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Impact of Wireless Backhaul Unreliability and Imperfect Channel Estimation on Opportunistic NOMA

Sunyoung Lee, Trung Q. Duong, Senior Member, IEEE, and Roger Woods, Senior Member, IEEE,

Abstract—We propose a new opportunistic non-orthogonal multiple access (NOMA) scheme under wireless backhaul unreliability and fronthaul channel uncertainty, where fronthaul represents regular radio access link. In particular, we propose two opportunistic methods for the proposed NOMA, which allow the best transmitter selection approaches based on either a near or a far-away receiver, considering the wireless backhaul unreliability and the fronthaul fading impairment. For the performance analysis, new closed-form expressions of exact and approximated outage probabilities of the grouped receivers are derived. The theoretical analysis provides an insight into the impact of wireless backhaul unreliability and imperfect channel estimation on the behaviour of outage probabilities at NOMA receivers. Furthermore, we analytically investigate how the number of multiple transmitters in the proposed opportunistic NOMA can determine the outage floors. We show that under unreliable wireless backhauls, a dominant receiver in the proposed NOMA scheme can achieve more than 3dB gain in outage performance, compared to the orthogonal multiple access. In addition, the outage probability at a non-dominant receiver is less influenced by imperfect channel information, while the outage probability at a non-dominant receiver is significantly sensitive. The analytical expressions and asymptotic results have been validated through Monte Carlo simulations, thus verifying the derived impact analysis of NOMA under wireless backhaul unreliability and imperfect channel estimation.

Index Terms—Non-orthogonal multiple access (NOMA), Nakagami-m fading, outage probability, wireless backhaul reliability, channel estimation error.

I. INTRODUCTION

In the fifth generation (5G) wireless communications, the demand for massive connectivity of devices requires new spectral efficient techniques. Non-orthogonal multiple access (NOMA) has received significant research interest over recent years and has been explored for various applications due to its spectrum efficiency [1], [2]. In NOMA, multiple users are allowed to share time and frequency resources at different power allocations. In particular, the users under better channel conditions perform successive interference cancellation (SIC) so that they remove the messages of other users, who are under worse channel conditions, and then decode their own messages [3], [4]. NOMA has been discussed in a cooperative scenario [5], where users who have good channel conditions work as relays for other users with poor channel conditions in order to enhance their performance. The relay-aided NOMA scheme was developed to improve the spectral efficiency as well as fairness for users [6]–[8]. In addition, work in [9] proposed the outage probability of the optimal relay selection for NOMA networks, while others investigated the random relay selection for full-duplex NOMA networks [10]. The NOMA scheme for cooperative spectrum-sharing networks over Nakagami-m fading channels was proposed in [11], where both primary and secondary users would be served by a base station simultaneously. For this, the primary user is allocated a high priority for its message. Furthermore, the performance of simultaneous wireless information and power transfer in cooperative NOMA has been investigated in [12].

Coordinated multipoint (CoMP) techniques have been proposed for multiple base stations (BSs) to jointly enhance the cell-edge users’ data rates [13]. The NOMA scheme together with the CoMP network has been studied to support both near and cell-edge users simultaneously [14], [15]. In addition, an opportunistic NOMA scheme where each user selects one access point (AP) or multiple APs in its preferred AP set to reduce the complexity of SIC, has been demonstrated for the CoMP network in [16]. However, with need for ultra-dense connectivity, randomly deployed wireless small cells are required to provide the core network with wireless backhauls. Therefore, wireless impairment inherent in backhaul reliability can be a key bottleneck in improving the system performance. In other words, wireless backhaul links will be often unreliable because of the communication channels’ wireless nature [17]. For this reason, heterogeneous cellular networks in presence of channel unreliability have emerged as an interesting research topic in the downlink CoMP networks. The impact of backhaul unreliability on CoMP-based cellular networks have been investigated [18]–[21]. Moreover, the performance of cooperative system under backhaul unreliability for non-cellular systems has been analyzed [22]. In [23], the authors studied a secrecy performance of cooperative single carrier systems under backhaul unreliability, where the existence of performance limits of outage probability and data rate have been verified for various backhaul scenarios. In [17], the selection combining-assisted cooperative system under backhaul unreliability over non-identical Nakagami-m fading channels has been studied.

Much existing research work in NOMA [5]–[8], [11] and cooperative wireless systems with unreliable backhauls [17]–
The contributions of this paper can be summarised as follows: The main certainties from wireless backhauls and fronthauls. The main small cell networks are derived. These allow us to provide an insight into the outage probability behaviours with the presence of realistic cases. New closed-form expressions for the outage probabilities of the opportunistically coordinated NOMA for backhauls for the connectivity with the core network [31]. A robust NOMA beamforming scheme for multiple-input single-output (MISO) channels where the channel uncertainties are considered has been proposed in [28]. Furthermore, the authors in [29] formulated a resource allocation algorithm and an optimal power allocation for a downlink NOMA with imperfect channel information to maximize the system energy efficiency. However, [27]–[29] focused on the performance analysis of a non-cooperative NOMA system with partial channel information, where wireless backhaul connection is not considered. In [30], the performance of NOMA is investigated in a cellular system with randomly deployed users. The authors in [30], have investigated the performance of the NOMA downlink system under perfect channel state information (CSI) with stochastic approach, while our proposed work focuses on the performance analysis of cooperative NOMA with wireless backhaul unreliability and imperfect CSI.

Due to the significant demand for a massive deployment of wireless devices, various NOMA concepts with a range of 5G applications (e.g., heterogeneous small cell networks) have emerged to increase the spectral efficiency. In particular, small cell base station with a randomly distributed deployment represents a critical development in the deployment of wireless backhauls for the connectivity with the core network [31]. However, wireless backhauls are often unreliable due to the random nature of wireless propagation environments. There is little research into NOMA designs for such small cells that can be robust to uncertainties in both wireless backhauls and fronthauls.

In this work, therefore, we propose a new NOMA-based scheme and investigate its practical application and impact on its performance with both wireless backhaul unreliability and fronthaul uncertainty. To the best of our knowledge, this is the first work to address the impact of uncertainties from both backhauls and fronthauls on outage-sensitive NOMA designs. Accordingly, the theoretical performance analysis of the proposed system is novel and is considered for two realistic cases. New closed-form expressions for the outage probabilities of the opportunistically coordinated NOMA for small cell networks are derived. These us to provide an insight into the outage probability behaviours with the presence of imperfect SIC and channel estimation noise over double uncertainties from wireless backhauls and fronthauls. The main contributions of this paper can be summarised as follows:

- New opportunistic NOMA schemes are introduced and have coordinated transmission under wireless backhaul unreliability and imperfect channel information. New closed-form expressions of the outage probabilities for the proposed schemes are derived. These allow the impact of backhaul unreliability and fronthaul channel uncertainty to be assessed and serve as a benchmark for outage performance of NOMA scheme with unreliable wireless backhauls.
- Opportunistic selection rules are proposed, jointly taking into account the random reliability of wireless backhauls and fading effects of fronthauls. For the random backhaul reliability, we adapt a Bernoulli process to take into account a successful or failed transmission from a central unit (CU) to multiple transmitters. It shows that wireless backhaul unreliability levels are jointly responsible for the outage floors.
- For the impact of fronthaul uncertainty, we consider imperfect channel estimation for fronthauls in the proposed system model. We theoretically analyse and show that the imperfect channel estimation causes imperfect SIC at NOMA receivers, producing residual interference. In particular, it is shown that the outage probability at a non-dominant receiver is very sensitive to imperfect channel information, whereas at a dominant receiver, it is not. Note that the dominant receiver is the one in relation to opportunistic selection scheduling (SS), i.e., $R_2$ in Case I and $R_3$ in Case II, whereas the non-dominant receivers are not. In particular, in Case I, the near receiver ($R_2$) acts as a dominant receiver and the best among $K$ transmitters is selected, exploiting both the backhaul reliability $I_k$ and the fronthaul channel only from $R_2$. In Case II, the far-away receiver ($R_3$) acts as a dominant receiver and the best transmitter is chosen, taking into consideration both $I_k$ and the fronthaul channel only from $R_3$. Therefore, the dominant receiver will have the best channel quality between CU and itself. We show that there are small error gaps between the outage probabilities with perfect channel estimation at a dominant receiver with imperfect channel estimation, whereas for a non-dominant receiver, there are big gaps which increase as the number of transmitters grows.
- For further insight, we theoretically analyze asymptotic outage probabilities. The numerical and simulation results clearly show that outage performance limits are determined by the wireless backhaul reliability and the number of transmitter.

