Crystal and magnetic structure of Mn₃IrSi

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A new ternary Ir-Mn-Si phase with stoichiometry Mn₃IrSi has been synthesized and found to crystallize in the cubic AlAu₄-type structure, space group P2₁3 with Z = 4, which is an ordered form of the β-Mn structure. The unit cell dimension was determined by x-ray powder diffraction to a = 6.4973(3) Å. In addition to the crystal structure, we have determined the magnetic structure and properties using superconducting quantum interference device magnetometry and Rietveld refinements of neutron powder diffraction data. A complex noncollinear magnetic structure is found, with magnetic moments of 2.97(4) μB at 10 K only on the Mn atoms. The crystal structure consists of a triangular network built up by Mn atoms, on which the moments are rotated 120° around the triangle axes. The magnetic unit cell is the same as the crystallographic and carries no net magnetic moment. The Neél temperature was determined to be 210 K. A first-principles study, based on density functional theory in a general noncollinear formulation, reproduces the experimental results with good agreement. The observed magnetic structure is argued to be the result of frustration of antiferromagnetic couplings by the triangular geometry.

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I. INTRODUCTION

The magnetic properties of different allotropes of elemental Mn and Mn-based compounds and alloys are conspicuous when compared to other transition-metal-based systems. With a few exceptions among the transition metals, it is for compounds containing Mn that one encounters noncollinear magnetism1—e.g., as witnessed in the α phase of Mn,1 fcc Mn,1 Mn₃Sn,1 and Pd₃Mn.2 Attempts have been made to understand why this is the case and first-principles calculations, based on a noncollinear formulation of density functional theory,1,3,4 reproduce the magnetic properties of, e.g., Pd₃Mn,5 fcc Mn,1 Mn₃Sn,1 and α-Mn.6 One view for why Mn dominates in the class of noncollinear magnets is that for the early part of the transition metal series the interatomic exchange interactions are antiferromagnetic whereas for the later part the exchange is ferromagnetic.7 Mn, being right in the middle of the series, hence has been suggested to balance these interactions by developing an exchange that is intermediate between ferromagnetism and antiferromagnetism—i.e., a noncollinear magnetic structure.8 This view has to some extent been challenged lately,9 with the argument that noncollinear magnetism is observed also for Fe (Ref. 1) and even Co-based (Ref. 10) compounds and alloys.

One of the allotropes of Mn, the β phase, reflects a very complex magnetic situation. Structurally the phase is stable between 725 °C and 1095 °C,11 but it can be preserved at low temperatures by quenching. The primitive cubic crystal structure consists of two crystallographically inequivalent sites: one 8-fold and one 12-fold position.12 For β-Mn several studies by both neutron diffraction,13 polarized neutron scattering,14 and NMR measurements14 have not found any evidence of magnetic long-range order down to 4.2 K. However, solid solutions with other elements—e.g., Al—give rise to antiferromagnetic couplings for the Mn atoms situated on the 12-fold positions, whereas the Mn atoms on the 8-fold positions carry no magnetism.14 There have been attempts to explain the lack of magnetic ordering by frustration of manganese spin moments on a three-dimensional triangular network built up by Mn atoms in 12-fold positions, which is suggested to result in a spin liquid that becomes spin glass like when other atoms are introduced.15 The Mn magnetic moments in β-(Mn₁₋ₓYₓ) range from 1μB to 1.5μB in size14 on the 12-fold site. This is in disagreement with a first-principles study that reports a moment of 0.2μB only on Mn atoms located on the type-I site.6 The type-I site would by common nomenclature correspond to the 8-fold position: however, this was not specified by the authors. Theoretical calculations for β-Mn have also been performed by, for instance, Sliwko et al. who within a theoretical model restricted to collinear couplings found a paramagnetic ground state, suggested to be close to ferrimagnetic order.15

Given the complexity of the magnetism of Mn-based compounds and, in particular, the β phase of Mn, it is of interest to examine the magnetic properties of other phases containing Mn, something we do here for the Ir-Mn-Si system for a phase with a crystal structure closely related to that of β-Mn. The ternary Ir-Mn-Si phase diagram has previously not been established: in fact, to our knowledge the only previous report in the literature of an ordered ternary compound of iridium, manganese, and silicon is a preliminary investigation of Mn₃IrSi by the present authors.16 In the current article we report the crystal and magnetic structures of the new phase Mn₃IrSi and also present magnetization measurements and theoretical electronic structure and total energy calculations.

