PARAMETER OPTIMISATION FOR WOODWIND SINGLE-REED MODELS

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ABSTRACT

Time-domain modelling of single-reed woodwind instruments usually involves a lumped model of the excitation mechanism. The parameters of this lumped model have to be estimated for use in numerical simulations. Several attempts have been made to estimate these parameters, including observations of the mechanics of isolated reeds, measurements under artificial or real playing conditions and estimations based on numerical simulations. In this study an optimisation routine is presented, that can estimate reed-model parameters, given the pressure and flow signals in the mouthpiece. The method is validated, tested on a series of numerically synthesised data. In order to incorporate the actions of the player in the parameter estimation process, the optimisation routine has to be applied to signals obtained under real playing conditions. The estimated parameters can then be used to resynthesise the pressure and flow signals in the mouthpiece. In the case of measured data, as opposed to numerically synthesised data, special care needs to be taken while modelling the bore of the instrument. In fact, a careful study of various experimental datasets revealed that for resynthesis to work, the bore termination impedance should be known very precisely from theory. An example is given, where the above requirement is satisfied, and the resynthesised signals closely match the original signals generated by the player.

1. INTRODUCTION

Physical modelling of musical instruments aims to translate a set of physical model parameters into audio. In order to synthesise a realistic sound, the value of the model parameters have to be estimated. When modelling single-reed woodwind instruments, special care should be taken regarding the reed excitation mechanism. Two types of non-linearities take place in the reed-mouthpiece system, where

- The first published attempt to measure reed stiffness under real playing conditions was by Boutillon and Gibiat [8]. Considering a saxophone reed, the authors managed to derive its
- Considering the simplest possible lumped model of the oscillating reed, namely a single mass-spring model, several attempts have been carried out in order to estimate the parameters that are related with the mechanical and fluid dynamical properties of the reed-mouthpiece system. In 1969 Nederveen measured the compliance of the reed by observing its deformation in static experiments [2]. Worman [3] managed to experimentally measure the parameters of an isolated reed and his results were adopted by many future authors [4, 5, 6]. Another measurement of the reed stiffness was provided by Gilbert [7] and was in agreement with the compliance measured by Nederveen. These approaches study the mechanics of reeds that are detached from the player, so any actions of the player that affect the system are neglected. In particular humidity might alter the material properties of the reed and it was not considered by the previous authors, besides Worman, who wetted the reed before carrying out his measurements.

Figure 1: Forward and inverse modelling of the reed-mouthpiece system, where $K_a$ is the effective stiffness, $S_r$ the effective reed surface, $y_m$, the closing position of the reed and $p$ and $u$ the pressure and flow inside the mouthpiece.

\[ \text{forward model} \]

\[ \begin{align*}
K_a & \quad \text{inverse model} \\
S_r & \\
y_m &
\end{align*} \]

physical model parameters

signals inside the mouthpiece
stiffness by establishing a balance between the reactive powers of the reed and the air-column. The obtained range of values was of the same order with those proposed by Nederveen and Gilbert. Gazengel [9] was the first one to suggest that varying lumped reed model parameters can be estimated using a distributed model of the reed. Using a simplified geometry for both the reed and the mouthpiece, he proposed that estimating non-constant lumped model parameters it would be possible to capture the effect of the reed-lay interaction. In 2003 Dalmont [10] provided experimental results for the internal reed damping and the reed stiffness, pointing out that when near closure a variable stiffness could be assumed in order to capture the non-linear interaction of the reed with the mouthpiece lay. More recently it has been shown how using a distributed model of the reed that captures the interaction with the lips and the mouthpiece lay, it is possible to estimate lumped reed model parameters, while considering a quite realistic geometry of the system [11, 12].

Concerning the flow in the mouthpiece, Backus [13, 14] used an artificial blowing method to obtain measurements for the vibrations of the clarinet in order to derive an expression between the volume flow and the reed opening. Nederveen also presented findings that were in qualitative agreement with those of Backus. A more extensive study on the fluid dynamics of the system along with experimental results [15, 16, 17, 18] observed the fact that the flow separates from the walls of the reed channel, forming an air jet. This fact was confirmed by numerical simulations of the flow in the mouthpiece [19] and incorporated in a refined reed model [20].

3. A LUMPED SINGLE-REEED MODEL

3.1. The forward model

The equation of motion for a single mass-spring model of the reed is

\[ m \frac{d^2 y}{dt^2} + m \frac{dy}{dt} + K_a (\Delta p) = \Delta p, \]  

where \( y \) is the reed displacement, \( m \) the mass per unit area and \( g \) the damping per unit area. The effective stiffness per unit area, \( K_a \), is treated as a function of \( \Delta p \), the pressure difference across the reed, thus rendering the model able to incorporate the quasi-static mechanical non-linear behaviour of the system.

The formation of the air jet in the reed channel can be taken as a wave propagating along the reed, thus rendering the model able to incorporate the effect of the reed-lay interaction. In 2003 Dalmont [10] provided experimental results for the internal reed damping and the reed stiffness, pointing out that when near closure a variable stiffness could be assumed in order to capture the non-linear interaction of the reed with the mouthpiece lay. More recently it has been shown how using a distributed model of the reed that captures the interaction with the lips and the mouthpiece lay, it is possible to estimate lumped reed model parameters, while considering a quite realistic geometry of the system [11, 12].