The paper is organized as follows. In Section II, the system model for opportunistic NOMA under wireless backhaul unreliability and channel estimation error are presented. In Section III, new outage probabilities of the grouped receivers are analyzed and two opportunistic selection scheduling rules for NOMA with wireless backhauls are developed. In Section IV, the outage probability performance for three relevant but special cases are investigated. In addition, various asymptotic performance based on backhaul reliability are discussed in Section V. Numerical and simulation results are presented in Section VI, followed by the conclusions in Section VII.

Notations: $f_x$ and $F_x$ denote the probability density function (PDF) and the cumulative distribution function (CDF) of the random variable $x$, respectively. $P_r(\cdot)$ denotes the probability of the random variable $x$, $E[\cdot]$ represents expectation, $\Gamma(\cdot)$, $\Gamma'(\cdot)$ and $\gamma(\cdot,\cdot)$ denote the Gamma function [32, Eq. (8.310.1)], the upper incomplete Gamma function [32, Eq.
(8.350.2)] and the lower incomplete Gamma function [32, Eq. (8.350.1)], respectively and $CN(w,R)$ represents the distribution of the complex Gaussian random variable with the mean $w$ and the covariance $R$.

II. SYSTEM MODEL

A. NOMA with Unreliable Wireless Backhauls

Consider a NOMA scheme with unreliable wireless backhauls, which consists of one CU, $K$ transmitters ($T_1$, $T_K$) and $M$ receivers. Specifically, receivers having similar distances are grouped into either a near or a far away cluster. The far-away cluster receivers and the near cluster receivers are denoted by $R_{1g} = R_{11}$ and by $R_{2g} = R_{2G}$, respectively where $M = G_1 + G_2$. For brevity and without loss of generality, we assume $G_1 = G_2 = G$ hereinafter. Such a proposed model is depicted in Fig. 1. Due to the high complexity issue of the CSI-based receiver grouping, we consider a receiver grouping based on measuring the intensity of the received signal strength indicator (RSSI). In particular, RSSI is the strength of the transmitter’s signal as seen by the receivers. Therefore, RSSI is used to approximate the distance between the transmitter and the receivers.

For every transmission, suppose that two receivers in a group are non-orthogonally served by one transmitter. For this scenario, let one receiver, i.e., the former receiver denoted by $R_{1g}$ be randomly chosen from the far-away cluster and the other, i.e., the latter one denoted by $R_{2g}$ from the near cluster, where $g \in \{1,\cdots,G\}$. Accordingly, each group can be supported by one dedicated transmitter in orthogonal channels among groups.

In particular, $K$ transmitters are wirelessly backhauled to the CU, and one out of the $K$ transmitters is opportunistically selected to transmit superimposed messages to two receivers in a group in a non-orthogonal manner. In this way, $K$ transmitters can opportunistically serve $G$ groups of two receivers. Inter-cluster interference can be handled by the NOMA operation, while the intra-cluster interference is avoided by the orthogonal access. Notice that the general cases with more than two NOMA receivers are not considered in this paper because the decoding complexity of the multiple NOMA receivers increases with the their number due to the SIC issue. To reduce the decoding burden, receivers are divided into multiple small groups where the decoding complexity of pairing two NOMA receivers per group is deemed reasonable enough to address this issue [33]. Accordingly, this work is focused more on investigating the impact of imperfect SIC and channel estimation noise on the NOMA performance, caused by unreliable backhauls and uncertain fronthaul conditions. For simplicity, we analyze one receiver’s group hereinafter because of independent operation across each group.

B. Wireless Backhauls Aided NOMA Transceivers

For the fronthauls, we employ the NOMA scheme between $T_k$ and the $g$-th grouped receivers where $k = 1, \cdots, K$. In particular, $T_k$ transmit the superposed information $x_k = \sqrt{a_1 \Pr_{T_k} x_1} + \sqrt{a_2 \Pr_{T_k} x_2}$ to the $g$-th grouped receivers ($R_{1g}$ and $R_{2g}$), where $x_1$ and $x_2$ are the messages for $R_{1g}$ and $R_{2g}$, respectively, $a_1$ and $a_2$ denote the power allocation coefficients subject to $a_1 > a_2$ and $a_1 + a_2 = 1$, and $\Pr_{T_k} = P$ is the transmit power at $T_k$. Provided that $T_k$ is backhauled with the CU, the received signals at $R_i$ are given by

$$y_i = h_{ik} \mathbb{I}_k x_k + w_{ik}$$

where $h_{ik}$ is the Nakagami-$m$ fading coefficient with a $m$ parameter to express a generalized channel fading between the $k$-th transmitter and the $i$-th receiver with $i \in \{1g, 2g\}$, $w_{ik}$ denotes the zero mean additive white Gaussian noise (AWGN), i.e., $w_{ik} \sim CN(0, \sigma^2_n)$. Due to the Nakagami-$m$ fading’s analytical tractability and flexibility, we consider the Nakagami-$m$ fading for fronthauls. Note that when the $m$ parameter of the Nakagami-$m$ equals 1, it represents Rayleigh fading. $\mathbb{I}_k$ is the backhaul indicator function which represents the random status of backhaul reliability. In particular, successful transmission in backhaul is represented as $\mathbb{I}_k = 1$ and unsuccessful transmission represented by $\mathbb{I}_k = 0$.

For practical systems, we consider heterogeneous characteristics of both fronthaul and backhaul links. Let $h_{ik}$ be independent and non-identical Nakagami-$m$ fading with $|h_{ik}|^2 \sim Ga(m_{ik}, n_{ik})$, where $m_{ik}$ is the shape of the gamma distribution and $n_{ik}$ is the scale factor, i.e., $n_{ik} = \mathbb{E}[|h_{ik}|^2]/m_{ik}$.

$$\mathbb{E}[|h_{ik}|^2] = \Omega_{ik}$$

where $\Omega_{ik} = n_{ik}(\Gamma(m_{ik}, n_{ik})/\Gamma(m_{ik}))$ heterogeneously represents the complete channel statistics that contain the path loss $d^{-\alpha}_{ik}$ and fading statistics $\sigma^2_{ik}$. Hence, the PDF and CDF of $|h_{ik}|^2$ are, respectively, given by

$$f_{|h_{ik}|^2}(x) = \frac{1}{\Gamma(m_{ik}) n_{ik}^{m_{ik}}} x^{m_{ik} - 1} e^{-x/n_{ik}},$$

$$F_{|h_{ik}|^2}(x) = 1 - \frac{\Gamma(m_{ik}, x/n_{ik})}{\Gamma(m_{ik})},$$

As for the backhauls, let $\mathbb{I}_k$ be independent and non-identical Bernoulli random process where at every backhaul interval, $Pr(\mathbb{I}_k = 1) = p_k$ and $Pr(\mathbb{I}_k = 0) = 1 - p_k$, where $p_k$ is the backhaul reliability probability of the $k$-th transmitter.