II. EXPERIMENT

A. Sample preparation

A sample of composition Mn₃IrSi was prepared by drop synthesis17 in a high-frequency induction furnace. Single-
crystal pieces of silicon (Highways International, purity 99.999%) and pressed pellets of iridium powder (Alfa Aesar, purity 99.95%) were melted in an Al₂O₃ crucible under 300 mbar argon pressure. Pieces of manganese (Cerac, claimed purity 99.999%, purified from manganese oxides by sublimation) were dropped into the melt. After solidification the ingot was crushed and pressed into pellets, which were annealed in evacuated fused silica tubes at 900 °C for 3 days and at 800 °C for 1 week and subsequently quenched. After fine grinding the sample was stress relieved at 800 °C for 30 minutes.

B. X-ray powder diffraction

X-ray powder diffraction photographs were recorded at room temperature using a Guinier-Hägg focusing camera with Cu Kα1 radiation and with silicon powder [a = 5.43088(4) Å at 295 K] added as the internal standard for calibration. All diffraction lines could be indexed with a primitive cubic unit cell. The cell parameter was refined to a = 6.4973(3) Å.

C. Neutron powder diffraction

Neutron powder diffraction data were collected at the Swedish research reactor R2 in Studsvik. Measurements were performed at 295 K, 80 K, and 10 K with the sample contained in a vanadium tube. The neutron beam was monochromated to 1.471 Å by two Cu(220) single crystals in parallel alignment. The diffractograms were recorded in the 2θ range 4° – 139.92° in steps of 0.08°. An absorption correction was applied using μR = 0.715, as calculated from a transmission measurement at 2θ = 0°.

D. Structure refinements

Structure refinements were performed according to the Rietveld method using the program FULLPROF (Ref. 18) with the scattering lengths Ir: 10.6 fm; Mn: –3.73 fm; Si: 4.149 fm. The background was modeled by interpolation between fixed points. A pseudo-Voigt profile function with a refined ratio of Gaussian and Lorentzian contributions was used to describe the peak shape. For iridium and silicon a common ratio of Gaussian and Lorentzian contributions was used to describe the peak shape. For iridium and silicon a common range 4° – 139.92° in steps of 0.08°. An absorption correction was applied using μR = 0.715, as calculated from a transmission measurement at 2θ = 0°.

FIG. 1. Observed (dotted line), calculated (solid line), and difference (bottom line) neutron diffraction profiles at 295 K. Tick marks indicate the positions of Bragg reflections. The (200) reflection, allowed in space group P2₁/a, is indicated.

in space group P1. The starting model for the magnetic structure was obtained by reverse Monte Carlo simulations using the program RMCPow.¹⁹

E. Magnetization measurements

The susceptibility was measured with a superconducting quantum interference device (SQUID) magnetometer in the temperature range from 50 K to 300 K with an applied field of 1000 G. Furthermore, the magnetization was measured as a function of applied field for T = 50 K and T = 300 K. The field was varied from –1500 G to 1500 G in steps of 100 G.
TABLE I. Refined structural parameters at 295 K. Estimated standard deviations in parentheses. Space group $P2_13$ (no. 198), unit cell parameter $a=6.4972(3)$ Å.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Wyckoff position</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$B_{iso}$ (Å$^2$)</th>
</tr>
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<tbody>
<tr>
<td>Mn</td>
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<td>0.2074(9)</td>
<td>0.4546(8)</td>
<td>0.49(5)</td>
</tr>
<tr>
<td>Ir</td>
<td>4a</td>
<td>0.6833(3)</td>
<td>0.6833(3)</td>
<td>0.6833(3)</td>
<td>0.46(2)</td>
</tr>
<tr>
<td>Si</td>
<td>4a</td>
<td>0.0654(9)</td>
<td>0.0654(9)</td>
<td>0.0654(9)</td>
<td>0.46(2)</td>
</tr>
</tbody>
</table>

$R_p = 4.29\%$, $R_wp = 5.46\%$, $R_{exp} = 4.64\%$, $\chi^2 = 1.39$, $R_{Bragg} = 5.71\%$

*The isotropic temperature factors of Ir and Si were restrained to have the same value.

