mean value of experimental to predicted failure load increased only slightly to 1.245, with the same coefficient of variation.

This correlation is very satisfactory in view of the wide variety of tests and the range of variables involved. It also means that an accurate but conservative estimate of the ultimate capacity of laterally restrained strips can be made.

A parameter search was carried out to determine if any of the major variables had a significant influence on the ratio of test to predicted failure load by the proposed method. Thus, the reinforcement ratio, span/depth ratio and the degree of restraint were examined independantly in respect of the predicted capacities. The best fit linear relationships for each of these parameters was found by means of the general library computer programme (Hewlett Packard-9845/REGD), and these are shown in Fig. 6.24. The study indicated that a slightly greater degree of conservatism can be associated with low levels of reinforcement. This would be expected as the degree of enhancement is reduced when the balanced reinforcement ratio is approached. In the limit, the strength is simply that of a fully reinforced section, which can be predicted with reasonable accuracy. From examination of the span/depth parameter, the tendency is again for less conservatism at higher slenderness ratios. However, values much higher than those normally found in practice would be necessary before unsafe predictions would result. This justifies the addition of the maximum arching component to the full bending resistance in all cases. No significant trend is apparent with the parameter of nominal relative stiffness, indicating that the 'three-hinged arch' analogy is a reliable but simple method of accounting for the degree of lateral restraint.
6.5 CONCLUSIONS

The following points of importance concerning the theory of arching action in slab strips can be concluded:

1. The assumption of a linear variation of strain along parallel fibres between the contact zone and the tensile crack has been verified. In reality, however, the compressive strains at the outer fibres will be greater than those calculated on this basis and hence, the predicted arching moment of resistance will be slightly conservative.

2. The stress-strain curve for the concrete can be idealised in the form of the elastic-plastic relationship. This defines the maximum compressive stress and the magnitude of the plastic strain in terms of the cylinder crushing strength.

3. The three-hinged arch analogy can be utilised to determine the arching load capacity of a strip with any degree of lateral restraint. In order to calculate the increased length of the 'affine' strip, the effective area of arch leg can be approximated as one half of the depth of section available for arching.

4. The elastic bending deformations can be neglected in the interaction criterion for bending and arching action. In order to ensure that this assumption does not result in unrealistic strain conditions, the ultimate flexural strength should be limited to the fully reinforced capacity.

5. The comparison with a wide range of test results from various sources, indicates that the theory will provide an accurate but conservative prediction of the ultimate capacity of a laterally restrained strip.

6. The direct procedure is simple to use and gives better correlation with test results than the more complex methods of other researchers.
<table>
<thead>
<tr>
<th>STRIP NO.</th>
<th>d (mm)</th>
<th>h (mm)</th>
<th>U (N/mm²)</th>
<th>ρ &amp; ̄ρ (%)</th>
<th>$f_y$ (N/mm²)</th>
<th>K (kN/mm²)</th>
<th>$P_T$ (kN)</th>
<th>$P_b$ (kN)</th>
<th>$P_p$ (kN)</th>
<th>$P_T$/$P_p$</th>
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<td>485</td>
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<td>8.36</td>
<td>10.27</td>
<td>1.098</td>
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<td>76.2</td>
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<td>88.9</td>
<td>28.3</td>
<td>0.523</td>
<td>485</td>
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$b = 152$ mm

strip 1 : span = 1829 mm

strips 2 - 4 : span = 1524 mm

**TABLE 6.2 STRIPS TESTED BY CHRISTIANSEN**
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<th>U (N/mm²)</th>
<th>μ &amp; ν (%)</th>
<th>f_y (N/mm²)</th>
<th>K (kN/mm²)</th>
<th>P_T (kN)</th>
<th>P_b (kN)</th>
<th>P_p (kN)</th>
<th>P_T/P_p</th>
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\[ b = 305 \text{ mm} \]

M1 - M8/M13 - M18 : span = 1829 mm

M9 - M10 : span = 2438 mm

M11 - M12 : span = 1219 mm

**TABLE 6.3 STRIPS TESTED BY LEIBENBERG**
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<th>h  mm</th>
<th>U N/mm²</th>
<th>p  %</th>
<th>$f_y$ N/mm²</th>
<th>kN/m²</th>
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<th>$P_b$ kN</th>
<th>$P_p$ kN</th>
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b = 229 mm
span = 1436 mm

TABLE 6.4 STRIPS TESTED BY ROBERTS.
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<th>f_y (N/mm²)</th>
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b = 50 mm
span = 200 mm

TABLE 6.5 STRIPS TESTED BY BIRKE (series 1)
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<th>d (mm)</th>
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<th>U (N/mm²)</th>
<th>$p%$</th>
<th>$f_y$ (N/mm²)</th>
<th>$K$ (kN/mm²)</th>
<th>$P_T$ (kN)</th>
<th>$P_b$ (kN)</th>
<th>$P_p$ (kN)</th>
<th>$P_T \over P_p$</th>
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<td>9.72</td>
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<td>25.08</td>
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<td>50.0</td>
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<td>5.02</td>
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<td>50</td>
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\( b = 150 \text{ mm} \)

\( \text{span} = 500 \text{ mm} \)

**TABLE 6.6 STRIPS TESTED BY BIRKE (series 2)**
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<th>d (mm)</th>
<th>h (mm)</th>
<th>U (N/mm²)</th>
<th>( \rho ) &amp; ( \bar{\rho} ) %</th>
<th>( f_Y ) N/mm²</th>
<th>( K ) kN/mm²</th>
<th>( P_T ) kN</th>
<th>( P_b ) kN</th>
<th>( P_p ) kN</th>
<th>( \frac{P_T}{P_p} )</th>
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<td>100</td>
<td>33.5</td>
<td>1.520</td>
<td>468</td>
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<td>98.0</td>
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<td>88.7</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
<td>40.0</td>
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<td>100</td>
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<td>468</td>
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<td>89.9</td>
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<td>1.520</td>
<td>468</td>
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<td>39.4</td>
<td>0.760</td>
<td>468</td>
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<td>35.3</td>
<td>22.6</td>
<td>29.4</td>
<td>1.201</td>
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<td>468</td>
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<td>28.3</td>
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<td>100</td>
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<td>0.760</td>
<td>468</td>
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<td>20.9</td>
<td>15.3</td>
<td>17.8</td>
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\( b = 250 \text{ mm} \)

1A - 1C : span = 1000 mm

2A - 2C : span = 2000 mm

3A - 3C : span = 3000 mm

TABLE 6.7 STRIPS TESTED BY CHATTOPADHYAY
<table>
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<th>STRIP NO.</th>
<th>d (mm)</th>
<th>h (mm)</th>
<th>U (N/mm²)</th>
<th>φ &amp; φ' (%)</th>
<th>f'y (N/mm²)</th>
<th>K (kN/mm²)</th>
<th>P_T (kN)</th>
<th>P_b (kN)</th>
<th>P_p (kN)</th>
<th>P_T/P_p</th>
</tr>
</thead>
<tbody>
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<td>39.1</td>
<td></td>
<td></td>
<td>1.40</td>
<td>5.9</td>
<td>-</td>
<td>4.7</td>
<td>1.255</td>
</tr>
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<td>A2</td>
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<td>51.0</td>
<td>40.7</td>
<td>0.833</td>
<td>530</td>
<td>-</td>
<td>8.1</td>
<td>6.7</td>
<td>6.7</td>
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<td>51.0</td>
<td>38.9</td>
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b = 87 mm
span = 640 mm

TABLE 6.8 STRIPS TESTED BY McCANN

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<th>d (mm)</th>
<th>h (mm)</th>
<th>U (N/mm²)</th>
<th>φ (%)</th>
<th>f'y (N/mm²)</th>
<th>K (kN/mm²)</th>
<th>P_T (kN)</th>
<th>P_b (kN)</th>
<th>P_p (kN)</th>
<th>P_T/P_p</th>
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b = 87 mm
span = 640 mm
A1 - D2 : φ = 0 ; D4 : φ = ρ

TABLE 6.9 STRIPS TESTED BY CLOSE
### Table 6.10 Comparison with Other Methods of Prediction

<table>
<thead>
<tr>
<th>STRIP NO.</th>
<th>P_T (kN)</th>
<th>P_T/P_PARK</th>
<th>P_T/P_C</th>
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P : proposed method neglecting elastic bending deformations (upper bound solution)

P* : proposed method including elastic bending deformations (lower bound solution)
Fig. 6.1 ARCHING ACTION

Fig. 6.2 LOAD CAPACITY OF A THREE-HINGED ARCH
Fig. 6.3  IDEALISED MODE OF ACTION

Fig. 6.4  ARCHING MOMENT OF RESISTANCE
a) finite element mesh

b) deformation of strip

c) strain distribution: A-A'

d) strain distribution: B-B'

e) theoretical strain distribution

f) assumed strain distribution

Fig. 6.5 STRAIN DISTRIBUTION IN DEFORMED STRIP
For $0 \leq R \leq 0.26$

$$M_r = 4.3 - 16.1 \sqrt{(3.3 \times 10^{-4} + 0.1243R)}$$

For $R \geq 0.26$

$$M_r = \frac{0.3615}{R}$$

**Fig. 6.6 VARIATION OF ARCHING MOMENT RATIO**

For $R \geq 0.26$: $u = 0.31$

$$u = -0.15 + 0.36 \sqrt{(0.18 + 5.6R)}$$

**Fig. 6.7 VARIATION OF CRITICAL DEFORMATION**
Fig. 68 ULTIMATE CONCRETE STRESS DISTRIBUTION

Fig. 69 IDEALISED STRESS-STRAIN RELATIONSHIP FOR CONCRETE
**Fig. 6.10** PROPERTIES OF ULTIMATE CONCRETE STRESS DISTRIBUTION

\[ k_1 k_3 = 0.89 - 0.0062 f'_c \]

\[ k_1 k_3 = k_1 k_3 f'_c c_1 \]

\[ k_2 c_1 = 0.85f'_c \]

\[ \epsilon_c = (1 - 0.4f'_c - 0.33f'_c^2) \times 10^{-6} \]

**Fig. 6.11** IDEALISED PLASTIC STRAIN RELATIONSHIP FOR CONCRETE
Fig. 6:12  THREE-HINGED ARCH ANALOGY

a) elastically restrained arch

b) equivalent rigidly restrained arch

Fig. 6:13  EFFECTIVE AREA OF ARCH LEG

a) assumed linear strain distribution

b) total average effective strain
Fig. 6.14 RELATIVE ARCHING RESISTANCE—DEGREE OF RESTRAINT

\[ \frac{M_a (\alpha = 0.845)}{M_a (\alpha = 1.0)} \]

\[ L_r = \sqrt{\frac{E_c \alpha d_1}{K L_e} + 1} \]

approximate method:
maximum contact area
\( \alpha = 1.0 \)

theoretical method:
minimum contact area
\( \alpha = 0.845 \)

Fig. 6.15 ERROR BOUNDS OF THEORETICAL / APPROXIMATE METHOD
Fig. 6.16 IDEALISED BEHAVIOUR OF LATERALLY RESTRAINED STRIPS

\[ P_p = P_a + P_b \]

- ultimate capacity
- yielding of reinforcement
- cracking of concrete

Deflection

Load

\[ P_a \]
\[ P_b \]

Fig. 6.17 ULTIMATE CAPACITY OF LATERALLY RESTRAINED STRIPS

- \( P_p = 4 \frac{M_a}{L} + 4 \frac{M_b}{L} \)  
  a) simply supported strip

- \( P_p = 4 \frac{M_a}{L} + 8 \frac{M_b}{L} \)  
  b) fixed-end strip
Fig. 6.18 STRESS CONDITIONS OF LOWER BOUND SOLUTION

Fig. 6.19 STRESS CONDITIONS OF UPPER BOUND SOLUTION
We IDEALISED SOLUT for laterally restrained strps.

Upper bound: $2d_1 = h - \frac{(p + \bar{p})\, f_y d}{0.85\, f_c'}$

Lower bound: $2d_1 = h - \frac{(p + \bar{p})\, f_y d}{0.85\, f_c'}$

Fig. 6.20 IDEALISED SOLUTIONS FOR LATERALLY RESTRAINED STRPS

\[ M_{a_{(\text{max})}} = 0.21\, f_c'\, h^2 \approx M_{u_{(\text{max})}} \]

$K = \infty$

$\rho = 0\%$

Fig. 5.21 VARIATION OF ARCHING RESISTANCE
Fig. 6.22 PREDICTED CAPACITY OF LATERALLY RESTRAINED STRIPS
Fig. 6.23 TEST ARRANGEMENTS FOR LATERALLY RESTRAINED STRIPS
Fig. 6.24 CORRELATION OF TEST AND PREDICTED FAILURE LOADS: RESTRAINED SLAB STRIPS
Chapter 7

THE ENHANCED PUNCHING STRENGTH OF LATERALLY RESTRAINED SLABS

7.1 INTRODUCTION

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7.2.1 Compressive membrane action
7.2.2 Flexural enhancement in large panel specimens
7.2.3 Concept of arching at the interior slab-column connection
7.2.4 Degree of restraint in the full panel specimen

7.3 INTEGRATED METHOD OF PREDICTION
7.3.1 Integrated approach
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7.5 CONCLUSIONS
7.1 INTRODUCTION

The present design procedures for the prediction of the punching strength of reinforced concrete slabs are empirical and based on the results from tests on conventional slab specimens. Consequently, the important influence of the slab boundary conditions is not recognised. In this regard, the ultimate capacity of the interior slab-column connection in the large panel specimen, is significantly greater than that of the equivalent conventional representation. Thus, in order to more realistically assess the punching strength of continuous slabs, the rational treatment of this enhanced capacity is pursued.

The enhanced punching strength of laterally restrained slabs is attributable to the phenomenon of compressive membrane action. From the examination of the distribution of membrane strain in the large panel specimen, a simple concept of the inherent mode of arching is presented.

The integrated method of prediction is developed from the rational approach for the conventional slab specimen in conjunction with the theoretical method for arching. On the basis of a conservative estimate of the relative arching resistance in the isolated full panel specimen, a simplified approach is presented for the prediction of the enhanced punching strength of the interior slab-column connection.

The general integrated procedure is validated by the good correlation with a wide range of test results from various sources. Furthermore, the simplified approach is shown to compare favourably with the available results from tests on isolated full panel specimens. The latter method is also shown to be more consistent than an alternative analytical approach and the present Code procedures.

Finally, the practical application of the integrated procedure and the implications for the design of continuous reinforced concrete slabs are briefly discussed.
7.2 THE ENHANCED CAPACITY

7.2.1 Compressive membrane action

It is widely accepted that the behaviour of reinforced concrete slabs under loading does not agree well with the predictions by elastic plate theory. This discrepancy arises from the characteristically inelastic properties of the composite material. In particular, the deformations associated with cracking of the concrete and yielding of the reinforcement can induce significant in-plane forces which are not accounted for in the analysis.

It has been shown that in reinforced concrete strips which are restrained against lateral expansion, the phenomenon of arching is of great importance. Similarly in continuous slabs, the development of internal arching forces in the form of a dome or membrane, can play a major role regarding the behaviour and ultimate capacity. This form of arching, which can be idealised as illustrated in Fig. 7.1, is known as 'compressive membrane action'. In effect, a proportion of the load is supported by the internal compression as distinct from the normal mode of bending, with some degree of lateral restraint being required to sustain the horizontal component of the compressive force. However, unlike the strips, which require external restraint, the reaction can be generated within the slab, owing to the two-dimensional continuity. Thus, under concentrated loading, the localised internal compression is restrained by tension in the surrounding region.

The present understanding of compressive membrane action is based largely on the empirical evidence of slab behaviour. Unfortunately, the difficulties involved with the rational treatment of this phenomenon are immense and consequently the existing methods of analysis vary widely in complexity, with few being suitable for practical application. For this reason, the development of a relatively simple and reliable procedure for
the prediction of the enhanced punching strength of laterally restrained slabs, was considered to be one of the major objectives of this part of the study.