C. Receiver Designs with Joint Backhaul and Fronthaul Impairment

Notice that in practice, $h_{ik}$ cannot be perfectly known to the receiver, i.e., the receiver estimates the channel information
\( \hat{h}_{ik} \), performing imperfect channel estimation of \( h_{ik} \). Let the relation between estimation of \( h_{ik} \) and \( h_{ik} \) be obtained as

\[
\hat{h}_{ik} = h_{ik} + e_{ik},
\]  

(3)

where \( e_{ik} \) denotes channel estimation error, i.e., \( e_{ik} \sim \mathcal{N}(0, \varepsilon_{ik}^2) \) and \( \hat{h}_{ik} \) is the estimation channel coefficient, i.e., \( \hat{h}_{ik} \sim \mathcal{N}(\hat{h}_{ik}, \hat{n}_{ik}) \) with \( \hat{n}_{ik} = (E[|h_{ik}|^2] - \varepsilon_{ik}^2)/m_{ik} \).

Based on imperfect channel estimation in [26], \( \varepsilon_{ik}^2 = E[|\hat{h}_{ik}|^2] - E[|h_{ik}|^2] \) can be attained by assuming that \( \hat{h}_{ik} \) and \( e_{ik} \) are statistically independent to each other. We assumed that the estimation channel coefficient is a true channel coefficient distorted with an additive channel estimation error that is caused by a number of uncertainties such as hardware impairment [34], background noise, etc. This assumption has been commonly used in the literature [26], [29]. Substituting (3) into (1), the received signal in the presence of backhaul unreliability and channel estimation error at \( R_i \), can be rewritten by

\[
y_i = (\hat{h}_{ik} + e_{ik})\bar{l}_kx_k + n_i = (\hat{h}_{ik} + e_{ik})\bar{l}_k\left(\sqrt{a_1P_2x_1} + \sqrt{a_2P_2x_2}\right) + \bar{n}_i = \sqrt{a_2P_2x_2} + \bar{n}_i,
\]  

(4)

where recall that \( i \in \{1, 2\} \) in (1). For simple presentation, we remove the group index notation \( g \) hereinafter.

Given (4), \( R_1 \) detects the designated signal \( x_1 \), treating \( x_2 \) in (4) as an interference. The instantaneous signal-to-interference-plus-noise ratio (SINR) at \( R_1 \) from (4) can be written by

\[
\gamma_{R_1} = \frac{a_1\hat{\rho}_{1k}\bar{l}_k}{a_2\hat{\rho}_{1k}\bar{l}_k + \varepsilon_{1k}^2\bar{l}_k\rho_o + 1},
\]  

(5)

where

\[
\hat{\rho}_{ik} = \frac{P[|\hat{h}_{ik}|^2]}{\sigma_n^2} \sim \mathcal{N}(m_{ik}, \eta_{ik}), \quad \eta_{ik} = \frac{P E[|\hat{h}_{ik}|^2]}{\sigma_n^2 m_{ik}},
\]

and \( \rho_o = P/\sigma_n^2 \). Notice that in the denominator of (5), the first term refers to the inter-cluster interference and the second term quantifies the channel estimation noise.

As for the detection of \( x_2 \), \( R_2 \) first performs SIC that decodes and removes \( R_1 \)'s message, followed by decoding its own message without interference. For this, the instantaneous SINR at \( R_2 \) for the detection of \( x_1 \) can be written as

\[
\gamma_{R_{12}} = \frac{a_1\hat{\rho}_{2k}\bar{l}_k}{a_2\hat{\rho}_{2k}\bar{l}_k + \varepsilon_{2k}^2\bar{l}_k\rho_o + 1}.
\]  

(6)

After the SIC in this context, producing a positive quantity for residual interference and channel estimation noise without the interference of \( x_1 \) by assuming perfect SIC, the instantaneous signal-to-noise ratio (SNR) at \( R_2 \) for the detection of \( x_2 \) can be represented as

\[
\gamma_{R_2} = \frac{a_2\hat{\rho}_{2k}\bar{l}_k}{\varepsilon_{2k}^2\bar{l}_k\rho_o + 1}.
\]  

(7)

As shown from (7), \( \gamma_{R_{12}} \) is affected by the residual interference and fronthaul channel estimation noise. The residual interference is quantified by \( \rho_o \), which is related to \( (a_1 + a_2)P \) rather than only \( a_2P \), while the channel estimation noise is represented by \( \varepsilon_{2k}^2 \).

### III. Outage Performance Analysis

We now provide the performance analysis of the proposed system which is suitable for outage sensitive applications. In particular, we focus on investigating the outages of the NOMA designs with regards to the impact of uncertainties raised from both the wireless backhauls and fronthauls. Consider an opportunistic SS, the best transmitter selection relies on a hybrid backhaul-fronthaul condition and this approach is related to either near receiver (\( R_2 \)) or far-away receiver (\( R_1 \)), in the proposed NOMA designs. Considering an outage sensitive system, case I should be considered as enhancing the system outage performance. On the other hand, if receiver fairness is considered to be a more important factor than system outage, then case II should be considered. For this, we investigate two different cases of an opportunistic SS: I) the best among the \( K \) transmitters is selected, exploiting jointly the backhaul reliability \( \bar{l}_k \) and the fronthaul channel only from the near receiver; II) the best transmitter is selected, taking into consideration jointly the backhaul reliability \( \bar{l}_k \) and the fronthaul channel only from the far-away receiver. In other words, the selected transmitter for the proposed NOMA acts as the best for one receiver, but not for both.

With regards to selection rules for the two cases, the index of the selected transmitter is given by

\[
k^* = \arg \max_{k=1, \ldots, K} |\hat{h}_{ik}|^2\bar{l}_k,
\]

(8)

where \( i = 2 \) is the index of near receiver \( R_2 \) for case I, and \( i = 1 \) the index of far-away receiver \( R_1 \) for case II.

This means that \( k^* \) indicates the index of the best transmitter in relation to \( R_2 \) in case II (or \( R_1 \) in case I). Next, we derive the outage probabilities of the grouped receivers for each case, providing a comprehensive insight into a relationship between the backhauled opportunistic SS and the imperfect channel estimation which leads to imperfect SIC at NOMA receivers by producing residual interference. The two receivers have their own target SINRs, which are denoted by \( \gamma_{th_i}, i = 1, 2 \). For simple analysis and without loss of generality, we assume that \( \gamma_{th_1} = \gamma_{th_2} = \gamma_{th} \).

#### A. Case I: Opportunistic SS for Near Receiver

We analyze the outage probability for Case I when the composite unreliable links of backhaul and fronthaul (in relation to \( R_2 \), i.e., the near receiver) are used by the opportunistic SS in determining the best transmitter.