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3.2. The inverse model

The first step towards the parameter optimisation is based on the simplifying assumption that the reed displacement \( y \) is proportional to the pressure difference \( \Delta p \) across it [22]:

\[ y = C \Delta p = C(p_m - p), \]

where \( C \) is the compliance of the reed [2] and \( p \) the pressure inside the mouthpiece. The reed opening \( h \) can be related to \( y \) as

\[ h = y_m - y, \]

with \( y_m \) the closing position of the reed. Under this assumption the total flow into the mouthpiece as a function of the reed displacement \( y \) is

\[ u = u_f + u_r = (-\lambda y + \lambda y_m)\sqrt{\frac{2(p_m - p)}{\rho}} - CS_r \frac{dp}{dt} \]

\[ = c_1 \sigma \sqrt{\frac{2}{\rho}} (p_m - p)^{3/2} + c_2 \sigma \sqrt{\frac{2}{\rho}} (p_m - p)^{1/2} + c_3 \frac{dp}{dt}, \]

where \( \sigma \) is the sign of \((p_m - p)\) and

\[ \begin{cases} c_1 = -\lambda \\ c_2 = y_m \lambda \\ c_3 = -CS_r \end{cases} \Rightarrow \begin{cases} K_a = -\lambda / c_1 \\ y_m = c_2 / \lambda \\ S_r = Nc_3 / c_1 \end{cases} . \]

Assuming that the pressure and flow signals in the mouthpiece are known, and taking as objective function the mean square error between the original flow signal and the estimated flow as calculated from equation (9), it is possible to use the Nelder-Mead optimisation algorithm [23], in order to get a first estimate for \( K_a \) (constant), \( S_r \), \( y_m \) and \( p_m \) [22]. These estimated parameters enable the synthesis of oscillatory signals; they lie within a range so that the simulation of sustained clarinet notes is possible. It remains to fine-tune them so that we get a better match between the original and the resynthesised signals.

For the fine-tuning process (second optimisation step) we use as objective function the mean square error of the pressure signals at the steady state:

\[ f_{obj} = ||p_{ori} - p_{res}||. \]
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Figure 2: Pressure (top) and flow (bottom) signals in the mouthpiece for the numerically synthesised sound (dotted-blue) and the resynthesised sound using the estimated parameters (red).

Figure 3: Schematic depiction of the experimental set-up.

Figure 4: Pressure and flow signals in the mouthpiece for the measured (blue) and the resynthesised sound (red).

Figure 5: Theoretical and experimental input impedance of the flanged pipe used for the experiments.

4. APPLICATION TO MEASURED DATA

The measured data is obtained from experiments with blowing a simplified clarinet, the schematic bore profile of which is shown in Figure 3. Travelling pressure waves are calculated at the reference plane \((p_0, u_0)\) and translated to pressure and flow signals in the mouthpiece. More details about the above process and concerning the model of the resonator of the instrument can be found in [25, 26, 1], as well as in [21, chapter 6].

The two-step optimisation routine is applied on a slice of the data that resembles the steady state of the measured sound. In order to obtain a meaningful estimation of the reed model parameters it is necessary to use an accurate model of the instrument bore. The complex geometry of the clarinet mouthpiece, as well as the radiation impedance of the cylindrical tube influence the input impedance of the resonator, thus affecting the parameter optimisation. Using a baffle termination of the cylindrical pipe allows for a more accurate representation of the termination impedance [27], resulting in a good match between theoretical and experimental input impedance, as depicted in Figure 5. After carrying out the optimisation routine, the estimated reed model parameters can be used to resynthesise the pressure and flow signals in the mouthpiece, as seen in Figure 4.

Table 1: Theoretical vs. estimated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>theoretical</th>
<th>estimated (I)</th>
<th>estimated (II)</th>
<th>units</th>
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<tr>
<td>(K_a)</td>
<td>8.66 · 10^4</td>
<td>8.97 · 10^4</td>
<td>8.67 · 10^4</td>
<td>N/m^2</td>
</tr>
<tr>
<td>(S_t)</td>
<td>7.61 · 10^5</td>
<td>8.42 · 10^5</td>
<td>8.33 · 10^5</td>
<td>m^2</td>
</tr>
<tr>
<td>(y_{in})</td>
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<td>3.2 · 10^{-4}</td>
<td>3.68 · 10^{-4}</td>
<td>m</td>
</tr>
<tr>
<td>(p_{in})</td>
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<td>1919</td>
<td>1825</td>
<td>Pa</td>
</tr>
<tr>
<td>(\lambda)</td>
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<td>—</td>
<td>0.0142</td>
<td>m</td>
</tr>
<tr>
<td>(m)</td>
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<tr>
<td>(g)</td>
<td>3000</td>
<td>—</td>
<td>3805</td>
<td>1/sec</td>
</tr>
</tbody>
</table>

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5. CONCLUSIONS

Estimating the reed model parameters from signals generated by a real player requires a model of the instrument that can adapt to the properties of the experimental set-up. Concerning the bore model, errors can arise during the acquisition of the signals at the reference plane, and their translation to the mouthpiece. Such errors can cause a mismatch between the input impedance of the model (obtained from theory) and that of the simplified clarinet used in the experiments. Taking special care in order to ensure that

\[ p^* = r_f \ast p^+ \quad (11) \]

where \( r_f \) is the reflection function of the tube, turns out to be a pre-requisite for a successful resynthesis. Remaining deviations from the original signals observed in Figure 4, especially in the case of the flow signal, are caused by uncertainties in the reed model, due to unpredictable fluid dynamical phenomena [19, 20] and a more complex reed-lay interaction than that predicted by the physical model.

6. REFERENCES