7.2.2 Flexural enhancement in large panel specimens

The experimental investigation into the influence of different boundary conditions on the behaviour of the slab-column specimens, revealed a significant enhancement in strength as the panel size was increased. In comparison with the equivalent contraflexure models, the ultimate capacities of the full panel specimens were between 30% and 50% greater. This gain in strength can be partially attributed to the different flexural characteristics of the larger panels, however, as shown by the following examination, the modified moment factors are responsible for only a small proportion of the enhancement in the punching strength.

The elastic distributions of radial and tangential bending moments within the full panel specimen are illustrated in Fig. 7.2. As a result of the stress in the slab beyond the line of the supports, the ratio of the applied load to internal moment at the column periphery is dependent upon the panel size. This variation in the elastic moment factor, determined from grillage analyses of the large panel specimens, is shown in Fig. 7.3. It can be seen that the overall increase in the elastic moment factor is approximately 15%, which is considerably less than the measured gain in strength.

The spread of the tangential yield zone in the large panel specimen is limited by the development of the circumscribing positive yield-line at the slab supports. In this case, the appropriate moment factor is the same irrespective of the extent of the slab beyond the yield zone. This implies
that the additional rotational capacity of the large panel specimen is not the source of the available increase in strength.

On the basis of the modified moment factors, it appears that the enhanced punching strength of the large panel specimens cannot be solely attributed to the superior flexural characteristics. For this reason, an alternative explanation for the measured increase in strength is required.

The experimental measurements of strain from the surface of the slab extending beyond the line of contraflexure, indicate the development of significant in-plane forces. In addition, the lateral expansion of the slabs suggests the action of localised internal compression against the outer panel region. Thus, the behaviour of the large panel specimen is influenced by the development of compressive membrane action. The recognition of this phenomenon is necessary to enable the realistic assessment of the enhanced punching strength of the interior slab-column connection.

7.2.3 Concept of arching of the interior slab-column connection

In order to form a basis from which the rational treatment of the enhanced punching strength of the interior slab-column connection can proceed, it is appropriate to examine the nature of compressive membrane action within the large panel specimen. Thus, from the evidence concerning the distribution of membrane forces, the concept of arching can be established.

The characteristic strain profiles in the zone of slabs beyond the line of contraflexure, are considered to be the result of combined in-plane bending and circumferential stress, as illustrated in Fig. 7.4. In the elastic range, the distribution of bending strains is linear and the circumferential strain diminishes towards the outer edge of the panel. The resultant strain profiles along the slab diagonal and column line are given by the superposition of
the component distributions in the appropriate proportions. In this respect, the following important features concerning the experimentally measured strain profiles can be noted:

a) on the column line, the strain is tensile and is minimum approximately halfway between the line of contraflexure and the panel edge.

b) on the slab diagonal, the maximum tensile strain is at the line of contraflexure.

c) at the panel corner, the strain is small, but compressive.

The experimental evidence suggests that the inherent restraint in the large panel specimen is provided by the zone of slab beyond the line of contraflexure. This simple concept is substantiated with the results of the analytical investigation into the mode of restraint, which is now described.

The membrane action within the zone of restraint in the full interior panel was examined by means of the finite element method. For the analysis, the plane stress programme (HECB/B/3) was utilised and the element mesh is shown in Fig. 7.5. Due to symmetry, it was only necessary to represent one quarter of the panel beyond the line of contraflexure with the appropriate boundary conditions. As the behaviour of the square slab and column can be idealised as axisymmetric, the loading applied to the analytical model was equivalent to uniform pressure along the line of contraflexure.

The distributions of strain along the column line and slab diagonal, obtained from the plane stress finite element analysis, are also shown in Fig. 7.5. In form, there is close resemblance between the analytical and experimental profiles of membrane strain, with the salient features being satisfactorily replicated. This good agreement supports the simplifying assumption that the internal arching forces develop within the line of contraflexure, beyond which the additional zone of slab provides the
necessary restraint. Thus, the behaviour of the large panel specimen can be considered equivalent to that of the laterally restrained portion of slab, which is circumscribed by the nominal line of contraflexure.

7.2.4 Degree of restraint in the full panel specimen

Having established the concept of arching in the large panel specimen, it is now possible to estimate the inherent degree of restraint, which in turn, determines the importance of compressive membrane action. The zone of slab which extends beyond the nominal line of contraflexure is considered to restrain the lateral expansion of the internal portion adjacent to the column. Thus, a simple theoretical method, by which the stiffness of the surrounding region can be assessed, is required.

As a result of the investigation into the distribution of membrane strain in the large panel specimen, the deformation of the zone of slab beyond the line of contraflexure is considered to be controlled by the combination of in-plane bending and circumferential stress. However, it is reasonable to assume that the square slab and column can be idealised as an axisymmetric system, in which case, the influence of in-plane bending deformations is neglected. Thus, the zone of restraint is analogous to a thick walled cylinder, for which the circumferential stress can be determined from the classical Lamé-Clapeyron equation:

$$\sigma_\theta = \frac{p_z}{\frac{\alpha}{r} - 1 \left(1 + \left(\frac{r_0}{r}\right)^2\right)}$$

Taking the radius to the nominal line of contraflexure as two fifths of the span, the maximum circumferential stress at the inner perimeter of the restraining ring is equal to:

$$\sigma_\theta = 1.38 \, p_z$$
The degree of restraint is determined by the lateral stiffness of the surrounding panel. From the stress-strain relationship for the thick walled cylinder, the radial displacement is given by:

\[ \Delta_r = \frac{r}{E} (\sigma_\theta - \nu \sigma_r) \]

Before proceeding, the suitability of the thick cylinder analogy can be assessed in relation to the results of the finite element analysis. The deformation of the zone of restraint subjected to uniform internal loading is illustrated in Fig. 7.6. In comparison are shown the positions of the inner and outer perimeters of the restraining ring under the same pressure. It can be seen that there is excellent agreement between the theoretical and analytical displacements, which supports the view that the lateral stiffness is primarily controlled by the circumferential strain. Furthermore, it can be shown that for matching theoretical and analytical stress distributions, the flexural rigidity which corresponds to the in-plane bending component is very much greater than the circumferential stiffness. Thus, although the stress distribution is significantly influenced by the particular shape of the zone of restraint, the lateral stiffness can be satisfactorily determined on the basis of the thick cylinder analogy.

In terms of the full panel specimen, the radial displacement at the line of contraflexure can be taken as:

\[ \Delta = 0.75p \cdot \frac{a}{E_c} \]

Thus, for the internal force per unit length of the inner perimeter, the lateral stiffness of the restraining ring is equal to:

\[ K = 1.33E_c \frac{h}{a} \]

The degree of restraint provided by the uncracked zone of slab beyond the line of contraflexure in the full panel specimen can be assessed on the
basis of this simple theoretical model. However, two additional factors must be considered before a realistic estimate can be made.

The observations of slab behaviour in the large panel tests, revealed that the zone of restraint remained uncracked at loads of less than 50% ultimate. Although the strain measurements indicate the same mode of restraint at higher load levels, the spread of radial cracking must significantly influence the lateral stiffness. In addition, the effective restraint is reduced, to some extent, by the rotation of the slab beyond the line of contraflexure. With the additional complexity involved, the accurate estimation of the inherent degree of restraint is practically impossible. However, it is worth examining the influence of anticipated stiffness reduction on the internal arching resistance.

The degree of restraint provided by the slab beyond the line of contraflexure can be assessed in terms of the relative stiffness parameter. In the case of the uncracked zone of restraint, this is equal to:

\[
\frac{K_a}{E_ch} = 1.33
\]

With reference to the theoretical relationship shown in Fig. 6.14, it can be seen that the arching resistance which corresponds to this degree of restraint is between 60% and 85% of the capacity for rigid lateral support.

The lateral stiffness after cracking is heavily dependent upon the level of reinforcement, which is normally fairly low in the slab beyond the line of contraflexure. Thus, a large reduction in stiffness and hence the arching capacity, can be expected to occur. Typically, for a reduction to between one quarter and one sixth of the uncracked stiffness, the relative arching resistance for the normal range of concrete strengths \( f_c' = 20 \text{ N/mm}^2 - 50 \text{ N/mm}^2 \) and span to depth ratios \( \frac{L}{d} = 20 - 40 \) is between approximately one quarter and one half.
In view of the difficulties involved with the accurate assessment of the degree of restraint inherent in the full panel specimen, a simpler approach is appropriate for practical purposes. This can be based on a conservative estimate of the relative arching resistance, thereby eliminating the need to determine the effective lateral stiffness.

One further consideration concerning the contribution of compressive membrane action, is the behaviour of the isolated full panel specimen in relation to the situation in a continuous floor slab. The theoretical radial displacement at the outer edge of the restraining ring is equal to approximately 60% of that at the inner perimeter. With continuity of panels, this movement is prevented and hence the degree of restraint at the line of contraflexure is significantly increased. Therefore, the results of tests on isolated full panel specimens can be considered lower bound to the actual behaviour in a real structure. In view of this, it is perhaps not worth pursuing the complex solution to the degree of restraint inherent in the isolated interior panel. The simplified approach, which is presented in a later section, proves to be adequate with respect to the available test results.

7.3 INTEGRATED METHOD OF PREDICTION

7.3.1 Integrated approach

The punching strength of the conventional slab specimen can be satisfactorily predicted by the rational method presented in Chapter 5. In addition, the direct procedure for the prediction of the ultimate capacity of laterally restrained strips has been formulated in Chapter 6. Thus, a suitable basis exists for the development of an integrated method for the prediction of the enhanced punching strength of laterally restrained slabs. The amalgamation of the two procedures into a reliable general
approach, which is further simplified for the analysis of the interior slab-column connection, is described in the following sections.

7.3.2. Ultimate flexural capacity

The flexural punching capacity is dependent upon the slab ductility and moment of resistance. In the case of the laterally restrained slab, compressive membrane action reduces the ductility and increases the flexural strength. However, unlike the plastic behaviour caused by yielding of the reinforcement in the conventional specimen, failure associated with internal arching is sudden and is usually accompanied by a marked decrease in the load capacity. Fortunately, this characteristic can be adequately incorporated into the existing flexural approach.

The ductility exhibited prior to failure is dependent upon the section moment of resistance. Thus, in the conventional slab specimen, the spread of the tangential yield zone is determined by the level of reinforcement. However, in the case of the laterally restrained slab, the moment capacity is increased by the resistance provided by internal arching. Thus, it is necessary to define an appropriate ductility parameter.

The radius of the tangential yield zone at failure is assumed to depend on the ratio of the moment at yield to the balanced moment of resistance. This expression of the ductility in terms of moment strength as opposed to the reinforcement ratio is convenient, in that the arching component can easily be included. However, the implications of incorporating the arching moment of resistance into the slab ductility parameter must first be considered.

The maximum possible arching resistance is approximately equal to the moment strength of the fully reinforced section. Thus, flexural failure is
precipitated by localised compression of the concrete. As there is no spread of the tangential yield zone, this brittle mode of failure is consistent with the rapid decrease in the initially high arching resistance. Conversely, with low arching resistance, the moment remains practically constant throughout a large range of deformation. Therefore, the integrated slab ductility is consistent with the spread of the tangential yield zone.

The arching moment of resistance can be derived in the normal manner by considering the action in an appropriate strip of slab. In the laterally restrained slab, the enhanced strength is the result of both radial and tangential arching. However, the primary cause of this compressive membrane action is the restraint provided against radial expansion. Thus, in order to determine the arching moment of resistance, it is appropriate to consider the radial strip of slab extending from the load to the zone of restraint.

Having considered the integrated slab ductility and arching moment of resistance to be applicable to the flexural criterion of failure, it therefore remains to define the moment factor at ultimate. The principal trajectories of membrane stress are in directions normal to the bending crack lines. A recent finite element investigation by Fujii (1981) has shown this to be the case. Thus, the normal moment factors can be considered applicable to the flexural criterion for slabs in which compressive membrane action develops.

The ultimate flexural capacity of the laterally restrained slab is given by:

\[ P_{vf} = \left[ k_y l - r_f (k_y - k_b) \right] \left( \frac{M_a + M_b}{M_{(ba)}} \right) (M_a + M_b) \] 7.1

This expression relates the interpolative moment factor to the integrated flexural strength in terms of the slab ductility and moment of resistance. It is applicable to simply supported slabs having lateral restraint in
which case the moment factors for bending and arching are the same. For slabs having rotational, in addition to lateral restraint, the interpolative moment factor and the integrated moment of resistance cannot be so easily combined, as the same moment factors are not applicable to both the bending and arching components. This can be appreciated by consideration of the different moment factors for a 'fixed-end' strip. However, the analysis of a rotationally restrained slab as an equivalent simply supported specimen will at least yield a lower bound estimate of the ultimate flexural capacity.

7.3.3 Ultimate shear capacity

The ultimate shear capacity of a laterally restrained slab is enhanced by the development of compressive membrane action. This is due to the deepening of the slab compression zone and the corresponding reduction in flexural crack width. Thus, the shear strength is increased by the additional resistance provided by the concrete and aggregate interlock. The method for the prediction of the ultimate shear capacity must adequately account for this enhancement.

In the theoretical method for arching, it is assumed that yielding of the reinforcement occurs prior to failure. However, for the shear mode of punching this assumption is invalid, as internal diagonal cracking develops before the reinforcement yields. Nevertheless, in reality the ultimate shear capacity is enhanced as the compressive membrane effect begins at an early stage, with the initiation of inelastic behaviour. Therefore, in order to avoid a conceptual conflict regarding the mode of failure, the ultimate shear capacity can be treated in a unique manner.

It is assumed that the compressive membrane action develops after flexural cracking but that the reinforcement is not active. That is, the section is considered to be unreinforced. Thus, the analysis for shear
is simplified yet the treatment of compressive membrane action is consistent with the assumptions in the theory of arching. An alternative approach in which the reinforcement is also neglected, has recently been presented by Kirkpatrick (1982).

The shear resistance is dependent upon the depth of the slab compression zone. In the laterally restrained slab, the contact depth decreases under vertical deformation. Thus, the capacity is minimum at the maximum vertical displacement. However, the ultimate arching moment is attained prior to this and hence, failure in shear can be considered to coincide with the development of the maximum arching capacity. Therefore, the contact depth at failure is governed by the critical deformation of the radial strip of laterally restrained slab, which can be estimated from the theoretical method for arching. On this basis, the ultimate shear criterion can be appropriately modified to incorporate the enhancing influence of compressive membrane action.

The nominal ultimate shear force per unit length on the perimeter at one half of the slab effective depth from the column perimeter in the conventional slab specimen is given by:

\[ V_u = 0.48\sqrt{f_c^T d(100\rho)^{0.25}} \]  

This semi-empirical expression accounts for the resistance provided by the concrete, aggregate interlock and the dowel action of the reinforcement. The component of shear in the concrete is dependent upon the neutral axis factor, which is approximated as:

\[ \frac{x}{d} = 0.35(100\rho)^{0.25} \]

Thus, in terms of the depth of the compression zone, the nominal ultimate force per unit length can be expressed by:

\[ V_u = 0.48\sqrt{f_c^T} \frac{x}{0.35} \]
In laterally restrained slabs, the depth of the compression zone is increased without a corresponding increase in dowel action. Therefore the shear force carried by the concrete and aggregate interlock must be considered independently of the dowel component.

The combined resistance of the concrete and aggregate interlock contributes to between 75% and 85% of the shear capacity in the conventional specimen. Due to the inclination of the failure plane, the dowel resistance is provided at some distance from the column perimeter. Thus, the proportion of load carried by dowel action can be expected to be relatively high. The total shear force in the unrestrained slab is assumed to be apportioned in the following manner:

\[ V_c + V_a = 75\% \, V_u \]
\[ V_d = 25\% \, V_u \]

On this basis, the contribution of each component to the shear capacity of the laterally restrained slab can be assessed. The nominal ultimate shear stress on the critical section is comprised of the enhanced resistance provided by the concrete and aggregate interlock, in addition to the normal dowel component. In terms of the depth of the contact zone and the level of reinforcement, the shear force per unit length is given by:

\[ V_u = 0.48\sqrt{F_c} \left[ 0.75 \frac{x}{0.35} + 0.25d(100\rho)^{0.25} \right] \]

7.2

The shear capacity of laterally restrained slabs having square or circular columns is given by the nominal ultimate shear stress in conjunction with the appropriate critical section and column shape factor.