1) Outage Probability at \( R_1 \): \( R_1 \) will be in outage when the transmission fails at \( R_1 \). Therefore, using (5), the outage probability at \( R_1 \) can be expressed as

\[
OP_{R_1}^i = \Pr(\gamma_{R_1} < \gamma_{th}) = \Pr\left(\hat{\rho}_{1k}\bar{l}_k < \frac{\varepsilon_{1k}^2\bar{l}_k\rho_o + \gamma_{th}}{a_1 - a_2\gamma_{th}}\right) = \Pr(\hat{\rho}_{1k}\bar{l}_k < \phi_1),
\]

(9)

where

\[
\phi_1 = \frac{\varepsilon_{1k}^2\bar{l}_k\rho_o + \gamma_{th}}{a_1 - a_2\gamma_{th}}.
\]

Note that PDF and CDF of a random variable, \( \hat{\rho}_{1k}\bar{l}_k \), which represents the product of the Bernoulli random process and the
Nakagami-m random process [17], are expressed, respectively, by
\[
\hat{f}_{\hat{\rho}_1:k}(x) = p_k \hat{f}_{\hat{\rho}_k}(x) + (1 - p_k)\delta(x),
\]
(10)
\[
F_{\hat{\rho}_1:k}(x) = 1 - \frac{p_k \Gamma\left(m_{ik}x, \frac{\eta_k}{\eta_k}\right)}{\Gamma(m_{ik})},
\]
(11)
where \(\delta(\cdot)\) denotes the Dirac delta function.

Employing the law of total probability, (9) can be given by
\[
OP_{R_1} = \sum_{k=1}^{K} \Pr(T_k = T_{k^*}, \hat{\rho}_{1:k} \neq \phi_1).
\]
(12)
It should be noted that \(k^*\) is the selected transmitter index which depends on \(\hat{h}_{2k}\) and is independent of \(\hat{h}_{1k}\).

After applying (10) in (12), \(OP_{R_1}\) can be re-expressed as
\[
OP_{R_1} = \sum_{k=1}^{K} \left\{ \Theta_1 \Pr(\hat{\rho}_{1:k} < \phi_1)p_k + \frac{1}{O_1} \prod_{u=1}^{K} (1 - p_u) \right\},
\]
(13)
where \(O_1\) is the outage probability that \(T_k\) (as the best transmitter) faces outage events under its reliable backhaul, \(O_2\) is the probability of the outage event when \(T_k\) is selected as the best under all the unreliable backhaul links where the outage always happens, and \(\Theta_1\) represents the probability that \(T_k\) becomes the best transmitter relying on its reliable backhaul and fronthaul links of \(R_2\) (not \(R_1\)), which can be formulated as
\[
\Theta_1 = \Pr(\hat{\rho}_{2:k} \leq \hat{\rho}_{2:k^*})
\]
\[
= \int_{0}^{\infty} \prod_{j=1}^{K} p_j \Gamma\left(m_j, \frac{\hat{\rho}_{2:k}}{\eta_j}\right) d\hat{\rho}_{2:k},
\]
(14)
where \(\hat{\rho}_{2:k} \geq \max_{j \neq k,j=1,\ldots,K} \hat{\rho}_{2:j}\). Due to mathematical intractability for \(\Theta_1\) in closed-form and highly complex integration, we develop an approximation. In particular, it should be noted that the selected transmitter index \(k^*\) is not related to \(R_1\). Thus, in the outage probability at \(R_1\), each \(T_k\), \(\forall k\) may be equally likely treated as the best transmitter, considering a normalized heterogeneous metric for \(\Theta_1\).

Substituting the series expansion of the upper incomplete gamma function [32, Eq. (8.352.4)], (13) can be approximately obtained as
\[
OP_{R_1} \approx \frac{1}{K} \sum_{k=1}^{K} \left\{ \left(1 - \frac{\phi_1}{\eta_k}\right) p_k + \prod_{u=1}^{K} (1 - p_u) \right\}
\]
\[
= \frac{1}{K} \sum_{k=1}^{K} \left\{ (1 - e^{-\eta_k}) \sum_{l=0}^{m-1} \frac{(\phi_1)^l}{l!} p_k + \prod_{u=1}^{K} (1 - p_u) \right\}.
\]
(15)
We validate the accuracy of (15) in the simulation section. There is a negligible gap between the exact and approximated outcomes for the outage probability of \(R_2\) even at small values for \(p_k\) (\(p_k \ll 1\)).

Remark 1: It is worth mentioning from (15) that \(OP_{R_1}\) is influenced by \(e_{2:k}^2\) via \(\phi_1\). That is, as \(e_{2:k}^2\) increases, \(\phi_1\) grows linearly, i.e., \(\phi_1 \approx (\gamma_{th}/(a_1 - a_2\gamma_{th})) \cdot e_{2:k}^2\). Thus, \(OP_{R_1}\) increases with \(e_{2:k}^2\) along with the presence of \(a_2\) in \(\phi_1\). This observation reveals the fact that the channel estimation noise and interference result in the composite distortion, leading to higher \(\phi_1\) (and thus, larger \(OP_{R_1}\)).

2) Outage Probability at \(R_2\): According to the NOMA scheme, \(R_2\) will be in outage when both the decoding message of \(R_1\) for SIC and the post-SIC message for \(R_2\) are in outage. Therefore, using (6) and (7), the outage probability at \(R_2\) can be determined as
\[
OP_{R_2} = 1 - \Pr(\gamma_{R_2} + \gamma_{R_2} > \gamma_{th}),
\]
\[
= 1 - \Pr\left(\hat{\rho}_{2:k^*}^* < \hat{\rho}_{2:k^*}^{\gamma_{th}} + \gamma_{th}, \hat{\rho}_{2:k^*}^* < \hat{\rho}_{2:k^*}^{\gamma_{th}} \right)
\]
\[
= 1 - \Pr\left(\hat{\rho}_{2:k^*}^* > \frac{\gamma_{th}}{a_2} \right)
\]
\[
= \Pr\left(\hat{\rho}_{2:k^*}^* < \frac{\gamma_{th}}{a_2} \right),
\]
(16)
where \(\phi_2 = (\gamma_{2:k}^2)^2 \eta_k \gamma_{th} + \gamma_{th})/a_2 \Phi = \max(\phi_1, \phi_2)\). \(k^*\) in (16) depends on \(h_{2k}\). According to the higher order statistics, the random variables of \(\hat{\rho}_{2:k^*}^*\) is the largest among \(K\) products of Bernoulli and Gamma distributions. After applying (11) in (16), the \(F_{\hat{\rho}_{2:k^*}^*}(\Phi)\) can be derived as
\[
OP_{R_2} = \prod_{k=1}^{K} \left\{ 1 - \frac{p_k \Gamma\left(m_k, \frac{\Phi}{\eta_k}\right)}{\Gamma(m_k)} \right\}.
\]
(17)
For the non-identical distribution of the \(F_{\hat{\rho}_{2:k^*}^*}(\Phi)\), we refer to the identity [17] as follows
\[
\prod_{k=1}^{K} (1 - x_n) = 1 + \sum_{k=1}^{K} (-1)^{K-k} \sum_{l=1}^{K} (1 - x_{nt}),
\]
(18)
where \(\sum = \sum_{n_1=1}^{K-k-1} \sum_{n_2=n+1}^{K-k-2} \cdots \sum_{n_k=n_k+1-1}^{K} \). Applying (18) and the series expansion of the upper incomplete gamma function, (17) can be rewritten as
\[
OP_{R_2} = 1 + \sum_{k=1}^{K} (-1)^{K-k} \sum_{l=1}^{K} \left\{ p_{nt} \frac{\Gamma\left(m_{nt}, \frac{\Phi}{\eta_{nt}}\right)}{\Gamma(m_{nt})} \right\}.
\]
(19)
Remark 2: It can be shown from (19) that \(OP_{R_2}\) is influenced by \(e_{2:k}^2\) via \(\Phi\), defined in (16), for given system parameters such as \(K, \gamma_{th}, a_1, a_2\). In particular, as \(e_{2:k}^2\) increases, \(\Phi\) increases, being dependent of either \(\phi_1\) or \(\phi_2\). Based on such dependency, the increase in \(OP_{R_2}\) can be determined by a proper choice of \(a_1\) and \(a_2\) as well as \(e_{2:k}^2\).
B. Case II : Opportunistic SS for Far-Away Receiver

We now address Case II when the opportunistic SS determines the best transmitter referring to the backhaul and fronthaul links of \( R_l \) (far-away receiver).

