Comment on "On the Uniqueness of Stable Marriage Matchings" [Economic Letters 69(1):1-8, 2000]

Accepted Manuscript


Mario Consuegra, Rajnish Kumar, Giri Narasimhan

PII: S0165-1765(13)00426-6
DOI: http://dx.doi.org/10.1016/j.econlet.2013.09.019
Reference: ECOLET 6076

To appear in: Economics Letters

Received date: 3 September 2013
Accepted date: 13 September 2013


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Mario Consuegra∗ Rajnish Kumar† Giri Narasimhan∗‡
September 3, 2013

Abstract

We prove with the help of a counterexample that Lemma 6 and Corollary 7 from Eeckhout [1] are incorrect.

Theorem 1 in Eeckhout [1] provides the following sufficient condition for the existence of unique stable matchings.

Condition 1 There exists an ordering of the set of females $F = (X_i)$ and an ordering of the set of males $M = (x_i)$ such that the preference profile satisfies

\[
\forall X_i \in F : x_i \succ X, x_j, \forall j > i \quad \text{and} \quad \forall x_i \in M : X_i \succ x_i X_j, \forall j > i
\]

It is claimed in Eeckhout [1] that Condition 1 is also necessary for uniqueness of stable matchings when $N \leq 6$. We observe here that although the claim holds for $N = 4$ (Lemma 5), it is incorrect for the case $N = 6$ (Lemma 6).

Claim 2 There exists a matching problem $(F, M, \succ)$ with $|N| = 6$ where the set of stable marriages $S$ is a singleton and the preference profile does not satisfy Condition 1.

Proof. Consider the matching problem $(F, M, \succ)$ with the following preference lists:

\[
\begin{align*}
(a,b,c)_A, & \quad (b,a,c)_B, (a,b,c)_C(B, A, C)_a, (A, C, B)_b,(A, C, B)_c
\end{align*}
\]

The only stable matching $\mu^*$ for $(F, M, \succ)$ is given by $\mu^*(A, B, C) = (b, a, c)$. This is easily verified using the Gale-Shapley algorithm by observing that the male and female optimal solutions are identical. However, one can confirm that it does not satisfy Condition 1 by considering all possible orderings of $F$ and $M$. An easier way to see that Condition 1 is not satisfied is by observing that none of the females get their top preference in the only stable matching. Therefore, none of the females can be first in the ordering $(X_i)$ from Condition 1. Furthermore, there also exists a ring $(A, a, B, b)$ which, together with Lemma 2 from [1], shows that Condition 1 cannot be satisfied for this example.

The proof of Lemma 6 in Eeckhout [1] argued that if Condition 1 is violated then either $\mu^*$ is not an equilibrium or there exists $\mu' \neq \mu^*$ that is also an equilibrium. The author’s statement that if $b >_A a$ and/or $B >_a A$ and the remaining conditions of Condition 1 hold then $\mu'(A, B, C) = (b, a, c) \in S$, is not true. Consider, for example, the following preference lists: $(b, a, c)_A, (b, a, c)_B, (a, b, c)_C(B, A, C)_a, (B, C, A)_b,(A, C, B)_c$. Clearly, we have $b >_A a$ and all other conditions of Condition 1 are satisfied. However, $\mu'(A, B,C) = (b, a, c) \notin S$. Indeed one can easily check that $\{B, b\}$ blocks $\mu'$.

Remark 3 Since none of females get their most preferred partners in the only stable matching in the example provided above, it is clear that Corollary 7 in Eeckhout [1] is also incorrect.

Acknowledgments

The work of GN was partly supported by NSF Grant # 1018262. MEC was supported by NSF Graduate Research Fellowship DGE-1038321. We also acknowledge Jan Eeckhout’s comments on an earlier version.

References


∗School of Computing and Information Sciences, Florida International University, Miami, Florida 33199, USA
†Queen’s University Management School, Queen’s University Belfast, 185 Stranmillis Road, Belfast, UK - BT9 5EE
‡Corresponding Author: giri.narasimhan@fiu.edu