The influence of stress dispersal is normally recognised by positioning the critical section at one half of the slab effective depth from the column perimeter. In the case of the laterally restrained slab, the actual
effective depth is not relevant as regards the depth of the compression zone. Thus, in order to maintain consistency within the method as applied to either restrained or unrestrained slabs, the effective depth can be replaced by an equivalent factor which is applicable to both cases. In normal reinforced concrete slabs, the average effective depth is approximately 80% of the overall depth. This hypothetical effective depth can be utilised to define the position of the critical section. Therefore, the ultimate shear capacity of the laterally restrained slab is given by:

a) square loaded area

\[ P_{Vs} = (c + 0.8h)\sqrt{\frac{F^c}{c}}[3.6x + 0.33h(100p)^{0.25}] \] 7.3a

b) circular loaded area

\[ P_{Vs} = (2r_c + 0.8h)\sqrt{\frac{F^c}{c}}[3.2x + 0.3h(100p)^{0.25}] \] 7.3b

The punching strength of the laterally restrained slab in the 'shear' mode of failure can be predicted on the basis of the preceding expressions, in which the compressive membrane enhancement is incorporated. The depth of the slab compression zone can be determined directly from the application of the theoretical method for arching to the radial strip of slab which extends to the zone of restraint. Although the influence of the tensile force in the reinforcement is neglected, the net result in the depth of the compressed concrete is not significantly affected. This assumption is consistent with the mode of failure and considerably simplifies the analysis.

7.3.4 Mode of failure

The influence of compressive membrane action is analogous to the addition of flexural reinforcement. In slabs having a low degree of restraint in conjunction with a low reinforcement ratio, the probable
mode of failure is flexural. The effect of either additional reinforcement or lateral restraint is to increase the susceptibility to the shear mode of punching. As the influence of the reinforcement is adequately recognised in relation to the predicted mode of failure, it remains to consider the suitability of the integrated procedure with regard to the degree of restraint.

In the case of less than rigid lateral restraint, the resistance provided by arching is determined by means of the three-hinged arch analogy. However, this method is based on the equivalent flexural capacity of the 'affine' strip of rigidly restrained slab and is not strictly applicable to the enhanced shear criterion. This anomaly can be appreciated by consideration of the arching moment of resistance in relation to the depth of the contact zone.

As the degree of lateral restraint is decreased there is a corresponding reduction in the arching moment of resistance, which eventually diminishes to zero. However, according to the theoretical method for arching, the critical deflection becomes constant (\(u = 0.31\)) with the occurrence of the triangular elastic stress distribution, in which case the minimum depth of the contact zone in the unreinforced section is equal to approximately 40% of the overall depth. Therefore, a level of constant shear resistance is reached, which corresponds to the minimum depth of the compression zone.

The nominal ultimate shear stress in the slab having a very low degree of lateral restraint can be compared with that corresponding to the conventional slab specimen, by the following ratio:

\[
\frac{V_u \text{ (conventional)}}{V_u \text{ (restrained)}} = \frac{(100d)^{0.25}}{[1.13 + 0.25(100d)^{0.25}]}.
\]

The disparity between the equivalent shear criteria is illustrated in Fig. 7.7. It can be seen that the largest discrepancy is associated with
low levels of reinforcement, for which the depth of the compression zone in the laterally restrained slab is overestimated. At high levels of reinforcement, the shear stress ratio approaches unity, which is desirable in view of the increased probability of punching in the shear mode.

Thus, although the theoretical method for arching is not, in principal, strictly applicable to the shear criterion of failure in slabs having less than rigid lateral support, it can actually be satisfactorily utilised for relatively low degrees of restraint. In practical situations, the lateral stiffness of the surrounding slab or supporting elements is usually of sufficient magnitude to fully warrant the use of the theoretical depth of the contact zone. Thus, the prediction of the mode of failure in slabs having a realistic degree of lateral restraint is unambiguous.

As in the case of the conventional slab specimen, the predicted punching strength of the laterally restrained slab is the lesser of the ultimate flexural and shear capacities.

7.3.5 Rigorous method for laterally restrained slabs

The integrated procedure for the prediction of the punching strength of laterally restrained slabs is outlined by the following flow chart:
An example of the integrated procedure applied to a laterally restrained specimen tested by Snowdon is presented in Appendix C. The following detailed steps constitute the rigorous method of analysis for the concentrically loaded, isotropically reinforced slab:

1. ARCHING PARAMETERS

   (i) square slab and loaded area: \( L_e = \frac{(a - c)}{2} \)

   (ii) circular slab and loaded area: \( L_e = r_a - r_c \)

\[ \varepsilon_c = (-400 + 60 f_c' - 0.33f_c'^2) \times 10^{-6} \]  

\[ E_c = 4.73\sqrt{f_c'} \]

2. INTEGRATED MOMENT OF RESISTANCE

\[ M_b = \rho f_y d^2 (1 - 0.59 \frac{\rho f_y}{f_c}) \]  

\[ 2d_1 = h - \frac{\rho f_y d}{0.85f_c'} \]  

\[ L_r = L_e \sqrt{\frac{E_c d_1}{KL_e} + 1} \]  

\[ R = \frac{\varepsilon c r^2}{4d_1^2} \]  

\[ R > 0.26 : M_r = \frac{0.3615}{R} \]  

\[ 0 < R < 0.26 : M_r = 4.31 - 16.11(3.3 \times 10^{-6} + 0.1243R) \]  

\[ M_a = 0.21f_c'^2 \frac{L_e}{r L_r} \]

Integrated moment = \( M_a + M_b \)
3. INTEGRATED SLAB DUCTILITY

\[ \beta_1 = 0.85 - \left( \frac{f'_c - 27.6}{6.9} \right) \times 0.05 + 0.85 \]  
(5.6)

\[ p_{(pal)} = \frac{0.85 f'_c \beta_1 (0.003 \times 200000)}{f_y \times (0.003 \times 200000 + f'_y)} \]  
(5.7)

\[ M_{(pal)} = p_{(pal)} f_y d^2 (1 - 0.59 \frac{p_{(pal)} f_y}{f'_c}) \]  
(5.5)

Integrated ductility parameter \[ \frac{M_a + M_b}{M_{(pal)}} \]  
(7.1)

4. MOMENT FACTOR

(i) square slab and loaded area \((r_f = 1.15)\)

\[ k_{y1} = 8 \left[ \frac{a}{a - c} - 0.172 \right] \]  
(5.2a)

\[ k_b = \frac{25}{\log_e \left( \frac{2.5a}{c} \right)}^{1.5} \]  
(5.3a)

(ii) circular slab and loaded area \((r_f = 1.0)\)

\[ k_{y1} = 2\pi \left[ \frac{r_a}{r_a - r_c} \right] \]  
(5.2b)

\[ k_b = \left[ \frac{8\pi}{210g_e \frac{r_a}{r_c} + \left( \frac{r_a^2 - r_c^2}{r_a^2} \right)} \right] \]  
(5.3b)

interpolative moment factor:

\[ k_t = \left[ k_{y1} - r_f (k_{y1} - k_b) \frac{M_a + M_b}{M_{(pal)}} \right] \]  
(7.1)
5. CONTACT DEPTH

assume i) \( d_1 = \frac{h}{2} \)

\[ L_r = L_e \sqrt{\frac{\frac{E_A}{C_{KL_e} + 1}}\]  

\[ R = \frac{\varepsilon_{\alpha} L_{r}^2}{4d_{1}^{2}} \]  

\[ R > 0.26 : u = 0.31 \]  

\[ 0 < R < 0.26 : u = -0.15 + 0.36 \sqrt{(0.18 + 5.6R)} \]  

\[ \alpha = 1 - \frac{u}{2} \]  

contact depth: \( x = \alpha d_1 \)

For theoretical refinement, assume \( A = \alpha d_1 \) and repeat this step

6. ULTIMATE FLEXURAL CAPACITY

\[ P_{vf} = k_f(M_a + M_b) + \frac{1}{r_y} k_b M_{(bal)} \]  

7. ULTIMATE SHEAR CAPACITY

(i) square loaded area

\[ P_{vs} = (c + 0.8h)\sqrt{F_{c}^{(3.6x + 0.33h(100p)^{0.25})}} \]  

(ii) circular loaded area

\[ P_{vs} = (2r_c + 0.8h)\sqrt{F_{c}^{(3.2x + 0.3h(100p)^{0.25})}} \]  

8. MODE OF FAILURE

\[ P_{vf} < P_{vs} : P_{p} = P_{vf} \text{ (flexural mode)} \]

\[ P_{vf} > P_{vs} : P_{p} = P_{vs} \text{ (shear mode)} \]
7.3.6. Prestressed flat slabs

The use of unbonded post-tensioned slabs is becoming increasingly popular owing to the many economic and structural advantages presented over conventional reinforced concrete construction. In general, considerable savings are to be made with the more efficient use of materials and reduced construction time. The possible advantages are described in a recent article by Khan (1979).

Unfortunately, in the United Kingdom, the technique has been employed in relatively few buildings, as compared to the much wider application in many other countries. As a result, the code of practice, CP110 (1972), contains no specific provisions for the design of post-tensioned flat slabs and the engineer must refer to other specialised literature such as the Concrete Society (1974) recommendations. In particular, the punching strength at slab-column connections, being of great importance, presents a difficult problem in design.

Although post-tensioned slabs can remain largely uncracked under working loads, there is likely to be a considerable amount of inelastic behaviour prior to failure. This is particularly true in the more common instance of the partially prestressed slab which contains supplementary bonded reinforcement. Thus, the development of compressive membrane action is inevitable and this may significantly increase the ultimate capacity. Fortunately, the prediction of the enhanced punching strength of the interior slab-column connection in the prestressed slab can be treated on the same basis as the reinforced concrete full panel specimen.

The punching strength of the post-tensioned slab is enhanced by both the prestress and compressive membrane action. In order to determine the influence of the prestress, a knowledge of the tendon stress at ultimate is required. From tests on full interior panel models, Franklin (1981)
found that the increase in the tendon stress prior to failure, is only of
the order of 10% - 15%. In view of this, the stress in the wire at ultimate
can be conservatively taken as the effective prestress in the tendon.
Thus, for post-tensioned slabs, the equivalent reinforcement ratio can be
defined as:
\[
\frac{p}{e} = \frac{p}{s} + \frac{p_{ps} f_{pe}}{f_y}
\]

For the prediction of the ultimate flexural capacity, it is necessary
to determine the combined bending and arching capacity of the radial strip
of laterally restrained slab. This can be both simply and effectively
achieved by the use of the equivalent reinforcement ratio in the enhanced
flexural criterion of failure.

In the shear mode of punching the prestress axial compression is
beneficial in that it delays the initiation of internal diagonal cracking.
By the use of the equivalent level of reinforcement in the enhanced shear
criterion, the ultimate shear capacity can be increased to allow for this.
The reinforcement parameter in the nominal ultimate shear stress is
representative of the dowel resistance provided by the steel. However,
as the dowel component contributes only marginally to the enhanced punching
strength, the use of the equivalent reinforcement ratio is likely to be
a conservative means of recognising the influence of the prestress on
the ultimate shear capacity.

The prediction of the punching strength of the interior slab-column
connection in the prestressed slab is based upon the integrated procedure
in which the equivalent reinforcement ratio is utilised. In view of the
paucity of reliable test results in this area, the method cannot be
developed further at present.
7.3.7 Eccentric loading of full panel specimens

The punching strength of the interior slab-column connection is reduced by the application of eccentric loading. For the conventional slab specimen, the capacity reduction factor, which is based on the elastic plate analysis, is equivalent to the following interaction criterion for combined loading:

$$\frac{P}{P_0} + \frac{M}{M_0} = 1$$

However, Long and Masterson (1974) have noted that the preceding relationship depends upon the type of test specimen. In the full panel specimen, a high level of eccentricity was associated with a greater enhancement of the lower bound flexural capacity, as shown in Fig. 7.8. This was attributed to an increased contribution of compressive membrane action, due to the smaller critical deformation under combined loading.

In the case of pure shear loading in the full panel specimen, the zone of restraint is considered to be the slab beyond the nominal line of contraflexure. However, for pure moment transfer, the restraint is provided by the slab in the immediate vicinity of the column, as illustrated in Fig. 7.9. This mode of resistance has been appropriately termed 'joggle action' by Stamenković (1970). The mutually opposed compressive forces in the concrete contribute to the moment capacity of the connection, irrespective of the position of the line of contraflexure. Thus, under combined loading, the contribution of compressive membrane action is increased and the capacity reduction factor which is appropriate for the conventional specimen, is more conservative for the full interior panel.

In view of the considerable difficulties involved with the analytical solution to the problem of combined loading, the following simple empirical adaptation is proposed. The punching strength of the full panel specimen
can be reduced in accordance with the relationship:

\[ p = \frac{p_{vo}}{1 + 10 \frac{e}{L}} \]

In this case, the modified capacity reduction factor corresponds to the empirical interaction criterion shown in Fig. 7.8.

It can be seen from Fig. 7.10, that the preceding expression is only significantly less conservative than the present British design method at high values of relative eccentricity. Thus, for the normal levels of moment transfer encountered in practice, the CP110 (1972) approach is considered to be satisfactory for both types of specimen.

7.3.8 Simplified approach for the interior slab-column connection

The major difficulty in the prediction of the enhanced punching strength of the interior slab-column connection lies with the assessment of the contribution of compressive membrane action to the ultimate capacity. In particular, the estimation of the degree of restraint in the full panel specimen is problematic and unfortunately, this can significantly influence the flexural resistance. Thus, in order to facilitate the practical application of the integrated procedure, some further simplifications are now introduced.

It has been shown that because of the cracking and rotation of the slab beyond the line of contraflexure, the inherent degree of restraint in the full panel specimen is relatively small. Consequently, the contribution of compressive membrane action is considered to be normally less than one half of that for rigid lateral support. In view of this, it is convenient to adopt a conservative estimate of the relative arching resistance which is suitable for the general case.
The simplified approach is similar to the use of an empirical restraint factor, as suggested by Hewitt and Batchelor (1975) in relation to the enhanced punching strength of bridge deck panels. In the isolated full panel specimen, the constant proportion which is considered appropriate is one fifth of the capacity for rigid lateral support. On this basis, the simplified flexural punching strength, which can be predicted from the interpolative moment factor in conjunction with the integrated moment of resistance, is given by:

\[ P_{vf} = \left[ k_{y1} - r_f(k_{y1} - k_b) \frac{M_b + 0.2M_{ar}}{M_{bal}} \right] (M_b + 0.2M_{ar}) \]  \hspace{1cm} 7.8

Unfortunately, a substantial amount of calculation is required for this prediction. Thus, in order to reduce the computational effort, the various components in the expression can be further simplified by approximate relationships. The following derivations are primarily to facilitate the analysis of interior slab-column connections and are not directly applicable to other forms of slab construction. In this respect, the relationships are based on the behaviour of the portion of slab circumscribed by the nominal line of contraflexure in the full interior panel. Thus, the prediction of the punching strength is based on the enhanced criteria for the conventional slab specimen.

For the normal variation in the ratio of the column size to slab span, the moment factor relationships in the preceding expression are shown in Fig. 7.11. It can be seen that the curvilinear forms are closely matched by the following simple linear approximations:

\[ k_{y1} = 6.1 + 33 \frac{C}{L} \]  \hspace{1cm} 7.9

\[ k_{y1} - k_b = 3.7 - 15 \frac{C}{L} \]  \hspace{1cm} 7.10
In addition to the moment factors, the integrated moment of resistance and ductility parameter can be expressed in simplified form. The flexural strength of the radial strip of slab extending to the line of contraflexure is dependent upon several material and geometric variables, which can be considered in terms of dimensionless parameters. In practice, the variation of these parameters is limited and the flexural strength is only of interest for the normal range.