1) Outage Probability at \( R_1 \): The outage probability at \( R_1 \) can be formulated as

\[
OP_{R_1}^2 = \Pr (\hat{p}_{1k} \hat{\eta}_k < \phi_1),
\]

where \( k^* = \arg \max_{k=1,...,K} |\hat{h}_{1k}|^2 \) and \( k^* \) in (20) depends on the channel coefficient \( \hat{h}_{1k} \). Thus, (20) can be derived as

\[
OP_{R_1}^2 = \prod_{k=1}^{K} \left\{ 1 - \frac{p_k \Gamma \left( m_k, \frac{\phi_1}{\eta_k} \right)}{\Gamma (m_k)} \right\} = 1 + \sum_{k=1}^{K} (-1)^k \sum_{l=1}^{K} p_{nl} e^{-\frac{\eta_n}{l!}} \sum_{l=0}^{m_l-1} \frac{\left( \frac{\phi_1}{\eta_n} \right)^l}{l!}. \tag{21}
\]

Remark 3: Similarly to \( OP_{R_1}^2 \), at \( R_2 \) in Case I, notice that \( OP_{R_2}^2 \) in Case II benefits from the best transmitter whose selection is related to backhaul-fronthaul links of \( R_1 \). Unlike \( OP_{R_1}^2 \), however, \( OP_{R_2}^2 \) is influenced by \( \phi_1 \), not by \( \Phi \). This reveals that \( OP_{R_1} \) always decreases faster than \( OP_{R_2}^2 \), due to \( \Phi \geq \phi_1 \). Such an observation can be observed in Figs. 2-3.

2) Outage Probability at \( R_2 \): The outage probability at \( R_2 \) can be expressed as

\[
OP_{R_2}^2 = \Pr (\hat{p}_{2k} \hat{\eta}_k < \Phi),
\]

where \( k^* \) in (22) is independent of \( \hat{h}_{2k} \). Thus, \( OP_{R_2}^2 \) can be given by

\[
OP_{R_2}^2 = \sum_{k=1}^{K} \left\{ \Theta_2 \left( 1 - \frac{\Gamma \left( m_k, \frac{\Phi}{\eta_k} \right)}{\Gamma (m_k)} \right) + \frac{1}{K} \prod_{u=1}^{K} (1 - p_u) \right\},
\]

where

\[
\Theta_2 = \Pr (\hat{p}_{1k} \hat{\eta}_k \geq \hat{p}_{1j} \hat{\eta}_j) \text{ with } \hat{p}_{1j} \hat{\eta}_j = \max_{j \neq k, j=1,...,K} \hat{p}_{1j} \hat{\eta}_j.
\]

Similar to \( OP_{R_1}^2 \) in Case I, (23) can be approximated as

\[
OP_{R_2}^2 \approx \frac{1}{K} \sum_{k=1}^{K} \left\{ \left( 1 - e^{-\frac{\Phi}{\eta_k}} \sum_{l=0}^{m_k-1} \frac{\left( \frac{\Phi}{\eta_k} \right)^l}{l!} \right) p_k + \prod_{u=1}^{K} (1 - p_u) \right\}.
\]

Remark 4: Notice that \( OP_{R_2}^2 \) in (24) is in a similar form to \( OP_{R_1}^2 \) in (15) and relies on \( \Phi \), not on \( \phi_1 \). This implies that \( OP_{R_2}^2 \) is worse than \( OP_{R_1}^2 \), because \( \Phi \) is always larger than \( \phi_1 \). Also, it is important to mention that \( OP_{R_2}^2 \) is influenced mainly by the channel estimation noise \( \varepsilon_{2k}^2 \), as \( \Phi = \phi_2 \) at \( a_1 \gg a_2 \).

Remark 5: As for the outage performance analysis of the proposed NOMA scheme, it is worth pointing out that for both Cases I and II, the outage probability benefits from the positive quantity of the backhaul reliability and the opportunistic SS. For example, when \( p_k = 0, \forall k \), \( OP_{R_1}^2 = OP_{R_2}^2 = 1 \). When \( p_k > 0, \forall k \), \( OP_{R_1}^2 \) can benefit from \( K \). In addition, notice that the uncertain fronthaul channel usage in the detector causes the imperfect SIC and channel estimation noise. Even after the SIC, interestingly, \( OP_{R_1}^2 \) in (19) and \( OP_{R_2}^2 \) in (24) are shown to suffer from the residual interference and the channel estimation noise, which are quantified by \( \varepsilon_{2k}^2 \) and \( a_1 \) in \( \Phi \). They become robust to the residual interference term \( \phi_2 \) (containing \( a_1 \)) relying only on \( \Phi = \phi_2 \), when \( a_1 \gg a_2 \).

IV. Special Cases of Outage Probability Analysis

We investigate the outage performance probability for the special cases, namely perfect channel estimation for the cooperative system with unreliable backhauls, the non-cooperative system with unreliable backhauls and then the cooperative system with completely reliable backhauls. In particular, to investigate the impact of channel estimation error on the outage performance, we consider the case when the receivers are provided with their perfect channel information. In addition, to investigate the impact of the non-opportunistic work of the transmitters which also can be a non-cooperative system, we consider the case when there is only one transmitter which supports two NOMA receivers. Finally, to investigate the impact of the residual interference on the backhauls, we consider the case when all backhauls are completely reliable. In this section, we focus on the outage performance for the special cases, considering only Case I as the outage probabilities for the special cases in Case II are straightforward.

A. Outage Performance with Perfect Channel Estimation

For the perfect SIC, we assume that the receivers are provided with their channel information perfectly, i.e., \( \varepsilon_{2k}^2 = 0 \). The outage probabilities of \( R_1 \) and \( R_2 \) with perfect channel information are, respectively, derived as

\[
OP_{R_1}^{per} \approx \frac{1}{K} \sum_{k=1}^{K} \left\{ \left( 1 - e^{-\frac{\phi_1}{\eta_k}} \sum_{l=0}^{m_k-1} \frac{\left( \frac{\phi_1}{\eta_k} \right)^l}{l!} \right) p_k + \prod_{u=1}^{K} (1 - p_u) \right\},
\]

\[
OP_{R_2}^{per} = 1 + \sum_{k=1}^{K} (-1)^k \sum_{l=1}^{K} p_{nl} e^{-\frac{\phi_1}{\eta_n}} \sum_{l=0}^{m_l-1} \frac{\left( \frac{\phi_1}{\eta_n} \right)^l}{l!},
\]

where \( \phi_1 = \gamma_{th}/(a_1 - a_2\gamma_{th}) \), \( \phi_2 = \gamma_{th}/a_2 \) and \( \Phi = \max(\phi_1, \phi_2) \).