IV. RESULTS

A. Crystal structure

The crystal structure of Mn$_3$IrSi was found to be of the AlAu$_4$ type,$^{26}$ space group $P2_13$, with manganese in a 12b site and iridium and silicon in two separate 4a sites, which corresponds to four formula units per unit cell. In this structure type, which is an ordered form of the $\beta$-manganese structure type,$^{12}$ space group $P4_132$, the 8c site of $\beta$-Mn is split into two sets of fourfold positions which are occupied by different elements, while the 12-fold site is retained. For Mn$_3$IrSi, the ordering of Ir and Si on two different fourfold sites is evidenced by an observable (200) reflection (see Fig. 1), which is symmetry forbidden in space group $P4_132$ but allowed in $P2_13$. Examples of ternary compounds with the AlAu$_4$-type structure are Mn$_{33}$Ni$_{10}$Si$_7$ (Ref. 27) and Cr$_3$Ni$_5$Si$_2$ (Ref. 28); both phases are reported to have some mixed occupancy of the sites.

The structural parameters of Mn$_3$IrSi, as refined from neutron powder diffraction data at 295 K, are listed in Table I. Within experimental errors all three atomic sites are fully occupied, with no evidence of mixed occupancy. The agreement factors after the refinements were $R_{profile} = 4.29\%$, $R_wp = 5.46\%$, $R_{exp} = 4.64\%$, $\chi^2 = 1.39$, and $R_{Bragg} = 5.71\%$ for the 295 K data set. Interatomic distances shorter than 3.5 Å were calculated and are listed in Table II.

Manganese is effectively 14 coordinated in an irregular polyhedron, as illustrated in Fig. 2(a). In the polyhedron the three shortest Mn-Mn distances have been marked with dashed bonds. As illustrated in Fig. 2(b) these Mn atoms form a set of three equilateral triangles that are linked by the central Mn atom, with slightly different side lengths for the different triangles. All manganese atoms have the same surrounding, since there is only one crystallographic site for Mn. Thus a three-dimensional network of corner-linked Mn triangles stretches through the crystal structure, with each Mn atom shared between three triangles, as illustrated in the unit cell of Mn$_3$IrSi in Fig. 3. Both the iridium and silicon atoms are 12 coordinated by 9 Mn atoms and 3 Si or 3 Ir atoms, respectively, in a somewhat distorted icosahedral arrangement.

B. Magnetization measurements

The magnetization versus applied magnetic field, measured at 50 and 300 K, is shown in Fig. 4. The curves are closely linear and have almost the same slope. However, there is a weak nonlinearity and hysteresis that indicates the existence of a minor amount of ferromagnetic impurity in the sample (not resolved in the diffractograms). Disregarding this marginal artifact, we plot the measured susceptibility ($M/H$) versus temperature in an applied field of 1 kG in Fig. 5(a). The temperature dependence of the susceptibility shows a weak maximum at about 225 K, indicating an antiferromagnetic state at low temperatures. From the maximum in the $d(\chi)/dT$ versus temperature curve, shown in Fig. 5(b), the antiferromagnetic transition is found to occur at about 210 K employing the relation $d(\chi)/dT \approx C_m$, where $C_m$ is the magnetic specific heat, derived by Fisher.$^{29}$ The susceptibility is quite weakly temperature dependent and decays.

![FIG. 2. Coordination geometry around manganese. (a) 14 CN polyhedron (central Mn atom enlarged). (b) The triangles of Mn atoms, building up the triangular network (distances at 295 K displayed).](image-url)
only very slowly at temperatures above the maximum. If we assume a Curie-Weiss behavior at temperatures above the antiferromagnetic transition temperature ($T_N$), the slow decay indicates that the Weiss temperature ($\theta_W$) is very large compared to $T_N$. Such a difference is characteristic of highly frustrated magnetic systems.\textsuperscript{30} Unfortunately, our measurement system does not allow high-temperature measurements, so we have no possibility to explore temperatures high enough to extract an experimental $\theta_W$ value.