The integrated moment of resistance, which is calculated by means of the direct procedure presented in Chapter 6, is related to the dependent parameters as shown in Fig. 7.12. As a result of the relatively low arching component, the relationships can be considered as being independent of the span/depth ratio and concrete strength. Thus, the moment of resistance is basically dependent upon the reinforcement index and can be determined from the following simple expression:

$$M_b + 0.2M_{ar} = 0.62f'_cd^2\left(\frac{f_y}{f_c}\right)^{0.67}$$  \hspace{1cm} 7.11

In order to ensure yielding of the reinforcement, the flexural strength is limited by the balanced moment of resistance. However, as an approximation, the following widely accepted empirical criterion for the capacity of the fully reinforced section can instead be adopted:

$$M_{u(max)} = 0.333f'_cd^2$$

Thus, the integrated ductility parameter is given by:

$$\frac{M_b + 0.2M_{ar}}{M_{u(max)}} = 1.88\left(\frac{f_y}{f_c}\right)^{0.67}$$  \hspace{1cm} 7.12

This expression corresponds to balanced resistance at a reinforcement index of 0.4, which is similar to the empirical value proposed by Whitney (1937) and is consistent with the predicted mode of failure in the conventional slab specimen.
The various components in the flexural criterion of failure can be replaced by the preceding simplified relationships to give the following single expression:

\[ P_{vf} = \left[ 6.1 + 33 \frac{C}{L} - 1.88f_c(3.7 - 15 \frac{C}{L})w^{0.67} \right]0.62f'_c d^2 \omega^{0.67} \quad 7.13 \]

Thus, the flexural punching strength of the full interior panel can be predicted without direct recourse to the theoretical method for arching.

The ultimate shear capacity is primarily governed by the depth of the compression zone at the critical section. In the laterally restrained slab, this is normally calculated by means of the theoretical method for arching and the compressed concrete due to bending is neglected. However, the variation in the contact depth is limited and consequently the influence upon the ultimate shear capacity is only marginal.

In view of the relatively low degree of restraint inherent in the isolated full interior panel, the minimum contact depth is appropriate for the prediction of the enhanced shear capacity. This can be incorporated into the nominal ultimate shear stress which, combined with the appropriate critical section and column shape factor results in the following expressions:

a) square column

\[ P_{vs} = 1.9\sqrt{f'_c d}(c + d) \left[ 1 + 0.22(100\rho)^{0.25} \right] \quad 7.14a \]

b) circular column

\[ P_{vs} = 1.7\sqrt{f'_c d}(2r_c + d) \left[ 1 + 0.22(100\rho)^{0.25} \right] \quad 7.14b \]

In this case, the critical section is defined with respect to the effective depth of slab, rather than to a proportion of the overall depth. This is appropriate because of the relatively small influence of compressive membrane action on the ultimate shear capacity.
The enhanced punching strength of the interior slab-column connection in the full panel specimen can be predicted on the basis of the preceding simplified approach. In the normal manner, the mode of failure is given by the lesser of the flexural and shear capacities. The simplifications inherent in the method are realistic and result in a close approximation to the more rigorous theoretical treatment. Thus, unless the degree of restraint in the full panel specimen can be assessed with reasonable confidence, the adoption of the simplified approach is recommended.

An example of the simplified approach applied to a typical slab tested by Masterson (1971) is presented in Appendix C.

7.4 COMPARISON WITH EXPERIMENTAL RESULTS

The procedure for the prediction of the punching strength of the conventional slab specimen has been validated by a comparison with a wide range of test results. In addition, the theoretical method for arching has been verified with regard to the tests on laterally restrained strips. Thus, the final assessment is the suitability of the integrated procedure for the prediction of the enhanced punching strength of laterally restrained slabs. In this case, the available test results are not so numerous and often the experiments have not been controlled in a manner from which the analytical parameters can be deduced.

The degree of lateral restraint is the principal factor which cannot be measured precisely. For this reason, the majority of tests have been concerned with slabs having a high degree of artificial lateral restraint, which can usually be considered as rigid for the purpose of analysis. Such tests provide valuable information regarding the influence of compressive membrane action in beam and slab structures, but do not reflect the behaviour of the interior slab-column connection, for which there is relatively little data.
In the following section, the predictions by the integrated procedure are compared with the results from many of the tests reported in the literature and the large panel specimens tested by the author. Where appropriate, the simplified approach is utilised in preference to the more rigorous method of analysis. In addition, the results of the tests on the large panel specimens tested by the author are compared with the predictions by an alternative analytical approach and the present Code procedures.

7.4.1 Tests reported in the literature

a) Tests by Taylor and Hayes

One of the earliest investigations into the influence of lateral restraint on the punching strength of slabs, was reported by Taylor and Hayes (1965). In this experimental programme, a series of tests was conducted on approximately $\frac{1}{3}$ scale models with edge restraint provided by a rigid steel frame. The principal variables in the tests were the column size and the level of reinforcement.

Of particular interest, are the results of the tests on the unreinforced slabs, which sustained a substantial load before failure. Details of the tests with the predicted punching strengths are presented in Table 7.1.

b) Tests by Long

The experimental technique for the more realistic appraisal of the strength of the interior slab-column connection was initially developed by Long (1967). In three of the $\frac{1}{4}$ scale models, the correct boundary conditions were successfully applied to simulate the behaviour of the full panel specimen subject to combined loading.
In view of the uncertainty as to the degree of restraint inherent in these panels, the simplified approach was utilised for the analysis. The influence of moment transfer was taken into account with the empirically modified capacity reduction factor. Details of the tests and the predicted failure loads are presented in Table 7.2.

c) Tests by Masterson

Further refinement of the experimental technique for the application of the correct boundary conditions to the isolated full panel specimen was achieved by Masterson (1971). Consequently, a comprehensive series of tests on \( \frac{1}{6} \) scale models, in which the principal variable was the level of moment transfer, was completed. Indeed, the variation in eccentricity covered much more than the normal range encountered in practice, extending from pure shear to almost pure moment loading.

The simplified approach was utilised for the analysis of these specimens and the influence of moment transfer was recognised by the incorporation of the modified capacity reduction factor. Details of these tests and the predicted punching strengths are presented in Table 7.3.

d) Tests by Aoki and Seki

The approximately \( \frac{2}{3} \) scale slabs tested by Aoki and Seki (1971) were subject to central concentrated loading over a circular area, with the lateral restraint provided by integral ring beams. The main variable in this series of tests was the level of flexural reinforcement, however, the slab span and the size of the boundary frame were also changed.

In view of the substantial proportions of the ring beams, the lateral restraint was assumed to be rigid for the purpose of the analysis. Furthermore, due to the arrangement of reinforcement at the slab boundary, the specimens were considered to represent simply supported restrained
The principal details of these tests and the predicted ultimate capacities are presented in Table 7.4.

e) Tests by Vanderbilt

An alternative type of specimen has been utilised by Vanderbilt (1972) for the investigation of the punching strength of the continuous flat plate. The approximately $\frac{1}{4}$ scale slabs were supported on a central column stub and the loading was applied uniformly over the panel. In order to simulate the continuity with adjacent panels, the slab edge was cast integrally with a surrounding ring beam. This was heavily over reinforced and provided a high degree of lateral restraint. The main variables in the tests were the column size and shape, and the level of reinforcement.

From elastic analyses, Vanderbilt determined the line of radial moment contraflexure to be located at approximately $\frac{1}{4}$ of the span from the column face. Therefore, the analysis of the laterally restrained portion of slab which was assumed to be either square or circular according to the particular column shape, was based on this nominal distance. Furthermore, in view of the presence of the heavily reinforced ring beam, the lateral restraint beyond the line of contraflexure was assumed to be rigid. The principal details of these tests and the predicted ultimate capacities are presented in Table 7.5.

f) Tests by Snowdon

In order to investigate the enhanced punching strength of the deck structure for the proposed floating airport at Maplin, Snowdon (1973) conducted a comprehensive series of tests on full scale specimens at the Building Research Station. The slabs were tested as interior panels in the surrounding heavily reinforced portion of the cellular runway and the concentrated loading was applied through a circular platen. In relation to the stiffness of the slab the lateral restraint was effectively rigid.
As the influence of several unorthodox variables was examined, not all of the specimens were amenable to the analysis. However, the majority of tests comprised concentrically loaded flat slabs, which were ideally suited. The principal variables were the level of reinforcement and the concrete strength, although the slab depth and the size of loading platen were also changed. For each test, two identical specimens were cast and the average results were utilised for the analysis. Details of the tests and the predicted punching strengths are presented in Table 7.6.

g) Tests by Smith and Burns

The three tests conducted by Smith and Burns (1974) on \( \frac{1}{3} \) scale models of the interior slab-column connection in a post-tensioned flat plate, were similar in nature to the full panel specimens tested by the author. In this respect, the slabs were supported on central square columns and the loading was applied along the nominal line of contraflexure. The only variable was the amount of supplementary bonded reinforcement located in the column region.

For the analysis of these specimens, the simplified approach was utilised and the influence of the prestress was taken into account by means of the equivalent reinforcement ratio. The principal details of these tests and the predicted punching strengths are presented in Table 7.7.

h) Tests by Long, Cleland and Kirk

Long et al (1978) reported tests on three \( \frac{1}{3} \) scale model slab-column sub-systems, in which the influence of moment transfer was investigated.

In view of the problematic nature of the specimen boundary conditions, the simplified approach, based on the longer span, was again adopted for the analysis. The principal details of these tests and the predicted punching strengths are presented in Table 7.8.
i) Tests by Franklin

The series of \( \frac{1}{3} \) scale models tested by Franklin (1981) were representative of the full interior panel of an unbonded post-tensioned flat slab. Of particular interest in this investigation, was the examination of the influence of the slab boundary conditions. This revealed little difference in the behaviour and strength of the uniformly loaded panels having floating edge moment restraint and the full panel specimens with free edges and loading applied at the nominal line of contraflexure.

In the direction of moment transfer, a bonded tendon arrangement was utilised, which resulted in very lightly reinforced sections away from the column location. The effect of bonding is to attract moment to, and reduce the ductility at, the critical section, thereby negating the increase in the moment of resistance. Thus, in the analysis, the punching strength was determined on the basis of the equivalent level of reinforcement corresponding to uniform spacing of tendons throughout the panel. The simplified approach was utilised in conjunction with the modified capacity reduction factor. Details of these specimens and the predicted ultimate capacities are presented in Table 7.9.

j) Tests by Holowka, Dorton and Csagoly

As part of the Ontario Ministry of Transportation investigation into the enhanced punching strength of bridge deck panels, Holowka et al (1979) conducted a series of tests on approximately \( \frac{1}{4} \) scale models. The isolated slabs were subjected to concentrated loading through a circular pad and the finite lateral restraint was provided by means of a carefully fitted steel ring. The two main variables were the level of reinforcement and the span/depth ratio, with the concrete strength remaining constant throughout.

For each combination of variables, three identical specimens were cast and the average results were utilised for the analysis. The degree of
lateral restraint was calculated by consideration of the stiffness of the steel restraining ring. The principal details of these tests and the predicted failure loads are presented in Table 7.10.

7.4.2 Predictions by other methods

The numerous procedures which are available for the prediction of the punching strength of the conventional slab specimen are not directly applicable to laterally restrained slabs. Indeed, few methods exist in which the influence of compressive membrane action is recognised and of these, many are too complex for hand calculation. In view of this, the results of the series of large panel specimens tested by the author are compared with the predictions by the integrated procedure, an alternative analytical approach due to Masterson (1971) and the present British (CP110, 1972) and American (ACI 318-77) building codes.

The proposed method of prediction is based upon the simplified integrated procedure, in which the appropriate magnitude of the relative arching resistance is incorporated. In the full panel specimens the arching moment of resistance is considered to amount to approximately one fifth of the capacity for rigid lateral support. Thus, in view of the relatively low degree of restraint, the arching component in the intermediate range of slabs can be assumed to depend directly upon the stiffness of the restraining ring. On the basis of the thick cylinder analogy, the relative arching resistance in the large panel specimens varies according to the relationship shown in Fig. 7.13.

The flexural punching strength is given by the integrated moment of resistance in conjunction with the interpolative moment factor. As the nominal line of radial moment contraflexure is located at the slab supports, the moment factor for overall tangential yielding can be considered as that
for the equivalent contraflexure specimen. To allow for the superior flexural characteristics of the large panel specimens, the elastic moment factor can be appropriately modified by the analytical relationship presented in Fig. 7.3. The ultimate shear capacity is given by the simplified shear criterion in which the minimum depth of the contact zone is incorporated.

In the analytical approach proposed by Masterson (1971), the influence of compressive membrane action is taken into account by the incorporation of the membrane moment into the lower bound flexural criterion of failure. The calculation of this additional moment of resistance is based on the method of Tong and Batchelor (1971), in which it is assumed that the magnitude is dependent on the critical slab deflection and the resistance of the surrounding tension ring to cracking. This method is ideally suited for the analysis of the large panel specimens, as the variable zone of restraint can be easily accommodated. The procedure is presented in detail in Appendix C.

The code methods of prediction have previously been described with reference to the punching strength of the conventional slab specimen. As the influence of compressive membrane action is not recognised, the comparison with the results of the large panel specimens is included to illustrate the inadequacies of the present procedures with respect to the enhanced punching strength of the interior slab-column connection.

7.4.3. Comparison with the author's test results

The full details of the \( \frac{1}{4} \) scale large panel specimens tested by the author have been presented in Chapters 3 and 4. These tests provide a comprehensive base from which the influence of the slab boundary conditions can be examined. In this respect, the careful control of the
major variables now enables the direct assessment of the importance of compressive membrane action in relation to the enhanced punching strength.

The failure loads of the large panel specimens are shown in comparison with the predicted ultimate capacities in Table 7.11. In addition, the results are compared with the predictions based on the average concrete strength of 40 N/mm², in Table 7.12. The correlation coefficients corresponding to each method of prediction, for the different concrete strengths, are shown in Tables 7.13 and 7.14.

The mean ratios of the experimental to the predicted punching strengths, by the Code procedures, are well in excess of unity. This is particularly so for the British method (CP110, 1972), and in terms of the design strength, the discrepancy is further increased by the inclusion of the partial factor of safety \( \gamma_m = 1.25 \). However, because of the recognition given to the influence of the level of reinforcement, the coefficient of variation is marginally better than for the American (ACI 318-77) approach.

Interestingly, the predictions given by CP110 for the full panel models, are reasonably consistent, although overly conservative. However, the modification of the present approach by the adoption of a less conservative shear criterion is unlikely to be satisfactory in view of the flexural nature of failure. Without recognition of such important parameters as the yield strength of the reinforcement and the slab span, the prediction of the punching strength on the basis of a nominal ultimate shear stress will result in a widely variable margin of safety. Thus, the present Code methods are inefficient in the use of material strength to overcome the problem of punching failure at the interior slab-column connection.

The flexural approach proposed by Masterson (1971), in which account is taken of the extent of the slab beyond the line of contraflexure, presents a much improved correlation with the test results. However, there are two points of particular interest concerning the enhanced criterion of failure.
a) The magnitude of the additional membrane moment is predicted as being the same irrespective of the level of reinforcement. This is not as would be expected from the understanding of the principles of arching and is contradictory to the experimental evidence. The danger lies in the prediction of the punching strength of slabs having fairly high levels of reinforcement ($\rho > 1\%$) for which the enhancement due to compressive membrane action may be overestimated.

b) If the tension in the surround can be sustained without cracking, the membrane moment is predicted as being infinite. Thus, the method is not applicable to slabs having rigid lateral restraint. This is somewhat illogical, as the analysis should become more effective with the removal of the uncertainty as to the degree of restraint.

Despite the preceding criticism, the method does provide a simple means by which the compressive membrane enhancement can be incorporated into the flexural approach to punching failure. Furthermore, there is good correlation with the results of the tests on the large panel specimens.

The integrated procedure for the prediction of the punching strength is shown to yield a significantly better correlation with the test results, than any of the alternative methods. The mean ratio of the experimental to the predicted failure load is 1.273 with a coefficient of variation equal to 6.8%. This correlation is similar to that obtained in the comparison of the method for the prediction of the punching strength of the conventional specimen with the results of the contraflexure models tested by the author. In terms of the variation, it represents an improvement of approximately 28% over that given by Masterson's method.

The correlation of the results with the predictions which are based on the average concrete strength, is of particular interest. In this case,
although the mean ratio of the experimental to the predicted failure load, by the proposed method, remains unchanged, the coefficient of variation is reduced to only 5.3%. This represents a dramatic improvement over the preceding correlation and is not reflected in the alternative procedures.