Remark 6: The outage probabilities of the receivers with perfect channel information are worth studying, since they show a difference between the outage performance of the opportunistic NOMA with perfect channel estimation and that of the opportunistic NOMA with imperfect channel estimation. Hence, the loss due to channel uncertainty can be investigated here. In particular, the channel estimation error, \( \varepsilon_{2k}^2 \), in \( \phi_1 \) \((\phi_1 = \frac{\varepsilon_{2k}^2 \eta_k \gamma_{th}}{a_1 - a_2 \gamma_{th}})\) between (15) and (25) are influenced by the power allocation coefficients, \( a_1 \) and \( a_2 \) in \( \phi_1 \) and \( \Phi \), and the value of estimation error. On the other hand, the outage probability of \( R_2 \), (26) provides error gaps from (19), and these gaps are independent of \( a_1 \) in \( \Phi \). Since the transmitter is selected based on the backhaul and the fronthaul links of \( R_2 \) which is a dominant receiver, there is a small difference between (19) and (26).
B. Non-opportunistic System with Unreliable Backhauls

Consider the outage probabilities of the non-opportunistic system with unreliable backhauls, where the number of transmitters, \( K = 1 \). Thus, the outage probabilities of (15) and (19) can be, respectively, evaluated as

\[
OP_{R_1}^{\text{non}} = \left\{ 1 - \frac{\Gamma \left( m_1, \frac{\phi_1}{\eta_1} \right)}{\Gamma (m_1)} \right\} p_1 + (1 - p_1)
\]

\[
= 1 - p_1 e^{-\frac{\phi_1}{\eta_1}} \sum_{l=0}^{m_1-1} \frac{\left( \frac{\phi_1}{\eta_1} \right)^l}{l!},
\]

\[
OP_{R_1}^{\text{non}} = 1 - p_1 e^{-\frac{\phi_1}{\eta_1}} \sum_{l=0}^{m_1-1} \frac{\left( \frac{\phi_1}{\eta_1} \right)^l}{l!}.
\]

Remark 7: The outage probabilities of the proposed system model in the non-opportunistic system provide that both outage probabilities rely on the power allocation coefficients, \( a_1 \) and \( a_2 \) in \( \phi_1 \) and \( \Phi \), since the opportunistic selection rule is not considered for the non-opportunistic system. From the equations, it can be seen that (27) quickly converges to a lower outage probability than (28), since \( \Phi \) is always bigger than \( \phi_1 \). Furthermore, for the outage probabilities of the non-opportunistic system for case II, the same outage probabilities can be obtained.

C. Cooperative System with Completely Reliable Backhauls

Now, we consider the special case of the outage probabilities of the cooperative system with completely reliable backhauls, where \( p_k = 1 \), \( \forall k \) in (15) and (17). Thus, the outage probabilities of \( R_1 \) and \( R_2 \) are, respectively, derived as

\[
OP_{R_1}^{\text{co}} \approx \frac{1}{K} \sum_{k=1}^{K} \left\{ 1 - \frac{\Gamma \left( m_k, \frac{\phi_1}{\eta_k} \right)}{\Gamma (m_k)} \right\}
\]

\[
= \frac{1}{K} \sum_{k=1}^{K} \left( 1 - e^{-\frac{\phi_1}{\eta_k}} \sum_{l=0}^{m_k-1} \frac{\left( \frac{\phi_1}{\eta_k} \right)^l}{l!} \right),
\]

\[
OP_{R_2}^{\text{co}} = \prod_{k=1}^{K} \left( 1 - e^{-\frac{\phi_1}{\eta_k}} \sum_{l=0}^{m_k-1} \frac{\left( \frac{\phi_1}{\eta_k} \right)^l}{l!} \right).
\]

Remark 8: In this case, only fronthaul links are considered to select the best transmitter for the opportunistic selection rule. From the above, note that both (29) and (30) decrease as \( K \) increases. In addition, interestingly, the outage probability of \( R_2 \) decreases much faster than the outage probability of \( R_1 \) since (30) depends on product of all the outage probabilities for various values of \( K \), while (29) depends on their average. In addition, it is verified that outage error floor depends on backhaul reliability for the opportunistic NOMA with channel estimation error.

V. ASYMPTOTIC ANALYSIS

In this section, we provide asymptotic outage probabilities of \( R_1 \) and \( R_2 \) in Case I for further insight. For simplicity, only Case I is considered in this section while the similar steps can be applied to Case II. Firstly, the extreme case of high channel estimation noise leading to the severe imperfect SIC outcomes is investigated and then, the case of high SNRs with unreliable backhauls is discussed, in terms of outage probability.

A. High \( \varepsilon_{ik}^2 \) and Imperfect SIC

Under imperfect channel estimation on the fronthaul links, we address asymptotically the outage probability behaviors associated with unreliable backhauls.

1) Asymptotic Outage Probability of \( OP_{R_1}^{\text{1}} \): When the wireless fronthauls are in high channel estimation noise (\( \varepsilon_{ik}^2 \gg 0 \)), (15) can be approximated as

\[
OP_{R_1}^{\text{1}} \approx \frac{1}{K} \sum_{k=1}^{K} \left\{ 1 - \left( 1 - \frac{\sum_{l=0}^{m_k-1} \left( \frac{A_1}{l!} \right)^l}{K} \right) p_k + \sum_{u=1}^{K} (1 - p_u) \right\},
\]

\[
(31)
\]

where \( A_1 = \varepsilon_{ik}^2 \log \rho_k \eta_{kth} \). As \( \varepsilon_{ik}^2 \to \infty \), (31) can be asymptotically given by

\[
OP_{R_1}^{\text{asy}} = \frac{1}{K} \sum_{k=1}^{K} \left\{ p_k + \sum_{u=1}^{K} (1 - p_u) \right\}.
\]

Interestingly, it can be shown from (32) that the asymptotic outage probability of \( R_1 \) with unreliable backhauls achieves the outage limit. For very high channel estimation noise, \( A_1 \) goes to zero. Hence, (32) determines the outage limit, in relation to the backhaul reliability \( p_k, \forall u \) and \( K \).

2) Asymptotic Outage Probability of \( OP_{R_2}^{\text{1}} \): When \( \varepsilon_{2k}^2 \gg 0 \), (17) can be approximated as

\[
OP_{R_2}^{\text{1}} \approx \prod_{k=1}^{K} \left\{ 1 - \left( 1 - \frac{\sum_{l=0}^{m_k-1} \left( \frac{B_1 \cdot \varepsilon_{2k}^2}{l!} \right)^l}{m_k} \right) p_k \right\},
\]

\[
(33)
\]

where \( B_1 = \max \left\{ \frac{1}{a_1 - a_2 \gamma_{2k}^2}, \frac{1}{a_2} \right\} \frac{\sum_{l=0}^{m_k-1} \left( \frac{B_1 \cdot \varepsilon_{2k}^2}{l!} \right)^l}{m_k} \). For high \( \varepsilon_{2k}^2 \), asymptotic outage probability of \( OP_{R_2}^{\text{1}} \) can be re-expressed as

\[
OP_{R_2}^{\text{asy}} = \prod_{k=1}^{K} \left\{ 1 - B_1 \cdot p_k \right\}
\]

\[
(34)
\]

As \( \varepsilon_{2k}^2 \to \infty \), \( B_1 \) goes to zero, leading to the increase in \( OP_{R_2} \). Note that the asymptotic outage probability of \( R_2 \) with high channel estimation noise is influenced by imperfect SIC that \( a_1 \) in \( B_1 \), and we find the outage performance limit which is determined by the backhaul reliability, \( p_k, \forall k \).