C. Experimental magnetic structure

The neutron powder diffractograms recorded at 10 K and 80 K both have magnetic contributions to the observed intensities; two peaks not present at room temperature emerge (indexed in Fig. 6) and some peaks display higher intensities at low temperatures. The magnetic contribution to the diffraction pattern is shown separately in the inset to Fig. 6, where the calculated nuclear contribution has been subtracted from the observed total intensity. It should however be noted that this plot also contains the differences between the observed and calculated nuclear diffraction intensities, which give rise to the somewhat noisy base line. All magnetic contributions can be explained by a magnetic unit cell of the same size as the crystallographic, but of lower symmetry, with magnetic moments only on the manganese atoms. The refined structural parameters and total magnetic moment at 10 K can be found in Table III. The agreement factors were $R_{\text{profile}}=4.59\%$, $R_{\text{wp}}=5.85\%$, $R_{\text{expt}}=4.75\%$, $\chi^2=1.52$, $R_{\text{Bragg}}=5.44\%$, and $R_{\text{mag}}=7.94\%$. No significant difference from the values obtained for the 10 K data set was noted in refinements of the 80 K data.

For the Mn atom with position coordinate $(0.1195, 0.2031, 0.4573)$ the total magnetic moment $M = 2.97(4) \mu_B$ at 10 K has the vector components $M_x = 1.3(1) \mu_B$, $M_y = 2.3(1) \mu_B$, and $M_z = -1.4(1) \mu_B$. The directions of the

FIG. 3. The crystal structure of Mn$_3$IrSi. The triangular network and the unit cell are indicated and one type of Mn triangle is shaded.

FIG. 4. The magnetization as a function of the applied field for $T=50$ K and 300 K.

FIG. 5. (a) The temperature dependence of the magnetic susceptibility ($\chi$) measured with an applied field of 1 kG. (b) $d(\chi T)/dT$ vs temperature. Note that the vertical axes have been truncated.

FIG. 6. Observed (dotted line), calculated (solid line), and difference (bottom line) neutron diffraction profiles at 10 K. Upper tick marks indicate the positions of Bragg reflections for the crystal structure, lower tick marks for the magnetic structure. Pure “magnetic” reflections, symmetry forbidden for the crystal structure, are indicated. The inset shows only the magnetic intensity, obtained by subtracting the calculated nuclear contribution from the observed total intensity.
magnetic moments on all 12 Mn atoms in the unit cell can be generated from this vector by symmetry operations of the crystallographic space group. For the atoms with coordinates generated by a $2_1$ screw operation, the components of the magnetic moments that are perpendicular to the $2_1$ axis are generated by a $2_1$ screw operation, the components of the crystallographic space group. For the atoms with coordinates generated from this vector by symmetry operations of the magnetic moments on all 12 Mn atoms in the unit cell can be generated by the symmetry operations described above. The four equivalent 2.68 Å triangles thus generated, with the net moments placed in the centers, are illustrated in Fig. 8.

### D. Theoretical magnetic structure

Several magnetic configurations for the Mn atoms in Mn$_3$IrSi have been investigated: ferromagnetic, collinear antiferromagnetic, and the experimental noncollinear antiferromagnetic arrangement described earlier in this paper. First-principles total energy calculations of the mentioned magnetic structures were performed in order to find the ground state of Mn$_3$IrSi. The calculations were achieved using the lattice parameters determined experimentally at room temperature, which are displayed in Table I. The ferromagnetic arrangement of the Mn atoms turned out to be the highest in energy. Four different kinds of collinear antiferromagnetic arrangements of the Mn atoms were considered in our investigation. All of them had lower energy than the ferromagnetic configuration. The noncollinear antiferromagnetic orientation of the spins, as suggested by experiments, is the configuration that has the lowest energy among all the studied magnetic orderings. Taking the energy of the experimental noncollinear state as a reference, we found that the ferromagnetic configuration is 17 mRy higher in energy and that the four different collinear antiferromagnetic orientations considered here are approximately 6–7 mRy above the experimental configuration, but below the ferromagnetic energy. In Fig. 9 we have schematically presented the energies

![FIG. 7. (a) Orientation of the magnetic moments on the three linked Mn triangles. Mn-Mn distances at 10 K displayed, with the magnetic moments illustrated as vectors of arbitrary length. (b) Triangular unit of 2.68 Å Mn-Mn distances.](image)

![FIG. 8. Net magnetic moments of the 2.68 Å Mn triangles illustrated as arrows pointing along four different (111) directions and thus canceling one another. The illustration box corresponds to one-half of the unit cell edges.](image)
of the configurations considered in the present study. It is interesting to note that the four collinear antiferromagnetic orderings have similar energies, which is consistent with frustrated antiferromagnetic interactions. It is well known that in some crystal structures it is not possible to satisfy all exchange interactions if they are antiferromagnetic and of nearest-neighbor nature—for instance, in the two-dimensional triangular network or the fcc lattice. In that case the system possesses a multiplicity of equally unsatisfied states and is said to show frustration. A way to lower the energy is then to form a noncollinear magnetic structure and we propose that this is the situation in Mn₃IrSi.