The reason for the more consistent predictions can perhaps be attributed to size effects in the concrete control specimens. It is widely recognised that the variability in the results of concrete compression tests increases as the specimen size is decreased (Mirza et al, 1972). In this respect, the assessment of the concrete strength in models by means of crushing tests on reduced scale cubes, may be the source of some inconsistencies. Thus, as the same concrete mix was utilised throughout the test programme, it may be more appropriate to base the analysis on the overall average cube strength, thereby minimising the influence of unrepresentative variations.

The predictions by the various methods, based on the average concrete strength, are further shown in comparison with a series of large panel test results, in Fig. 7.14. It can be seen that the predicted trend by the integrated procedure is consistently lower bound to the entire range of results. Although Masterson's method also predicts the correct trend it is unconservative for the more heavily reinforced specimens, the results of which are not illustrated. Some margin of safety is desirable in view of the possibility of size effects in the models. As the Code procedures do not recognise the enhanced capacity of the larger panels, the punching strength is predicted as being constant.

The uniformity with which the trend of the results is predicted by the integrated procedure, suggests that the strength enhancement is consistent with the relative arching resistance which is considered to develop. It is interesting to note that the predicted mode of failure for the full panel specimens is flexural. This is in agreement with the experimental measurements of strains and deformations in the models, which indicate some degree of plasticity prior to failure.
Although the trend of the results suggests further enhancement for even larger panels, the punching strength is limited by the enhanced shear capacity, which is also shown in Fig. 7.14. In this respect, the available increase in strength requires verification by means of further test results, perhaps from isolated full panel specimens with additional lateral restraint provided at the outer perimeter.

7.4.4 Overall correlation

The effectiveness of the integrated procedure for the prediction of the punching strength of laterally restrained slabs can be appreciated from the good correlation with various test results. For the 109 specimens analysed, the mean ratio of the experimental to the predicted failure load is 1.131 with a coefficient of variation equal to approximately 13%. As shown in Fig. 7.15(a) there is no significant trend with the results in relation to the reinforcement ratio, which ranges from zero to beyond the balanced level.

Although the variation in the predictions is marginally greater than for the individual methods from which the integrated procedure is derived, the difference is to some extent due to the difficulty of prescribing the correct analytical boundary conditions. This is particularly so for the specimens tested by Aoki and Seki (1971) and Vanderbilt (1972), in which the lateral restraint is provided by integral ring beams. However, even with the uncertainties involved, the prediction of these results is reasonably consistent, although less satisfactory than the excellent correlation obtained for the comprehensive test series of Snowdon (1973) and Holowka et al (1979).

The predicted modes of failure for the various test results are shown in Fig. 7.15(b). It can be seen that, unlike the conventional
specimens, the modes of failure in the laterally restrained slabs cannot be segregated according to the reinforcement index. There is a considerably greater proportion of the total number of slabs predicted to fail in the 'brittle' modes of shear and flexural compression, even at low reinforcement ratios. This is to be expected, as the enhancing influence of compressive membrane action is, in some respects, similar to the addition of flexural reinforcement. Thus, un reinforced slabs having rigid lateral restraint can exhibit strengths which are comparable to the equivalent heavily reinforced, conventional specimens.

It is interesting to note that all of the interior slab-column connections are predicted to fail in the 'yield' mode, which is associated with some ductility prior to punching. This is because of the relatively low degree of restraint inherent in the isolated full panel specimen.

The validity of the simplified approach for the prediction of the punching strength of the interior slab-column connection is illustrated in Fig. 7.15(c). In comparison to the more rigorous analysis of the laterally restrained slabs the correlation is slightly better, with a mean ratio of the experimental to the predicted failure load of 1.218 and a coefficient of variation equal to approximately 11%. This correlation is very satisfactory in view of the simplified treatment of the complex behaviour at the interior slab-column connection. In addition, because of the possibility of size effects in the model slabs, the inherent degree of conservatism is not unreasonable. That the margin of safety is not increased by the integration of the two individually conservative procedures can be attributed to several factors, of which the following is perhaps the most significant.

For simplicity, the analysis of many of the tests is based on the assumption of rigid lateral restraint, whereas in reality, this cannot be the case due to the elasticity of the restraining ring. This is
particularly so for the specimens having concrete surrounds, which can compress radially. Consequently, in the case of lightly reinforced slabs, in which the influence of compressive membrane action is most significant, the margin of safety is diminished by the reduction in the degree of restraint. Conversely, at high levels of reinforcement, for which the theoretical method for arching is less conservative, the resultant margin of safety is more consistent with that for the conventional slab specimen. Thus, over the full range of the reinforcement ratio, the degree of conservatism is approximately constant and is consistent with the individual methods of analysis.

The treatment of the prestressed slabs and the adoption of the empirically modified capacity reduction factor to account for eccentric loading, are shown to be satisfactory in respect of the available test results. In both cases, the simplified approach for the prediction of the punching strength of the isolated full panel specimen is appropriate.

In view of the wide range of material and geometric variables encountered in the analysis of the test specimens, the overall correlation with the integrated procedure is very satisfactory. The rigorous method is suitable for analysis of slabs for which the boundary restraints can be appropriately defined. In the case of the interior slab-column connection, for which the degree of restraint in the isolated full panel specimen is difficult to deduce, the simplified approach is also satisfactory. Thus, the integrated procedure is suitable for the prediction of the enhanced punching strength of various forms of slab, in which the influence of compressive membrane action is apparent.
7.4.5 Discussion

The integrated procedure has been shown to yield consistent and generally conservative predictions of the enhanced punching strength of laterally restrained slabs. Some consideration is now given to the practical application of the method and the implications for the design of continuous slabs subjected to concentrated loading.

In practice, the necessary lateral restraint is unlikely to be provided by artificial means, as this would be an inefficient form of reinforcement, but by the inherent continuity of the slab, or an integral beam system. Thus, in order to calculate the enhanced punching strength of a continuous slab, the designer must assess both the length of the laterally restrained radial strip of slab and the degree of restraint. This assessment may not be too difficult in the case of a beam and slab system, for which the zone of restraint is self-evident and the assumption of rigid lateral support is usually realistic. However, for continuous flat slab structures, the extent of the restrained portion of slab and the degree of lateral restraint are not so easy to define.

This research had demonstrated the validity of assuming that the punching strength can be determined on the basis of the enhanced capacity of the conventional slab specimen, which is equivalent to the concept of lateral restraint around the nominal line of contraflexure. In the majority of practical situations, the position of the nominal line of zero radial moment can be estimated with sufficient accuracy by means of a simple elastic analysis. As the redistribution of moments which accompanies inelastic behaviour will tend to tighten the ring of radial moment contraflexure, this solution can be expected to yield a conservative prediction of the enhanced punching strength, provided the degree of restraint is realistically assessed. This latter problem can be resolved by the use of some simple technique to determine the elastic stiffness
of the restraint, and making an appropriate allowance for the effective reduction in stiffness due to cracking and rotation. In this respect, it may be necessary to conservatively estimate the relative arching resistance in slabs having poorly defined lateral support.

The practical implications of adopting the more economical design procedures proposed in this investigation are considerable. Although the use of the rigorous method for the prediction of the enhanced punching strength of laterally restrained slabs would require some additional effort on the part of the designer, the expense incurred would be far outweighed by the reduced cost of construction. Furthermore, as the punching strength of the interior slab-column connection is often the decisive factor in the design of a flat slab structure, the adoption of the simplified approach, which is straightforward to use, would improve the viability of this form of construction.

The flexural nature of punching failure at the interior slab-column connection in slabs having normal reinforcement ratios means that the ultimate capacity can be effectively increased by the provision of additional negative reinforcement or the use of higher yield steel. However, by making use of the full ultimate strength, including the compressive membrane effect, it may be necessary to introduce higher load factors in order to avoid problems concerning the slab serviceability under high levels of sustained loading. Alternatively the enhancement in strength due to compressive membrane action could be regarded as an additional safeguard against punching failure under overload, in which case, the capacity of the conventional slab specimen could be more fully utilised.
7.5 CONCLUSIONS

The following points of importance concerning the enhanced punching strength of laterally restrained slabs can be concluded:

1. The enhancement in the punching strength of the large panel specimen is primarily attributable to the effect of compressive membrane action.

2. The lateral restraint in the large panel models can be considered to be provided by the zone of slab beyond the nominal line of contraflexure. The inherent degree of restraint in the isolated full panel specimen is normally relatively small.

3. The integrated moment of resistance is compatible with the flexural criterion of failure. Furthermore, the normal moment factors are applicable to simply supported laterally restrained slabs.

4. The enhanced ultimate shear capacity can be determined by approximating the depth of the slab compression zone as the contact depth of the unreinforced section. This assumption is consistent with the mode of failure and considerably simplifies the analysis.

5. The integrated procedure can be utilised for the analysis of slabs with any degree of lateral support. In this approach, it is necessary to consider the effect of arching action in the radial strip of slab which extends from the loaded area to the zone of restraint.

6. The satisfactory treatment of prestressed slabs is possible by utilising the equivalent level of reinforcement in the integrated procedure.

7. The increased contribution of compressive membrane action in the full panel specimens subjected to eccentric loading can be taken into account by adopting the empirically modified capacity reduction factor.
8. The use of the overall average cube strength in the analysis of the large panel models improves the correlation of the predicted ultimate capacities with the test results.

9. The integrated procedure for the prediction of the enhanced punching strength of laterally restrained slabs gives good correlation with a wide range of test results from various sources.

10. The simplified approach for the prediction of the enhanced punching strength of the interior slab-column connection is more consistent and generally less conservative than the present code procedures.
### TABLE 7.1 LATERALLY RESTRained SLABS TESTED BY TAYLOR AND HAYES

<table>
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<tr>
<th>SLAB NO.</th>
<th>c</th>
<th>d</th>
<th>U</th>
<th>p</th>
<th>f_y</th>
<th>P_T</th>
<th>p_p</th>
<th>p_T/p_p</th>
<th>MODE OF FAILURE</th>
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</thead>
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<td>1R2(a)</td>
<td>51</td>
<td>-</td>
<td>36.8</td>
<td>-</td>
<td>-</td>
<td>84.9</td>
<td>80.7</td>
<td>1.052</td>
<td>s</td>
</tr>
<tr>
<td>1R2(b)</td>
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<td>-</td>
<td>32.5</td>
<td>-</td>
<td>-</td>
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<td>76.1</td>
<td>1.168</td>
<td>s</td>
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<td>-</td>
<td>-</td>
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<td>116.0</td>
<td>1.291</td>
<td>s</td>
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<td>-</td>
<td>27.3</td>
<td>-</td>
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<td>s</td>
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<td>110.0</td>
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<td>s</td>
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<td>1.570</td>
<td>377</td>
<td>139.3</td>
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slab size = 864 mm
overall depth = 76 mm
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<th>$f_y$</th>
<th>$P_T$</th>
<th>$P_p$</th>
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<td>y</td>
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<td>44.5</td>
<td>33.5</td>
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<td>90.5</td>
<td>94.9</td>
<td>0.954</td>
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<td>1.750</td>
<td>359</td>
<td>53.5</td>
<td>49.7</td>
<td>1.076</td>
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slab span = 1600 mm
column size = 152 mm

TABLE 7.2 FULL PANEL SPECIMENS TESTED BY LONG

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<th>SLAB NO.</th>
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<th>$d$</th>
<th>$f'_c$</th>
<th>$\rho$</th>
<th>$f_y$</th>
<th>$P_T$</th>
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<th>$P_T/P_p$</th>
<th>MODE OF FAILURE</th>
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<td>DM-1</td>
<td>0.095</td>
<td>30.2</td>
<td>30.5</td>
<td>1.040</td>
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<td>15.0</td>
<td>1.427</td>
<td>y</td>
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<td>0.000</td>
<td>31.8</td>
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<td>30.9</td>
<td>1.110</td>
<td>y</td>
</tr>
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<td>29.5</td>
<td>28.2</td>
<td>1.060</td>
<td>333</td>
<td>3.1</td>
<td>2.2</td>
<td>1.409</td>
<td>y</td>
</tr>
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<td>0.200</td>
<td>29.5</td>
<td>30.2</td>
<td>1.210</td>
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slab span = 1016 mm
column size = 76 mm

TABLE 7.3 FULL PANEL SPECIMENS TESTED BY MASTERSON
| SLAB NO. | \( \phi_c \) mm | \( d \) mm | \( f'_c \) N/mm² | \( p \) % | \( f_y \) N/mm² | \( P_t \) kN | \( \frac{P}{P_t} \) | \( P_t \) kN | \( \frac{P}{P_t} \) | MODE OF FAILURE |
|----------|----------------|-----------|----------------|----|-------------|----------|-------------|----------|-------------|----------------|----------------|
| XC-2     | 190            | 78        | 35.7           | 0.910 | 408         | 275      | 293         | 0.939    | s           |                |
| XC-3     | 190            | 71        | 33.8           | 1.000 | 408         | 275      | 235         | 1.170    | c           |                |
| XC-4     | 190            | 62        | 24.5           | 2.300 | 372         | 147      | 138         | 1.065    | c           |                |
| XC-5     | 190            | 73        | 24.2           | 1.300 | 369         | 206      | 189         | 1.090    | c           |                |
| XC-6     | 190            | 63        | 23.8           | 1.130 | 379         | 153      | 138         | 1.109    | c           |                |
| FC-1     | 190            | 75        | 33.4           | 0.950 | 354         | 256      | 270         | 0.948    | c           |                |
| FC-2     | 190            | 75        | 23.5           | -     | -           | 131      | 119         | 1.101    | y           |                |
| FC-3     | 190            | 90        | 23.0           | -     | -           | 132      | 147         | 0.898    | y           |                |
| FC-4     | 190            | 73        | 23.0           | 0.390 | 391         | 186      | 167         | 1.114    | c           |                |
| FC-5     | 190            | 70        | 22.5           | 0.400 | 391         | 185      | 150         | 1.233    | c           |                |
| FC-6     | 190            | 73        | 22.5           | 0.770 | 391         | 177      | 163         | 1.086    | c           |                |
| FC-7     | 190            | 70        | 20.1           | 0.810 | 391         | 186      | 134         | 1.388    | c           |                |
| FC-8     | 190            | 74        | 29.4           | 0.380 | 391         | 177      | 213         | 0.831    | y           |                |

slab size: XC-2 to FC-1 = 1400 mm  
FC-2 = 1200 mm  
FC-3 to FC-8 = 1600 mm  
overall depth (excluding FC-2 and FC-3) = 100 mm

**TABLE 7.4 LATERALLY RESTRAINED SLABS TESTED BY AOKI AND SEKI**
<table>
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<th>SLAB NO.</th>
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<th>$d$ (mm)</th>
<th>$f'_c$ (N/mm²)</th>
<th>$f_y$ (N/mm²)</th>
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<td>43.0</td>
<td>50.7</td>
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<td>388</td>
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<td>86.7</td>
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slab span = 2896 mm
overall depth = 51 mm