B. High SNR and Unreliable Backhauls

For high SNRs on the fronthaul links, we address asymptotically the outage probability behavior associated with unreliable backhauls.
1) Asymptotic Outage Probability of $OP_{R_1}^1$: In high SNRs ($\rho_o \gg 0$), (15) can be rewritten as

$$OP_{R_1}^1 \approx \frac{1}{K} \sum_{k=1}^{K} \left\{ 1 - e^{-\bar{A}_2} \sum_{l=0}^{m_k-1} \frac{(\bar{A}_2)^l}{l!} \right\} p_k + \prod_{u=1}^{K} (1 - p_u),$$  

(35)

where $\bar{A}_2 = (\varepsilon_{ik}^2 k \gamma_{th} m_k)/(\{a_1 - a_2 \gamma_{th}\} E[|\hat{h}_{1k}|^2])$. $OP_{R_1}^1$ can be asymptotically given by

$$OP_{R_1}^{asy} = \frac{1}{K} \sum_{k=1}^{K} \left\{ A_2 \cdot p_k + \prod_{u=1}^{K} (1 - p_u) \right\}.$$  

(36)

Notice that $A_2$ does not depend on $\rho_o$. Hence, (36) determines the outage limit, in relation only to the backhaul reliability $p_u$, $\forall u$ and $K$. In addition, the asymptotic outage probability of $R_1$ in the high SNRs is influenced by the power allocation coefficients, $(a_1$ and $a_2$) and the estimation error variance $\varepsilon_{ik}^2$.

2) Asymptotic Outage Probability of $OP_{R_2}^1$: In high SNRs ($\rho_o \gg 0$), (17) can be rewritten as

$$OP_{R_2}^1 \approx \prod_{k=1}^{K} \left\{ 1 - p_k e^{-\bar{B}_2} \sum_{l=0}^{m_k-1} \frac{\bar{B}_2^l}{l!} \right\},$$  

(37)

where $\bar{B}_2 = \max \left\{ \frac{1}{a_1 - a_2 \gamma_{th}} \cdot \frac{1}{a_2}, 1 \right\} \varepsilon_{ik}^2 k \gamma_{th} m_k E[|\hat{h}_{1k}|^2]$. (37) can be approximated in terms of $K$ and $p_k$ as

$$OP_{R_2}^{asy} = \prod_{k=1}^{K} \left\{ 1 - B_2 \cdot p_k \right\}.$$  

(38)

It can be asymptotically shown from (38) that $OP_{R_2}^{asy}$ (and $B_2$) does not depend on $\rho_o$. Instead, the asymptotic outage probability of $R_2$ with high SNRs is determined by the backhaul reliability and $K$ for given $B_2$. Thus, the more $K$ transmitters along with high $p_k$, the lower that outage limit is. In addition, $B_2$ in the outage limit is also influenced by the power allocation coefficients, $a_1$ and $a_2$, $m_k$ and the estimation error variance $\varepsilon_{ik}^2$.

Remark 9: The asymptotic outage probabilities of $R_1$ and $R_2$ for both extreme conditions (very high channel estimation noise and high SNRs) have outage performance limits, in relation to the backhaul reliability and $K$. Interestingly, the condition of high $\varepsilon_{ik}^2$ reveals that the outage limits at $R_2$ in Case I is influenced by simply $p_k$, $\forall k$ and $K$, underperforming $R_1$. In turn, this means there is no benefits to opportunistic SS.

VI. NUMERICAL AND SIMULATION RESULTS

Now, we present numerical and simulation results to show our theoretical analysis on the outage probability of the grouped two receivers under both the impact of wireless backhaul unreliability and fronthaul channel uncertainty. In order to see improved outage performance of our proposed system, we consider an OMA scheme in presence of wireless backhaul unreliability with perfect channel information, and classical NOMA scheme, which is a non-cooperative NOMA scheme with perfectly reliable backhaul ($K = 1$ and $p_1 = 1$) as benchmark system models. Moreover, the proposed system for non-cooperative case is presented.

In the considered system, the power allocation coefficients are fixed to $a_1 = 0.8$ and $a_2 = 0.2$. For simplicity and without any loss of generality, we assume the same target SNR’s at $R_1$ and $R_2$, $\gamma_{th} = 0.1$ dB. For simulations, we use non-identical backhaul reliability and non-identical Nakagami-$m$ fading channels. For this, we assume the backhaul reliability for the $k$-th transmitter, $p_k$, with $p_1 = 0.96$, $p_2 = 0.95$, $p_3 = 0.94$ and the Nakagami-$m$ fading $m$ parameter for the $k$-th transmitter, $m_k$, with $m_1 = 1$, $m_2 = 2$, $m_3 = 3$. In order to see the outage performance of the grouped two receivers in the two opportunistic cases, we consider the outage probabilities of the proposed system model with perfect channel estimation for the first three figures (Figs. 2-4) with the channel estimation error set as $\varepsilon_{ik}^2 = 0$. Fig. 2 depicts the outage probabilities of the grouped two receivers in Case I (opportunistic SS based on a near receiver, $R_2$) for $K = 1, 2, 3$ with various backhaul reliability, $p_k$, and various values of $m_k$. As observed from Fig. 2, in non-cooperative system ($K = 1$), $R_1$ outperforms $R_2$ due to its allocated higher power coefficient. In contrast, $R_2$ which is a dominant receiver based on the opportunistic selection rule obtains better outage performance as the number of transmitter increases in cooperative system ($K > 1$), as shown in (25) and (26). In addition, both receivers can obtain a significant outage performance gain by increasing the number of transmitters. In other words, compared to the non-cooperative NOMA scheme ($K = 1$), the cooperative NOMA scheme provides a better outage performance. Interestingly, outage floors can be observed and their dominances are shown in high SNR’s. We also see the outage floors’ convergence rate which depends...
Outage Probability

**Fig. 3:** Impact of backhaul reliability on outage probability (Case II) for $K = 1, 2, 3$ under perfect channel estimation ($\varepsilon_{ik}^2 = 0$) and various levels of backhaul reliability, $p_k$, with various values of $m_k$.

**Fig. 4:** Impact of various values of $m_k$ on outage probability (Case I) for $K = 3$ under perfect channel estimation ($\varepsilon_{ik}^2 = 0$) with unreliable and reliable backhauls.

**Fig. 5:** Comparison of perfect and imperfect channel estimation ($\varepsilon_{1k}^2 = 0$ and $0.1$) at $R_1$ for $K = 1, 2, 3$ and various levels of backhaul reliability, $p_k$, with various values of $m_k$.

On $m$ parameter. For example, the outage probability of $R_1$ obtains a slower convergence rate as $m$ parameter increases, whereas that of $R_2$ obtains a faster convergence rate as $m$ parameter increases.

In order to compare the outage performance of different cases of opportunistic SS, Fig. 3 depicts the outage probabilities in Case II (opportunistic SS based on a far-away receiver, $R_2$) for $K = 1, 2, 3$ with various backhaul reliability and various values of $m_k$. As seen from Fig. 3, $R_1$ achieves better outage performance than $R_2$ in Case II. Moreover, outage probability of $R_1$ quickly converges to a constant in medium and high SNRs as $m$ parameter increases, while the outage probability of $R_2$ converges slowly to a constant in high SNRs. For fully comparison, OMA has been included in this figure. As observed from Fig. 3, compared with the traditional OMA\(^1\) with unreliable wireless backhauls, our proposed NOMA scheme can enhance the outage performance and also demonstrate the motivation of opportunistic NOMA under wireless backhaul unreliability. In particular, the figure shows that under unreliable wireless backhauls, a dominant receiver ($R_1$) in the proposed opportunistic NOMA scheme can achieve more than 3dB gain in outage performance, compared to the OMA. Interestingly, in non-cooperative system ($K = 1$), where only one transmitter supports two receivers with NOMA scheme, the outage performance of $R_1$ obtains better outage performance due to its allocated higher power coefficient. Moreover, the outage performance of a dominant receiver ($R_1$) based on the opportunistic selection rule obtains better outage performance as the number of transmitter increases in cooperative system.