We proceeded the theoretical analysis by self-consistent calculations, where not only the charge and spin densities were allowed to be updated from iteration to iteration, but also the polar angles determining the direction of the magnetic moments to the experimental values for every manganese atom in the unit cell. The agreement between the experimental values and those calculated theoretically is remarkable, especially considering the complexity of the experimental magnetic structure. The manganese atoms ordered in this noncollinear manner have a calculated magnetic moment of \( \sim 3.0 \mu_B \) and the Ir and Si atoms have small magnitudes of 0.03\( \mu_B \) and 0.01\( \mu_B \), respectively. Hence we conclude that not only the directions of the magnetic moments, but also the magnitudes are well reproduced by theory. It should be noted that for ferromagnetic compounds and alloys, a similar, excellent agreement between experiment and theory is normally found. Although less experience has been gained for noncollinear systems, the data that are available suggest that DFT-based methods reproduce the magnetic moments of these systems with great accuracy.

In order to present a complete picture of the new phase Mn₃IrSi, we show in Fig. 10 the density of states (DOS) of Mn₃IrSi. Note that negative densities represent the spin-down states. All Si states (\( s, p, \) and \( d \)) are summed and depicted in Fig. 10 (dashed line): we also show the Ir 5\( d \) states, which correspond to the thinner solid line. The thickest solid line represents the Mn 5\( d \) states. From the figure it is possible to see that the spin splitting is approximately 0.3 Ry for Mn and that no significant exchange splitting is found.

TABLE IV. Comparison between calculated and experimental values of the \( x, y, \) and \( z \) components (\( M_x, M_y, M_z \)) of the magnetic moment. The Mn atoms are numbered as the equivalent coordinates of the 12\( b \) site in space group \( P2_13 \) in the International Tables for Crystallography.

<table>
<thead>
<tr>
<th>Atom</th>
<th>( M_x )</th>
<th>( M_y )</th>
<th>( M_z )</th>
<th>( M_x )</th>
<th>( M_y )</th>
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<td>1.3(1)</td>
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</table>
for Si and Ir. The large spin splitting in Mn is consistent with the large magnetic moments found on those atoms. It is also interesting to note that there is some hybridization between the Ir and Mn states, whereas the Si states are distributed over a large energy interval.

V. CONCLUSIONS

A new phase, Mn$_3$IrSi, has been synthesized and found to crystallize in the AlAu$_4$-type structure, an ordered form of the β-Mn structure. In contrast to pure β-Mn, Mn$_3$IrSi is magnetically ordered below the Neél temperature (210 K). Localized magnetic moments in the β-Mn structure have previously been observed—e.g., on Al substitution (Mn$_{1-x}$Al$_x$).$^{14}$ The magnetic structure of Mn$_3$IrSi is complex with noncollinear moments of approximately $3\mu_B$ only on the Mn atoms. The Mn atoms form a triangular network on which the moments are rotated 120° around the triangle axes. The magnetic unit cell is of the same size as the crystallographic and has no net magnetic moment. Our first-principles calculations reproduce with good agreement the experimental magnitude and orientation of the atomic magnetic moment. The microscopic origin of the exchange interactions has been found to be antiferromagnetic couplings between Mn atoms. The observed magnetic structure is argued to be the result of the frustration of these antiferromagnetic interactions by triangular geometry. Comparing the magnitude of the interaction energies involved in the ordering of Mn$_3$IrSi (20 mRy) with the corresponding energies and transition temperatures for some unfrustrated systems (Fe, Co, and Ni),$^{13}$ we find that the ordering temperature of Mn$_3$IrSi would have been of the order of 700–1000 K in an unfrustrated geometry. The much lower value of the measured transition temperature (210 K) confirms that geometric frustration is of crucial importance for magnetic ordering in Mn$_3$IrSi.

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9 R. Lázárraga et al. (unpublished).
24 The computer code was made in-house, based on the original method published in Ref. 22. For details see www.fysik.uu.se/theomag.