TABLE 7.5 LATERALLY RESTRAINED SLABS TESTED BY VANDERBILT


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<th>SLAB NO.</th>
<th>$\phi_c$ mm</th>
<th>$d$ mm</th>
<th>$U$ N/mm²</th>
<th>$\rho$ %</th>
<th>$f_y$ N/mm²</th>
<th>$P_T$ kN</th>
<th>$P_p$ kN</th>
<th>$P_T/P_p$</th>
<th>MODE OF FAILURE</th>
</tr>
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<tbody>
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<td>1/2</td>
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<td>82</td>
<td>54.1</td>
<td>0.610</td>
<td>530</td>
<td>340</td>
<td>310</td>
<td>1.097</td>
<td>y</td>
</tr>
<tr>
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<td>82</td>
<td>54.7</td>
<td>0.610</td>
<td>530</td>
<td>302</td>
<td>312</td>
<td>0.968</td>
<td>y</td>
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<td>82</td>
<td>50.3</td>
<td>0.610</td>
<td>530</td>
<td>309</td>
<td>295</td>
<td>1.047</td>
<td>y</td>
</tr>
<tr>
<td>7/8</td>
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<td>82</td>
<td>70.5</td>
<td>0.610</td>
<td>530</td>
<td>374</td>
<td>362</td>
<td>1.033</td>
<td>s</td>
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<td>400</td>
<td>359</td>
<td>356</td>
<td>1.008</td>
<td>c</td>
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<td>400</td>
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<td>337</td>
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<td>c</td>
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<td>107</td>
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<td>0.470</td>
<td>530</td>
<td>413</td>
<td>420</td>
<td>0.983</td>
<td>s</td>
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<td>400</td>
<td>537</td>
<td>501</td>
<td>1.072</td>
<td>s</td>
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<td>456</td>
<td>0.969</td>
<td>s</td>
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<td>55.0</td>
<td>0.850</td>
<td>395</td>
<td>427</td>
<td>447</td>
<td>0.955</td>
<td>s</td>
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<td>489</td>
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<td>107</td>
<td>64.5</td>
<td>0.470</td>
<td>530</td>
<td>448</td>
<td>471</td>
<td>0.951</td>
<td>s</td>
</tr>
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<td>400</td>
<td>433</td>
<td>452</td>
<td>0.958</td>
<td>s</td>
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<td>101</td>
<td>78.3</td>
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<td>400</td>
<td>507</td>
<td>549</td>
<td>0.923</td>
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<td>107</td>
<td>55.5</td>
<td>0.470</td>
<td>530</td>
<td>451</td>
<td>439</td>
<td>1.027</td>
<td>s</td>
</tr>
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<td>35/36</td>
<td>200</td>
<td>107</td>
<td>67.0</td>
<td>0.470</td>
<td>530</td>
<td>508</td>
<td>480</td>
<td>1.058</td>
<td>s</td>
</tr>
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<td>45/46</td>
<td>200</td>
<td>-</td>
<td>45.7</td>
<td>-</td>
<td>-</td>
<td>409</td>
<td>345</td>
<td>1.186</td>
<td>s</td>
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<tr>
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<td>107</td>
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<td>530</td>
<td>523</td>
<td>533</td>
<td>0.981</td>
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<td>49/50</td>
<td>100</td>
<td>107</td>
<td>44.7</td>
<td>0.470</td>
<td>530</td>
<td>291</td>
<td>264</td>
<td>1.102</td>
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<td>51/52</td>
<td>200</td>
<td>107</td>
<td>59.0</td>
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<td>530</td>
<td>438</td>
<td>452</td>
<td>0.969</td>
<td>s</td>
</tr>
</tbody>
</table>

slab size = 1720 mm

overall depth: 1/2 - 11/12 = 100 mm
13/14 - 51/52 = 125 mm

TABLE 7.6 LATERALLY RESTRANDED SLABS TESTED BY SNOWDON
### Table 7.7 Prestressed Panels Tested by Smith and Burns

<table>
<thead>
<tr>
<th>SLAB NO.</th>
<th>c (mm)</th>
<th>d (mm)</th>
<th>f'_c (N/mm²)</th>
<th>P_e (%)</th>
<th>f_y (N/mm²)</th>
<th>P_T (kN)</th>
<th>P_p (kN)</th>
<th>P_T/P_p</th>
<th>MODE OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1</td>
<td>203</td>
<td>54.4</td>
<td>30.1</td>
<td>1.076</td>
<td>386</td>
<td>112.6</td>
<td>103.3</td>
<td>1.090</td>
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<td>203</td>
<td>54.4</td>
<td>29.0</td>
<td>1.226</td>
<td>386</td>
<td>121.8</td>
<td>108.7</td>
<td>1.121</td>
<td>y</td>
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<td>S-3</td>
<td>203</td>
<td>54.4</td>
<td>32.0</td>
<td>1.316</td>
<td>386</td>
<td>135.6</td>
<td>118.1</td>
<td>1.148</td>
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slab span = 2743 mm

### Table 7.8 Full Panel Specimens Tested by Long, Cleland and Kirk

<table>
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<th>SLAB NO.</th>
<th>e/L</th>
<th>d (mm)</th>
<th>U (N/mm²)</th>
<th>P_e (%)</th>
<th>f_y (N/mm²)</th>
<th>P_T (kN)</th>
<th>P_p (kN)</th>
<th>P_T/P_p</th>
<th>MODE OF FAILURE</th>
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<td>34.3</td>
<td>0.970</td>
<td>316</td>
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<td>61.8</td>
<td>0.987</td>
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<tr>
<td>2</td>
<td>0.016</td>
<td>51.5</td>
<td>32.0</td>
<td>0.970</td>
<td>316</td>
<td>66.5</td>
<td>63.2</td>
<td>1.052</td>
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<td>3</td>
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<td>26.5</td>
<td>0.970</td>
<td>316</td>
<td>64.3</td>
<td>64.6</td>
<td>0.995</td>
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</table>

long span = 2000 mm

column size: 1 and 2 = 150 mm, 3 = 225 mm

### Table 7.9 Prestressed Panels Tested by Franklin

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<th>SLAB NO.</th>
<th>e/L</th>
<th>d (mm)</th>
<th>U (N/mm²)</th>
<th>P_e (%)</th>
<th>f_y (N/mm²)</th>
<th>P_T (kN)</th>
<th>P_p (kN)</th>
<th>P_T/P_p</th>
<th>MODE OF FAILURE</th>
</tr>
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<td>44.5</td>
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<td>0.070</td>
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<td>1.337</td>
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<td>1.330</td>
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<td>46.7</td>
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<td>1.293</td>
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slab span = 2540 mm

column size = 169 mm
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<th>SLAB NO.</th>
<th>$\phi_c$ mm</th>
<th>$d$ mm</th>
<th>$f'_c$ N/mm²</th>
<th>$\rho$ %</th>
<th>$f_y$ N/mm²</th>
<th>$P_T$ kN</th>
<th>$P_p$ kN</th>
<th>$P_T/P_p$</th>
<th>MODE OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1-A3</td>
<td>63.5</td>
<td>38.1</td>
<td>27.4</td>
<td>0.200</td>
<td>310</td>
<td>42.4</td>
<td>40.0</td>
<td>1.060</td>
<td>s</td>
</tr>
<tr>
<td>B1/B2</td>
<td>63.5</td>
<td>31.8</td>
<td>27.7</td>
<td>0.200</td>
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<td>36.8</td>
<td>31.4</td>
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<td>25.2</td>
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</tr>
</tbody>
</table>

slab diameter = 572 mm

A, D, G, I : $h = 44.5$ mm, $K = 3.273$ kN/mm²
B, E, H, J : $h = 38.1$ mm, $K = 2.732$ kN/mm²
C, F, K, : $h = 31.8$ mm, $K = 2.182$ kN/mm²

**TABLE 7.10** LATERALLY RESTRAINED SLABS TESTED BY HOLOWKA, DORTON AND CSAGOLY
<table>
<thead>
<tr>
<th>SLAB NO.</th>
<th>$P_P$ kN</th>
<th>$P_M$ kN</th>
<th>$P_{CP110}$ kN</th>
<th>$P_{ACI}$ kN</th>
<th>$P_T/P_P$</th>
<th>$P_T/P_M$</th>
<th>$P_T/P_{CP110}$</th>
<th>$P_T/P_{ACI}$</th>
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<tbody>
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<td>35.7</td>
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<td>1.164</td>
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<td>73.5</td>
<td>40.6</td>
<td>42.9</td>
<td>1.109</td>
<td>0.949</td>
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<td>0.999</td>
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<td>1.225</td>
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**Table 7.11** LARGE PANEL SPECIMENS TESTED BY THE AUTHOR
(actual cube strength)
<table>
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<tr>
<th>SLAB NO.</th>
<th>$P_p$ kN</th>
<th>$P_m$ kN</th>
<th>$P_{CP110}$ kN</th>
<th>$P_{ACI}$ kN</th>
<th>$P_T$</th>
<th>$P_T$</th>
<th>$P_T$</th>
<th>$P_T$</th>
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<td>1.146</td>
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<td>1.131</td>
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<td>1.251</td>
<td>1.137</td>
<td>2.525</td>
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</table>

**TABLE 7.12** LARGE PANEL SPECIMENS TESTED BY THE AUTHOR  
(average cube strength = 40 N/mm²)
### TABLE 7.13 CORRELATION OF VARIOUS METHODS WITH AUTHOR'S TEST RESULTS (actual cube strength)

<table>
<thead>
<tr>
<th>METHOD</th>
<th>MEAN (16 TESTS)</th>
<th>STANDARD DEVIATION</th>
<th>COEFFICIENT OF VARIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>author</td>
<td>1.273</td>
<td>0.087</td>
<td>0.068</td>
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<tr>
<td>Masterson</td>
<td>1.156</td>
<td>0.100</td>
<td>0.087</td>
</tr>
<tr>
<td>CP110</td>
<td>2.015</td>
<td>0.204</td>
<td>0.101</td>
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<tr>
<td>ACI</td>
<td>1.708</td>
<td>0.191</td>
<td>0.112</td>
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</table>

### TABLE 7.14 CORRELATION OF VARIOUS METHODS WITH AUTHOR's TEST RESULTS (average cube strength = 40 N/mm²)

<table>
<thead>
<tr>
<th>METHOD</th>
<th>MEAN (16 TESTS)</th>
<th>STANDARD DEVIATION</th>
<th>COEFFICIENT OF VARIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>author</td>
<td>1.273</td>
<td>0.068</td>
<td>0.053</td>
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<td>Masterson</td>
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<td>0.095</td>
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<tr>
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</tr>
<tr>
<td>ACI</td>
<td>1.710</td>
<td>0.205</td>
<td>0.120</td>
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</table>
a) sectional elevation

b) plan

Fig. 7.1 IDEALISED COMPRESSION MEMBRANE ACTION
Fig. 7.2 ANALYTICAL DISTRIBUTIONS OF ELASTIC BENDING MOMENTS

Fig. 7.3 VARIATION OF ELASTIC MOMENT FACTOR
Fig. 7.4 MEMBRANE STRAINS IN LARGE PANEL SPECIMEN
Fig. 7.5  ANALYTICAL DISTRIBUTIONS OF MEMBRANE STRAIN

Fig. 7.6  DEFORMATION OF THE ZONE OF RESTRAINT
Fig. 7.7 CONVERGENCE OF NOMINAL SHEAR CRITERIA

\[ \frac{V_u}{(100p)^{0.25}} \leq \frac{[1.13 + 0.25(100p)^{0.25}]}{1.0} \]

% reinforcement

\[ \frac{P_T}{P_0} = \frac{P_0}{1 + 10^8/L} \]

\[ \frac{P_T}{P_0} = \frac{P_0}{1 + 15^8/L} \]

\[ \frac{P_T}{P_0} = \frac{M_T}{M_0} = 1 \]

Fig. 7.8 INTERACTION OF MOMENT AND SHEAR LOADING

(LONG AND MASTERS, 1974)
Fig. 7.9 'JOGGLE ACTION' IN SLAB UNDER PURE MOMENT LOADING

Fig. 7.10 COMPARISON OF CAPACITY REDUCTION FACTORS FOR COMBINED LOADING OF FULL PANEL SPECIMENS
Fig. 7.11 APPROXIMATE MOMENT FACTOR RELATIONSHIPS

simplified relationship:

\[
\frac{M_b + 0.2M_{ar}}{d^2 f'_c} = 0.62 \left( \frac{\rho f_y}{f'_c} \right)^{0.67}
\]

\( f'_c = 20 \text{N/mm}^2 - 40 \text{N/mm}^2 \)

L/h = 20 - 40

Fig. 7.12 INTEGRATED MOMENT OF RESISTANCE IN FULL PANEL SPECIMEN
**Fig. 7.13** ARCHING COMPONENT IN LARGE PANEL SPECIMENS

Relative arching resistance

- Relative arching resistance vs. Slab size (mm)
- Thick cylinder analogy
- Restraining ring

**Fig. 7.14** COMPARISON OF VARIOUS METHODS OF PREDICTION

- Ultimate shear capacity
- Masterson (1971)
- Author
- ACI (318-77)
- CP110 (1972) \(*\)
- *γ_m = 1.25* removed

R-08 tests:
- Contraflexure models
  - \(L/h = 31\)
  - \(ρ = 0.8\%\)
- Full panel models

Load (kN) vs. Slab size (mm)
Fig. 7.15 CORRELATION OF TEST AND PREDICTED FAILURE LOADS: LATERALLY RESTRAINED SLABS
Chapter 8

CONCLUSIONS

8.1 INTRODUCTION

8.2 GENERAL CONCLUSIONS

8.3 RECOMMENDATIONS FOR FUTURE RESEARCH
8.1 INTRODUCTION

The principal objectives of this research into punching failure and compressive membrane action in reinforced concrete slabs have been attained. As the major points of importance have been concluded in detail at the end of the relevant chapters, it is now appropriate to take an overall perspective of the work. Therefore, in the following sections, the more general conclusions of the study are presented and finally some recommendations are made for future research on this subject.

8.2 GENERAL CONCLUSIONS

1. The punching strength of the interior slab-column connection is significantly influenced by the flexural capacity of the portion of slab within the nominal line of contraflexure. Therefore, in normal flat slabs, the failure load can be effectively increased by the provision of additional flexural reinforcement, the use of higher yield steel or reducing the span/depth ratio.

2. The enhanced punching strength of the interior slab-column connection in specimens having more realistic boundary conditions, is primarily attributable to the effect of compressive membrane action. Typically, the enhancement in strength may amount to between 30% and 50% of the capacity of the equivalent conventional representation.

3. High levels of sustained loading are unlikely to adversely affect the ultimate capacity of the interior slab-column connection. However, in order to utilise the enhanced punching strength in the design of flat slab structures, it may be necessary to adopt higher load factors if problems concerning slab serviceability are to be avoided.
4. The rational method for the prediction of the punching strength of the conventional slab specimen is more consistent and generally less conservative than the present code procedures. Furthermore, this approach delineates between the various modes of punching, enabling the ductility or brittleness of failure to be assessed.

5. The theory of arching action in slab strips has been justified by the good agreement between the predicted ultimate capacities and the results of tests from various sources. The direct procedure can be satisfactorily utilised for the ultimate load analysis of strips having a variety of boundary conditions, including finite lateral restraint.

6. The assumptions upon which the integrated procedure for the prediction of the enhanced punching strength of laterally restrained slabs is based, have been validated by the good correlation obtained from the comparison of the predicted failure loads with a wide range of test results. The rigorous method is suitable for the analysis of slabs for which the degree of lateral restraint can be accurately assessed and the simplified approach is applicable to the interior slab-column connections of continuous flat slabs.

7. The integrated procedure is not limited to the prediction of the punching strength of isotropically reinforced slabs subjected to concentrated loading. The effects of concentrating the reinforcement in a narrow band passing directly over the column region, eccentricity of loading and prestress can be satisfactorily included in the method of analysis.
8. The practical implications of utilising the effect of compressive membrane action in the design provisions for punching failure are considerable. The most immediate benefits are likely to accrue from the introduction of more economical design procedures for heavily restrained panels subjected to localised transient loadings. However, the adoption of a more rational method for the prediction of the punching strength of the interior slab-column connection may eventually be of even greater importance, as it could significantly improve the viability of flat slab construction. The procedures presented in this thesis provide a basis for the development of a more realistic design approach to punching failure and compressive membrane action in reinforced concrete slabs.