In order to see impact of $m_k$, Fig. 4 depicts the outage probabilities of the grouped receivers in Case I when $K = 3$ with various values of $m_k$. As seen from Fig. 4, the outage probabilities show that their convergence rate can be seen to be determined by minimum value of $m_k$ parameters. In addition, to clearly see their convergence behaviour on the outage probability, we also consider the identical links for both receivers where all $m$ parameters are 2. Based on the observation, the value of the $m$ under cooperated transmitters influences the convergence rate of the grouped receivers in opportunistic NOMA with unreliable backhauls. Moreover, NOMA schemes with reliable and unreliable backhauls are separately considered in this figure. Interestingly, the outage

\(^1\)For OMA scheme, we consider opportunistic orthogonal frequency division multiple access (OFDMA) where one of the selected transmitters serves each receiver with the full transmit power. For comparison and avoiding high complexity, the power allocations for OMA receivers and the power allocation coefficients for NOMA receiver are not optimized in this paper.
probability clearly shows that backhaul reliability is responsible for the outage floor.

Fig. 5 shows the outage probabilities of $R_1$ for various values of $K$ when an imperfect channel estimation, $\varepsilon^2_{1k} = 0.1$ for the case $I$. The differences due to the estimation error between the outage probabilities with perfect channel estimation and the outage probabilities with imperfect channel estimation for various values of $K$ are presented. In addition, outage probabilities of $R_1$ which is a non-dominant receiver based on the best selection rule provide bigger error gaps, and these gaps can be increased as $K$ increases. The figure clearly shows that each receiver obtains different channel estimation error gaps which can be influenced by the number of $K$.

Fig. 6 shows similar outage probabilities of $R_2$ for various values of $K$ when an imperfect channel estimation, $\varepsilon^2_{2k} = 0.1$. Compared to the outage probabilities of $R_1$, the outage probabilities of $R_2$ which is a dominant receiver based on the best selection rule, provide small error gaps which are the same for all values of $K$. As we discussed in IV, the best transmitter is selected based on the backhaul and the fronthaul links of $R_2$. Hence, there is small difference between the outage probability of $R_2$ with perfect channel estimation and that of $R_2$ with imperfect channel estimation.

In order to see the impact of channel estimation error, Figs. 7 and 8 plot the outage probabilities of $R_1$ and $R_2$ with various values of channel estimation error when $K = 3$, for the case $I$. The normalized channel estimation error (CEE), $\varepsilon^2_{1k}$, varies from 0.001 to 0.5. For the outage probabilities of $R_1$, it is clearly shown that the outage performance is significantly improved as the estimation error values decrease while the outage probabilities of $R_2$ are not changed that much in the low value of channel estimation errors. But, interestingly, as the channel estimation noise grows very large ($\varepsilon^2_{1k} \to \infty$) as shown in (32) and (34), the asymptotic outage probability of $R_2$ is higher than that of $R_1$, because there is no benefits of opportunistic SS on the outage probability of $R_2$.

In order to see the impact of number of transmitters on the outage performance, Fig. 9 plots the outage probabilities under unreliable backhauls and imperfect channel estimation for the case $I$. As seen from Fig. 9, outage floors are shown from the medium SNR regions. In addition, the convergence rate to the outage floors of $OP_{R_1}$ is much faster than that in Fig. 2 while the convergence rate to the outage floors of $OP_{R_2}$ in Fig. 2 and that in Fig. 9 is similar. In addition, Fig. 9 verifies that the outage probability limits of $R_1$ and $R_2$ are determined by backhaul reliability. Based on the observations from Fig. 9, it is shown that the asymptotic outage performance of $R_1$ is much worse than the asymptotic outage performance of $R_2$ for the cooperative system when backhauls are unreliable in the medium and high SNR regions, because interference of $R_2$ in $OP_{R_2}$ also increases as $P/\sigma^2_n \to \infty$. 
We investigate the opportunistic NOMA under wireless backhaul unreliability and fronthaul channel uncertainty. For this, we have proposed two opportunistic selection methods, which reveal the impact of the wireless backhaul reliability and the impact of channel uncertainty. The closed-form outage probabilities of the grouped NOMA receivers have been derived. In addition, we have investigated the outage probability in the special cases and the asymptotic outage performance for further insights. These provided several insights into the impact of wireless backhaul unreliability and fronthaul uncertainty on the opportunistic NOMA. For example, the unreliability levels of multiple wireless backhauls are jointly responsible for the outage floors. The outage probability at a dominant receiver is shown to be less influenced by imperfect channel information, while that at a non-dominant receiver is highly sensitive. These provide an insight into that the dominant receiver is shown to be less influenced by imperfect channel uncertainty compared to the opportunistic SS.

Moreover, with the traditional OMA, it has been shown that the proposed opportunistic NOMA can improve the outage performance under wireless backhaul unreliability. The theoretical analysis expressions of the opportunistic NOMA under unreliable wireless backhauls can be applied to evaluate the outage performance for various NOMA designs under unreliable backhaul and imperfect channel uncertainty.

REFERENCES


Sunyoung Lee (S’15) received her Ph.D. degree in the Electronics, Communications and Information Technology (ECIT) Institute at Queen’s University Belfast in 2019. She is currently a research fellow at Warwick Manufacturing Group (WMG), University of Warwick. Her research interests include non-orthogonal multiple access, relay network, cognitive radio network, vehicular communications.

Trung Q. Duong (S’05, M’12, SM’13) received his Ph.D. degree in Telecommunications Systems from Blekinge Institute of Technology (BTH), Sweden in 2012. Currently, he is with Queen’s University Belfast (UK), where he was a Lecturer (Assistant Professor) from 2013 to 2017 and a Reader (Associate Professor) from 2018. His current research interests include Internet of Things (IoT), wireless communications, molecular communications, and signal processing. He is the author or co-author of over 330 technical papers published in scientific journals (196 articles) and presented at international conferences (135 papers). Dr. Duong currently serves as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON COMMUNICATIONS, IEEE COMMUNICATIONS LETTERS, and a Lead Senior Editor for IEEE COMMUNICATIONS LETTERS. He was awarded the Best Paper Award at the IEEE Vehicular Technology Conference (VTC-Spring) in 2013, IEEE International Conference on Communications (ICC) 2014, IEEE Global Communications Conference (GLOBECOM) 2016, and IEEE Digital Signal Processing Conference (DSP) 2017. He is the recipient of prestigious Royal Academy of Engineering Research Fellowship (2016-2021) and has won a prestigious Newton Prize 2017.

Roger Woods (M’95–SM’01) received the B.Sc. degree (Hons.) in electrical and electronic engineering and the Ph.D. degree from Queen’s University Belfast in 1985 and 1990, respectively. He is currently a Full Professor there and has founded a spin-off company, Analytics Engines Ltd which develop data analytics software. His research interests are in heterogeneous programmable systems, data analytics, and wireless communications. He holds four patents and has authored over 230 papers. He is a member of the IEEE Signal Processing and Industrial Electronics Societies and sits on the Advisory Board for the IEEE SPS Technical Committee on the Design and Implementation of Signal Processing Systems. He is on the Editorial Board for the Journal of VLSI Signal Processing Systems and the IET Proceedings on Computer and Digital Techniques. He is on the program committees of a number of IEEE conferences.