8.3 RECOMMENDATIONS FOR FUTURE RESEARCH

a) In order to predict the ultimate flexural capacity of slabs subjected to concentrated loading, it is necessary to compute the appropriate moment factors. Although this is a relatively straightforward procedure for simple slab specimens it is more difficult in such cases as asymmetric or perforated slabs. Thus, it would be advantageous to produce an improved relationship for the nominal ultimate shear stress on a critical perimeter, which would be generally applicable. The parameters which influence the punching strength could be derived from the flexural analysis of the simple specimen and included in a semi-empirical expression based on the critical section at one half of the slab effective depth from the perimeter of the loaded area.

b) The serviceability criteria for flat slab structures requires careful investigation if the compressive membrane effect is to be utilised in design. Ideally, a programme of service load tests on prototype floor
slabs, in conjunction with ultimate load tests on the full panel models, would provide the necessary information upon which the appropriate load factors could be based.

c) The apparent 'size effect' in the results of the reduced scale models requires further investigation. In this respect one factor which may be significant is that high yield steel was utilised in the specimens tested as part of this study. As this was likely to contribute to an increase in strength by virtue of improved dowel resistance, it may be appropriate to allow for this effect in the prediction of the ultimate shear capacity.

d) The effect of sustained loading on the behaviour and ultimate capacity of the interior slab-column connection is worth some consideration from a theoretical viewpoint. It is envisaged that as the plasticity of the compressed concrete increases with time, the degree of restraint becomes relatively larger and hence, the integrated moment of resistance in the isolated full panel specimen is not significantly reduced.

e) The procedures which have been developed are at present only in a form suitable for application to square or circular slabs and columns. Therefore, it is necessary to extend and verify the methods to enable the analysis of rectangular areas. This might be accomplished simply by utilising the average column dimensions and slab span, although slightly higher reduction coefficients to allow for stress concentrations may be appropriate for very rectangular columns.

f) As the influence of eccentric loading on the punching strength of the full panel specimens has been allowed for by the empirical modification of the capacity reduction factor, it would be useful to further examine the interaction of moment transfer and compressive membrane action on
an analytical basis. The range of variables for which the empirical approach is applicable should also be extended.

g) Due to the flexural nature of punching failure at the interior slab-column connections in normal flat slabs, the incorporation of stirrups or shearheads may not increase the ultimate capacity as much as the present design recommendations suggest. Consequently, there is a need for tests on typical full panel specimens to determine the influence of shear reinforcement.

h) The theory of arching action in slab strips can be utilised for the ultimate load analysis of laterally restrained strips or beams subjected to various types of loading. Therefore, it may be possible to develop a method for the prediction of the enhanced flexural capacity of uniformly loaded continuous slabs. This would be of value in the justification of reduced design moments for flat slab structures and two-way beam and slab construction.

i) It is desirable to ensure adequate residual shear strength of slab-column connections, in order to prevent a progressive collapse of the complete building in the event of one initial punching failure. In this respect, as the effect of compressive membrane action in a real floor slab is to enhance the ultimate capacity, it is futile to develop design recommendations for post-punching resistance on the basis of the results from tests on conventional slab specimens.

The provision of continuous bottom steel through the slab-column connections appears to be the most effective means of ensuring adequate post-punching capacity. Thus, tests on full panel specimens are required to determine the quantity of bottom reinforcement which is necessary to absorb the energy released during failure and support the applied loading.
j) The resistance of continuous slabs to punching failure under concentrated impact loading is often of importance and is of particular interest in the design of offshore structures. Therefore, it would be of practical value to conduct impact loading tests on specimens with realistic boundary conditions in order to examine the interaction of compressive membrane action and dynamic effects. A major incentive for this study is that the use of less reinforcement and greater concrete cover would improve the durability performance of slabs in a marine environment.
APPENDICES
APPENDIX A  ANALYSIS OF CONVENTIONAL SLAB SPECIMEN

Al Elastic-plastic solution for circular plate with concentric ring load

The general solution for an edge supported circular plate with a concentric ring load has been developed from small deflection theory by Brotchie (1960). The following exposition is presented to facilitate the examination of the spread of tangential yield in relation to the flexural mode of punching failure. It is assumed that the Poisson's ratio for the material is equal to zero.

In the elastic range, the deflections, moments and shear force are given by:

\[ w = \frac{1}{D} \left[ C_1 + C_2 r^2 + C_3 \log_e r + C_4 r^2 \log_e r \right] \]

\[ M_{rm} = -\left[ 2C_2 - \frac{C_3}{r^2} + C_4(2\log_e r + 3) \right] \]

\[ M_{tm} = -\left[ 2C_2 + \frac{C_3}{r^2} + C_4(2\log_e r + 1) \right] \]

\[ Q_r = -\frac{C_4}{r} \]

In the case of tangential yielding, the deflections, moments and shear force are given by:

\[ w = \frac{1}{D} \left[ C_1 + C_2 r + C_3 r^2 + C_4 r \log_e r \right] \]

\[ M_{rm} = -2C_3 - \frac{C_4}{r} \]

\[ M_{tm} = M_y \]

\[ Q_r = -\left[ \frac{2C_3}{r} + \frac{M_y}{r} \right] \]
The coefficients $C_1 - C_4$ for the elastic and plastic zones of the partially yielded plate can be determined by the application of the following boundary conditions:

(i) $r = r_c$

$$M_{rm} = M_{tm} = M_y$$

$$Q_r = -\frac{p}{2\pi r_c}$$

(ii) $r = r_y$

$$M_{tm} = M_y = -\frac{DN}{r} dw$$

(iii) $r = r_s$

$$w = 0$$

$$M_{rm} = 0$$

$$Q_r = -\frac{p}{2\pi r_s}$$

In the elastic range, the coefficients can be derived as:

$$C_1 = \frac{Pr_s^2}{16\pi} \left[3 - 4\log e r_s - r^2 r_s \frac{P(\log e r - \log e r_s + 1) - M_y}{2(r^2 + r_s^2)} (1 + 2\log e r_s) \right]$$

$$C_2 = \frac{P}{16\pi} \left[2\log e r_s - 3 \right] + r^2 \frac{P(\log e r - \log e r_s + 1) - M_y}{2(r^2 + r_s^2)}$$

$$C_3 = r^2 r_s^2 \frac{P(\log e r - \log e r_s + 1) - M_y}{(r^2 + r_s^2)}$$

$$C_4 = \frac{P}{8\pi}$$
In the case of tangential yielding the coefficients can be derived as:

\[
C_1 = \frac{P}{2\pi} \left[ r_s - r_c \left( 1 + \log_e r - \log_e r_s \right) \right] - r_s^2 \left[ \frac{P}{4\pi} - \frac{M_y}{2} \right]
\]

\[
C_2 = \frac{P}{2\pi} \left[ r_c - r + \log_e r \right]
\]

\[
C_3 = \frac{P}{4\pi} - \frac{M_y}{2}
\]

\[
C_4 = -\frac{pr_c}{2\pi}
\]

The preceding coefficients can be used to determine the deflections, moments and shear force at radii within the plate. Therefore, to ensure compatibility between the elastic and plastic regions of the plate, the deflections at the boundary of the tangential yield zone can be equated to give the following elastic-plastic solution:

\[
\frac{r_y^2 \left[ \frac{P}{4\pi} \left( \log_e r_s - \log_e r_y + 1 \right) - M_y \right]}{2(r_y^2 + r_s^2)} (r_y^2 + 2r_y^2 \log_e r_y - 2r_s^2 \log_e r_y - r_s^2)
\]

\[
+ \frac{P}{16\pi} \left[ 3r_y^2 - r_y^2 (2\log_e r_s - 2\log_e r_y + 3) \right] =
\]

\[
\frac{P}{2\pi} \left[ r_s r_y + r_c r_y - r_y^2 - r_s r_c \left( 1 + \log_e r_y - \log_e r_s \right) \right]
\]

\[
+ \left[ \frac{P}{4\pi} - \frac{M_y}{2} \right] (r_y^2 - r_s^2)
\]

This expression relates the applied load to the yield moment for the radius of tangential yielding, and is of the form:

\[
P = k_t M_y
\]

Therefore the moment factor \( k_t \) can be determined for the defined radius of the yield zone. For the commencement of tangential yield at
the edge of the load, the factor \( k_t \) is approximately equal to that given by the solution for ring type loading, which has been derived by Timoshenko and Woinowsky-Kreiger (1959) as:

\[
\frac{p}{M} = \frac{8\pi}{2\log_e \frac{r_s}{r_c} + \left( \frac{r_s^2 - r_c^2}{r_s^2} \right)} \quad (5.3b)
\]

Alternatively, \( k_t \) may be compared with the moment factor for an equivalent square slab, which is given by the simplified relationship derived from the finite element analyses:

\[
\frac{p}{M} = \frac{25}{\log_e \left( \frac{L_c}{L_s} \right)^{1.5}} \quad (5.1)
\]

In the case of overall tangential yielding, the factor \( k_t \) is equal to the moment factor given by the yield line solution:

\[
k_t = k_{y1} = \frac{2\pi r_s}{r_s - r_c} \quad (5.2b)
\]

The variation of the moment factor for partial tangential yielding is as shown in Fig. 5.11.
A2 Example by direct procedure

The direct procedure for the prediction of the punching strength of conventional slab specimens is illustrated by the following worked example. The typical slab 'H1' tested by Moe (1961) is considered:

Data:

\[ s = 1829 \text{ mm} \quad f'_c = 26.1 \text{ N/mm}^2 \]
\[ a = 1778 \text{ mm} \quad p = 0.0115 \]
\[ d = 114.3 \text{ mm} \quad f_y = 328 \text{ N/mm}^2 \]
\[ c = 254 \text{ mm} \quad P_T = 372 \text{ kN} \]

1. \[
M_b = 0.0115 \times 328 \times 114.3^2 (1 - 0.59 \times \frac{0.0115 \times 328}{26.1}) \quad (5.5)
\]

\[
M_b = 45.08 \text{ kNmm/mm}
\]

\[ f'_c < 27.6 \text{ N/mm}^2 \] therefore \( \beta_1 = 0.85 \) \quad (5.6)

\[
\rho_{(bal)} = \frac{0.85^2 \times 26.1 \times (0.003 \times 200000)}{328 \times (0.003 \times 200000 + 328)} \quad (5.7)
\]

\[
\rho_{(bal)} = 0.0372
\]

\[
M_{(bal)} = 0.0372 \times 328 \times 114.3^2 (1 - 0.59 \times \frac{0.0372 \times 328}{26.1}) \quad (5.5)
\]

\[
M_{(bal)} = 115.44 \text{ kNmm/mm}
\]

\[
\frac{M_b}{M_{(bal)}} = \frac{45.08}{115.44} = 0.391
\]
2. 

\[ k_{y1} = 8 \left[ \frac{1829}{1778 - 254} - 0.172 \right] \]  
\[ k_{y1} = 8.225 \]  

\[ k_b = \frac{25}{\log_e \left( \frac{4445}{254} \right)^{1.5}} \]  
\[ k_b = 5.163 \]  

\[ k_t = [8.225 - 1.15(8.225 - 5.163)^{0.391}] \]  
\[ k_t = 6.848 \]  

3. 

\[ P_{vf} = 6.848 \times 45.08 \]  
\[ P_{vf} = 309 \text{ kN} \]  

4. 

\[ P_{vs} = 1.66 \sqrt{26.1} \times (254 + 114.3) \times 114.3 \times (1.15)^{0.25} \]  
\[ P_{vs} = 370 \text{ kN} \]  

5. 

\[ P_{vf} < P_{vs} \]  
Therefore \[ P = 309 \text{ kN} \]  

\[ \frac{P_t}{P} = \frac{372}{309} = 1.204 \]  

mode of failure is 'yield'
A3 Other methods of prediction

The following procedures are outlined for the general case of the conventional isotropically reinforced square slab and column specimen.

(i) CP110 (1972)

area of critical section = \( d(4c + 3nh) \)

nominal ultimate shear stress = \( \frac{0.27}{\gamma_m} (100f_{cu})^{1/3} \)

slab depth factor: \( \varepsilon_s = 1.6 - \frac{h}{500} \)

\( 1.0 \leq \varepsilon_s \leq 1.3 \)

for laboratory controlled tests the partial factor of safety for concrete \( (\gamma_m = 1.25) \) is removed, giving:

\[ P_{CP110} = 0.27\varepsilon_s (100f_{cu})^{1/3} d(4c + 3nh) \]

(ii) ACI (318-77)

area of critical section = \( 4d(c + d) \)

nominal ultimate shear stress = \( 0.333\sqrt{f_c^T} \)

Without the strength reduction factor, the capacity is given by:

\[ P_{ACI} = 1.33\sqrt{f_c^T} d(c + d) \]

(iii) Long (1975)

'flexural' punching capacity: \( P'_V = \frac{\rho_f y d^2 (1 - 0.59\frac{\rho_f y}{f_c})}{(0.2 - 0.9\frac{c}{L})} \)

'shear' punching capacity: \( P''_V = \frac{1.66\sqrt{f_c^T} d(c + d)(100\rho)^{0.25}}{(0.75 + 4\frac{c}{L})} \)

\( P_L \) is the lesser of \( P'_V \) and \( P''_V \)

(iv) Regan (1981)

\[ P_R = 0.13 \sqrt{\frac{300}{d}} \gamma_{100f_{cu}} 2.69d(4c + 7.85d) \]
APPENDIX B  ANALYSIS OF ARCHING ACTION IN SLAB STRIPS

B1. Analysis of a three-hinged arch

Considering the equilibrium and compatibility conditions of the three-hinged arch shown in Fig. 6.12(a), the following theory is applicable for shallow arches.

At the deformed position:

\[ p = \frac{2Fh^*}{a} \]  \hspace{2cm} (1)

stress in each leg = \( \frac{F}{A} \)

strain in each leg = \( \frac{\delta L_e}{L_e} \)

\[ F = \frac{E A \delta L_e}{L_e} \]  \hspace{2cm} (2)

The lateral displacement at the base hinges equals:

\[ \Delta = \frac{F}{K} \]  \hspace{2cm} (3)

From the geometry of the deformed arch:

\[ \Delta + \sqrt{L_e^2 - h_a^2} = \sqrt{(L_e - \delta L_e)^2 - h_a^2} \]

Eliminating the negligible terms gives:

\[ 2\Delta L_e \left( 1 - \frac{h_a^2}{L_e^2} \right) - h_a^2 = -2L_e \delta L_e - h_a^2 \]

By expanding this:

\[ \delta L_e = \left( \frac{h_a^2 - h_a^2}{2L_e} \right) - \Delta \]  \hspace{2cm} (4)
Combining (2), (3) and (4) gives:

\[
F = \frac{EA(h_a^2 - h^2_a)}{2L_e^2(EA/KL_e + 1)} \tag{5}
\]

Therefore from (1)

\[
P = \frac{h^*EA(h_a^2 - h^2_a)}{L_e^3(EA/KL_e + 1)} \tag{6}
\]

Maximising \( P \) with respect to \( h^*_a \):

\[
\frac{dP}{dh^*_a} = 0 \quad \text{if} \quad h^*_a = \frac{h_a}{\sqrt{3}}
\]

critical deflection = 0.423 \( h_a \)

B2. Example by direct procedure

The proposed procedure for predicting the ultimate capacity of laterally restrained reinforced concrete strips is illustrated by the following worked example.

Considering the typical strip 'M12' tested by Leibenberg (1966). Load application was by means of a concentrated force at midspan and the end conditions were such as to induce 'fixed end' moments.

Data:

\begin{align*}
L &= 1219 \text{ mm} & \rho &= 0.485\% \\
b &= 305 \text{ mm} & f_y &= 270 \text{ N/mm}^2 \\
h &= 84.6 \text{ mm} & U &= 31.2 \text{ N/mm}^2 \\
d &= 64.3 \text{ mm} & K &= 0.23 \text{ kN/mm}^2 \\
\text{assume } f'_c &= 0.8U
\end{align*}
1. 
\[ M_b = 0.00485 \times 270 \times 64.3^2 \left(1 - 0.59 \times \frac{0.00485 \times 270}{25}\right) \]  

\[ M_b = 5247 \text{ Nmm/mm} \]

\[ k_b = \frac{8 \times b}{L} = \frac{8 \times 305}{1219} = 2.00 \]

\[ P_b = 2 \times 5.247 = 10.49 \text{ kN} \]

2. 
\[ 2d_1 = 84.6 - \left(\frac{2 \times 0.00485 \times 270 \times 64.3}{0.85 \times 25}\right) \]

\[ d_1 = 38.3 \text{ mm} \]

3. 
\[ E_c = 4.73\sqrt{25} \]

\[ E_c = 23.7 \text{ kN/mm}^2 \]

\[ L_r = \frac{1219^3 \sqrt{23.7 \times 38.3}}{2 \sqrt{0.23 \times \frac{1219}{2}}} + 1 \]

\[ L_r = 1191 \text{ mm} \]

4. 
\[ \varepsilon_c = (-400 + 60 \times 25 - 0.33 \times 25^2) \times 10^{-6} \]

\[ \varepsilon_c = 0.000894 \]

\[ R = \frac{0.000894 \times 1191^2}{4 \times 38.3^2} \]

\[ R = 0.2161 \]

5. 
\[ 0 < R < 0.25 \]

\[ u = -0.15 + 0.36\sqrt{(0.18 + 5.6 \times 0.2161)} \]

\[ u = 0.274 \]
6.

\[ \alpha = 1 - \frac{0.274}{2} \]
\[ \alpha = 0.863 \]
\[ ad_1 = 33.0 \text{ mm} \]

\[ L_r^{(1)} = \frac{1219}{2} \sqrt{\frac{23.7 \times 33.0}{0.23 \times \frac{1219}{2}} + 1} \]
\[ (6.7) \]

\[ L_r^{(1)} = 1141 \text{ mm} \]

\[ R^{(1)} = \frac{0.000894 \times 1141^2}{4 \times 38.3^2} \]
\[ (6.1) \]

\[ R^{(1)} = 0.1984 \]

\[ u^{(1)} = -0.15 + 0.36\sqrt{0.18 + 5.6 \times 0.1984} \]
\[ (6.5b) \]

\[ u^{(1)} = 0.259 \]

\[ u^{(1)} = u \text{ : do not repeat cycle} \]

7.

\[ 0 < R^{(1)} < 0.25 \]

\[ M_r = 4.3 - 16.1\sqrt{(3.3 \times 10^{-4} + 0.1243 	imes 0.1984)} = 1.755 \]
\[ (6.5a) \]

\[ M_a = 0.21 \times 25 \times 38.3^2 \times 1.755 \times \frac{609.5}{1141} \]
\[ (6.12) \]

\[ M_a = 7.22 \text{ kNmm/mm} \]

\[ k_a = \frac{4 \times b}{L} = \frac{4 \times 305}{1219} = 1.00 \]

\[ P_a = 1.0 \times 7.22 \]
\[ (6.9) \]

\[ P_a = 7.22 \text{ kN} \]
8.

\[ P_p = 10.49 + 7.22 \]

\[ P_p = 17.71 \text{ kN} \]

\[ \frac{P_T}{P_p} = \frac{21.40}{17.71} = 1.208 \]

Note: By using the first approximation for \( L_r \) (1191 mm):

\[ M_r = 4.3 - 16.1 \sqrt{(3.3 \times 10^{-4} + 0.1243 \times 0.2161)} \]  

(6.5a)

\[ M_r = 1.645 \]

\[ M_a = 0.21 \times 25 \times 38.3^2 \times 1.645 \times \frac{609.5}{1191} \]  

(6.12)

\[ M_a = 6.48 \text{ kNm/mm} \]

\[ P_a = 1.0 \times 6.48 \]  

(6.9)

\[ P_a = 6.48 \text{ kN} \]

\[ P_p = 10.49 + 6.48 \]

\[ P_p = 16.97 \text{ kN} \]

\[ \frac{P_T}{P_p} = \frac{21.40}{16.97} = 1.261 \]
B3. Other methods of analysis

The method proposed by Christiansen (1963) for predicting the ultimate capacity of laterally restrained strips can be outlined as follows:

The depth available for arching is given by:

\[ K_1 h = h - w_e - \frac{(\rho + \sigma) f_y d}{0.67U} \]

where \( w_e \) = midspan elastic deflection at yield

The ratio of the outward movement of the support to the elastic shortening of the strip is equal to:

\[ k = \frac{2E_c h}{K_L} \]

and \( K_2 = \frac{(1 + k)0.67UL^2}{8E_c h^2} \)

The maximum arching resistance is obtained by the solution of the following polynomial to give the minimum relative plastic deflection \( y \):

\[ 3y^4 + (8K_2 - 4K_1)y^3 + (6K_2^2 - 12K_1K_2)y^2 - 12K_1K_2^2y + 4K_1^2K_2^2 = 0 \]

From this, the relative depth of additional compression which causes arching is found from the expression:

\[ x_1 = \frac{K_1y - 0.5y^2}{2(K_2 + y)} \]

The maximum arching moment is given by:

\[ M_a = x_1(K_1 - y - x_1)h^20.67U \]

Thus the load carried by arching can be found and added to the bending capacity to give the total ultimate strength.
The method presented by Park and Gamble (1980) involves the computation of the load-deflection response from which the ultimate load can be extracted. The procedure is therefore suited for programming by computer.

For a plastic mechanism with positive hinges at a distance of $\beta L$ from the negative hinges, the axial shortening and lateral displacement is given by:

$$
\varepsilon + \frac{2\Delta}{L} = \frac{1}{\frac{1}{hE_C} + \frac{2}{LK}} \left[ 0.85f'_c\beta_1 \left( \frac{h}{2} - \frac{\delta}{4} - \frac{\bar{T} - \bar{T} - \bar{C}_s + C_s}{1.7f'_c\beta_1} \right) + C_s - \bar{T} \right]
$$

The sum of the moments of the internal forces about one end is:

$$
\bar{m} + m_u - n_u \delta = 0.85f'_c\beta_1 h \left[ (1 - \frac{\beta_1}{2}) + \frac{\delta}{4}(\beta_1 - 3) \right]
$$

$$
+ \frac{B L^2}{4\delta} (\beta_1 - 1)(\varepsilon + \frac{2\Delta}{L}) + \frac{\delta^2}{8h} \left( 2 - \frac{\beta_1}{2} \right)
$$

$$
+ \frac{B L^2}{4h} (1 - \frac{\beta_1}{2})(\varepsilon + \frac{2\Delta}{L}) - \frac{\beta_1 B^2 L^4}{16h^2 \delta^2} \left( \varepsilon + \frac{2\Delta}{L} \right)^2
$$

$$
- \frac{1}{3.4f'_c^2} (\bar{T} - \bar{T} - \bar{C}_s + C_s)^2 + (\bar{C}_s + C_s) \left( \frac{h}{2} - d - \frac{\delta}{2} \right)
$$

$$
+ (\bar{T} + \bar{T})(d - \frac{h}{2} + \frac{\delta}{2})
$$

For a virtual rotation $\theta$, the virtual work done by these actions is equal to:

$$
(\bar{m}_u + m_u - n_u \delta) \theta
$$

Thus, the external work done can be equated to the internal work done by the actions at the yield sections to relate the deflection of the strip to the load carried.
APPENDIX C  ANALYSIS OF LATERALLY RESTRAINED SLABS

Cl Examples by integrated procedure

a) rigorous method

The rigorous method for the prediction of the punching strength of laterally restrained slabs is illustrated by the following worked example.

The average results of the typical specimens '35 and 36' tested by Snowdon (1973) are considered. Concentric loading was applied to the simply supported square slabs through a circular platen and the lateral restraint was effectively rigid.

Data:

- slab size = 1720 mm
- \( \rho = 0.47\% \)
- \( f_c = 200 \text{ mm} \)
- \( f_y = 530 \text{ N/mm}^2 \)
- \( d = 107 \text{ mm} \)
- \( U = 67.0 \text{ N/mm}^2 \)
- \( h = 125 \text{ mm} \)
- \( K = \infty \)

Assume:

i) \( f'_c = 0.8U = 53.6 \text{ N/mm}^2 \)

ii) equivalent column size: \( c = 157 \text{ mm} \)

1. ARCHING PARAMETERS

\[ L_e = \frac{1720 - 157}{2} \]

\[ L_e = 782 \text{ mm} \]

\[ \varepsilon_c = (-400 + 60 \times 53.6 - 0.33 \times 53.6^2) \times 10^{-6} \]

\[ \varepsilon_c = 0.001868 \]

\[ E_c = 4.73\sqrt{53.6} \]

\[ E_c = 34.6 \text{ kN/mm}^2 \]
2. INTEGRATED MOMENT OF RESISTANCE

\[ M_b = 0.0047 \times 530 \times 107^2 \times (1 - 0.59 \times \frac{0.0047 \times 530}{53.6}) \]  \hspace{1cm} (5.5)

\[ M_b = 27.74 \text{ kNmm/mm} \]

\[ 2d_1 = 125 - \frac{0.0047 \times 530 \times 107}{0.85 \times 53.6} \]  \hspace{1cm} (6.11)

\[ d_1 = 59.6 \text{ mm} \]

\[ L_r = L_e \sqrt{\frac{3}{\frac{34.6 \times 59.6}{782} + 1}} \]  \hspace{1cm} (6.7)

\[ L_r = L_e = 782 \text{ mm} \]

\[ R = \frac{0.001868 \times 782^2}{4 \times 59.6^2} \]  \hspace{1cm} (6.1)

\[ R = 0.0804 \]

\[ 0 < R < 0.26: \]

\[ M_r = 4.3 - 16.1\sqrt{(3.3 \times 10^{-4} + 0.1243 \times 0.0804)} \]  \hspace{1cm} (6.5a)

\[ M_r = 2.664 \]

\[ M_a = 0.21 \times 53.6 \times 59.6^2 \times 2.664 \times 1 \]  \hspace{1cm} (6.12)

\[ M_a = 106.52 \text{ kNmm/mm} \]

\[ M_a + M_b = 134.26 \text{ kNmm/mm} \]

3. INTEGRATED SLAB DUCTILITY

\[ \beta_1 = 0.85 - \frac{53.6 - 27.6}{6.9} \times 0.05 \]  \hspace{1cm} (5.6)

\[ \beta_1 = 0.66 \]

\[ \rho(ba1) = \frac{0.66 \times 0.85 \times 53.6 \times (0.003 \times 200000)}{530 \times (0.003 \times 200000 + 530)} \]  \hspace{1cm} (5.7)

\[ \rho(ba1) = 0.03 \]
\[ M_{(bal)} = 0.03 \times 530 \times 107^2 \times (1 - 0.59 \times \frac{0.03 \times 530}{53.6}) \]  \hspace{1cm} (5.5)

\[ M_{(bal)} = 150.18 \text{ kNmm/mm} \]

\[ \frac{M_a + M_b}{M_{(bal)}} = \frac{134.26}{150.18} = 0.894 \]  \hspace{1cm} (7.1)

4. Moment Factor:

\[ k_{y1} = 8 \left[ \frac{1720}{1720 - 157} - 0.172 \right] \]  \hspace{1cm} (5.2a)

\[ k_{y1} = 7.427 \]

\[ k_b = \left[ \frac{25}{\log_e \left( \frac{1720 \times 2.5}{157} \right)} \right]^{1.5} \]  \hspace{1cm} (5.3a)

\[ k_b = 4.151 \]

\[ k_t = \left[ 7.427 - 1 \times (7.427 - 4.151) \times 0.894 \right] \]  \hspace{1cm} (7.1)

\[ k_t = 4.498 \]

5. Contact Depth

\[ d_1 = \frac{125}{2} \]

\[ d_1 = 62.5 \text{ mm} \]

\[ L_r = \frac{34.6 \times 62.5}{\omega \times 782} + 1 \]  \hspace{1cm} (6.7)

\[ L_r = 782 \text{ mm} \]

\[ R = \frac{0.001868 \times 782^2}{4 \times 62.5^2} \]  \hspace{1cm} (6.1)

\[ R = 0.0731 \]
\[ 0 < R < 0.26: \]

\[ u = -0.15 + 0.36\sqrt{(0.18 + 5.6 \times 0.0731)} \]

\[ u = 0.126 \]

\[ a = 1 - \frac{0.126}{2} \]

\[ a = 0.937 \]

\[ x = 0.937 \times 62.5 \]

\[ x = 58.6 \text{ mm} \]

6. **ULTIMATE FLEXURAL CAPACITY**

\[ P_{vF} = 4.498 \times 134.26 \]

\[ P_{vF} = 604 \text{ kN} \]

7. **ULTIMATE SHEAR CAPACITY**

\[ P_{vs} = (200 + 0.8 \times 125) \times \sqrt{53.6} \times [3.2 \times 58.6 + 0.3 \times 125 \times (0.47)^{0.25}] \]

\[ P_{vs} = 480 \text{ kN} \]

8. **MODE OF FAILURE**

\[ P_{vF} > P_{vs}: P_p = P_{vs} \]

\[ P_p = 480 \text{ kN} \]

\[ \frac{P_T}{P_p} = \frac{508}{480} = 1.058 \]

mode of failure is 'shear'
b) simplified approach

The simplified method for the prediction of the punching strength of the interior slab-column connection in the full panel specimen is illustrated by the following worked example.

The concentrically loaded model 'DM-2' tested by Masterson (1971) is analysed.

Data: 

\[ L = 1016 \text{ mm} \]
\[ c = 76 \text{ mm} \]
\[ d = 31.8 \text{ mm} \]
\[ h = 38.1 \text{ mm} \]
\[ \rho = 0.98\% \]
\[ f_y = 333 \text{ N/mm}^2 \]
\[ f'_c = 29.5 \text{ N/mm}^2 \]
\[ P_T = 34.3 \text{ kN} \]

1. ULTIMATE FLEXURAL CAPACITY

\[ \omega = \frac{0.0098 \times 333}{29.5} = 0.111 \]
\[ \omega^{0.67} = 0.229 \]

\[ M_a + M_b = 0.62 \times 31.8^2 \times 29.5 \times 0.229 = 4.235 \text{ kNmm/mm} \quad (7.11) \]

\[ \frac{M_a + M_b}{M_{u(max)}} = 1.88 \times 0.229 = 0.430 \quad (7.12) \]

\[ k_{y1} = 6.1 + 33 \times \frac{76}{1016} = 8.568 \quad (7.9) \]

\[ k_{y1} - k_b = 3.7 - 15 \times \frac{76}{1016} = 2.578 \quad (7.10) \]

\[ P_{vf} = \left[8.568 - 1.15 \times 2.578 \times 0.430\right] \times 4.235 \quad (7.13) \]

\[ P_{vf} = 30.9 \text{ kN} \]
2. ULTIMATE SHEAR CAPACITY

\[ P_{Vs} = 1.9 \sqrt{29.5} \times 31.8 \times (76 + 31.8) \times \left[ 1 + 0.22 \times (0.98)^{0.25} \right] \] (7.14a)

\[ P_{Vs} = 43.1 \text{ kN} \]

3. MODE OF FAILURE

\[ P_{Vf} < P_{Vs} : p_p = P_{Vf} \]

\[ P_{p} = 30.9 \text{ kN} \]

\[ \frac{P_T}{P_p} = \frac{34.3}{30.9} = 1.110 \]

mode of failure is 'yield'

C2 Masterson's method of prediction

\[ M_b = f_y d^2 (1 - 0.59 \frac{f_y}{f_c}) \]

\[ M_m = 0.3 h^2 f_p \left[ \frac{r_0^2 - r_z^2}{r_0^2 + r_z^2} \right] \]

for full panel specimens: \( r_0 = 0.5L, r_z = 0.2L \)

\( k_b = \) elastic moment factor from finite element analysis

\[ P_M = k_b \left( 1.3 M_b + M_m \right) \]
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Plate 3.1 EXTREME LEVELS OF REINFORCEMENT
(a) Bottom steel

(b) Top and bottom steel

Plate 32 REINFORCEMENT IN LARGE PANEL MODEL
Plate 3.3  SWEDISH REINFORCEMENT

Plate 3.4  VIBRATING WIRE STRAIN GAUGES
Plate 3.5 CONTRAFLEXURE MODEL IN TEST RIG

Plate 3.6 LOADING ARRANGEMENT
Plate 4.1  CRACK DEVELOPMENT (ρ = 0.4%)

(a) $L/h = 28$

(b) $L/h = 31$
Plate 4.2 CRACK DEVELOPMENT (ρ = 2.0%)
Plate 4.3 CRACK OPENING
($\rho = 0.4\%$)

Plate 4.4 FAILURE ZONE
($\rho = 0.4\% - 2.0\%$)
Plate 4.5 CRUSHING ALONG SLAB DIAGONALS

Plate 4.6 SHEARING AROUND COLUMN PERIPHERY
Plate 4.7 COLUMN STUB

Plate 4.8 INCLINED FRACTURE SURFACE
Plate 49  SOFFIT OF LARGE PANEL MODEL

Plate 410  FULL PANEL SPECIMEN AFTER FAILURE
Plate 4.11  FAILURE PATTERNS  ($L/h = 35 - 25$)
Plate 4.12 CRACK DISTRIBUTION IN FULL PANEL SPECIMENT