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Phonological awareness and mathematical difficulty: A longitudinal perspective

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The present longitudinal study sought to investigate the impact of poor phonology on children’s mathematical status. From a screening sample of 256 five-year-olds, 82 children were identified as either typically achieving (TA; \( N = 31 \)), having comorbid poor phonology and mathematical difficulties (PDMD; \( N = 31 \)), or having only poor phonology (phonological difficulty, PD; \( N = 20 \)). Children were assessed on eight components of informal and formal mathematics achievement at ages 5–7 years. PD children were found to have significant impairments in some, mainly formal, components of mathematics by age 7 compared to TA children. Analysis also revealed that, by age 7, approximately half of the PD children met the criteria for PDMD, while the remainder exhibited less severe deficits in some components of formal mathematics. Children’s mathematical performance at age 5, however, did not predict which PD children were more likely to become PDMD at age 7, nor did they differ in terms of phonological awareness at age 5. However, those PD children who later became PDMD had lower scores on verbal and non-verbal tests of general ability.

Although the comorbidity of reading difficulty and mathematical difficulty is well documented (Dirks, Spyer, Van Lieshout, & de Sonneville, 2008; Miles, Haslum, & Wheeler, 2001; Von Aster, 2000), less is known about the mathematical status of children with poor phonology. Intuitively, it would make sense that phonological processing should impact on at least some mathematical skills, since speech sounds are known to play an important role in tasks such as mathematical computation (Bull & Johnston, 1997; Geary, 1993; Hecht, Torgesen, Wagner, & Rashotte, 2001; Rourke & Conway, 1997).

It has been suggested that weak phonological ability may directly influence mathematics achievement because mathematics involves the retrieval and retention of verbal number codes (Robinson, Menchetti, & Torgesen, 2002). For example, in solving single digit arithmetic problems, depending on the age of the child, children will either retrieve a phonologically based numerical code directly from long-term memory (LTM; Geary, Hoard, & Hamson, 1999; Siegler & Shipley, 1995), or they will reconstruct the answer by counting phonological name codes of numbers (Geary, 1993; Hecht et al.,

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2001; Logie & Baddeley, 1987). Indeed, there is evidence to suggest that phonological ability may be a better predictor of mathematics achievement than reading ability per se (Hecht et al., 2001; Simmons, Singleton, & Horne, 2008). More recently, Krajewski and Schneider (2009) reported that phonological awareness uniquely predicted basic mathematical skills in typically developing children.

Several researchers have reported evidence of associations between phonological and mathematical skills. Bryant, MacLean, Bradley, and Crossland (1990) reported significant correlations between measures of phonological awareness in 4- to 6-year-olds and their later performance on mathematical computation tasks at approximately age 7 years.

In a longitudinal study, Hecht et al. (2001) investigated the relationship between phonological processing, reading ability and mathematical ability in children from age 8 to 11 years. They reported that, although there was a significant association between reading and computational skills, almost all of the variance in the association was explained by three phonological components - phonological working memory, rate of access to phonological name codes in LTM, and phonological awareness. They also found that these components, and in particular phonological awareness, were uniquely associated with growth in general computational skills across the longitudinal period.

Other studies have found phonological awareness to predict arithmetical ability in children aged 7 years (Leather & Henry, 1994; Simmons et al., 2008), and several researchers have stressed the importance of phonological working memory during mental calculation (e.g. De Jong, 2006; Geary, 1993; Lemaire, Abdi, & Fayol, 1996). Hecht et al. (2001) and Noel, Seron, and Trovarelli (2004) reported a relationship between phonological loop function and arithmetic in school age children, while Rasmussen and Bisanz (2005) found a similar relationship in preschoolers. Durand, Hulme, Larkin, and Snowling (2005) failed to find any such association.

Several explanations have been put forward to explain why children with poor phonology may exhibit difficulties in specific components of mathematics. For counting, it has been suggested that children with phonological processing deficits have slower counting speed because this skill requires the direct manipulation of verbal number codes (Simmons & Singleton, 2008). Specifically, when counting, a child must retrieve the phonological representation of number words and operate on these in phonological working memory (Logie & Baddeley, 1987).

Such weaknesses in counting may partially explain why so many children with poor phonology have difficulty with fact retrieval. Counting is commonly used by young children to solve arithmetic problems and, in turn, correctly solving these problems gradually strengthens the association between problem and solution (Siegler & Shrager, 1984). Since children with poor phonology are less efficient counters, there are simply fewer opportunities over time for a problem to be associated with its correct answer.

For arithmetic calculation, it has been argued that the more effective the storage and manipulation of number representations in phonological memory, the more working memory capacity there will be for calculation procedures (Bull & Johnston, 1997). Therefore, children with weaker phonological ability are likely to have fewer resources left over to allocate to calculation than typically achieving (TA) children. In more complex tasks such as written multi-digit arithmetic involving many steps such as aligning digits and carrying, children are also required to maintain and transform information about a problem in phonological memory, and those with poor phonology are less likely to have sufficient spare capacity to solve these complex problems than for simpler tasks such as single digit fact retrieval.
**Reading and mathematical difficulty**

Although the focus of the present study is on children with poor phonology, the subtyping methodology used draws on previous studies of older children who have undergone formal instruction in reading (e.g. Hanich, Jordan, Kaplan, & Dick, 2001; Jordan, Hanich, & Kaplan, 2003). Since phonological ability has been found to predict reading proficiency (e.g. De Jong, 2006; Liberman, Shankweiler, Fischer, & Carter, 1974; Simmons et al., 2008), some of the findings from these studies are relevant to the present research. However, it is important to note that the relationship between phonological processing and reading ability is far from perfect, and poor phonology does not always lead to poor reading (Liberman et al., 1974). Indeed, many children identified as being at familial risk of reading impairment, and who tend to have poor phonology, do not systematically go on to develop poor reading (Pennington & Lefly, 2001; Snowling, Gallagher, & Frith, 2003).

Children with comorbid reading and mathematics difficulty (abbreviated in the literature to MDRD) have been found to perform significantly worse in mathematics than children with singular reading (RD) or mathematical deficits (MD), and any difference in mathematics performance between MDRD and MD may be considered to be the result of reading difficulty *per se* on mathematical achievement.

In considering the relationship between reading, or indeed phonological processing, on mathematical ability, it should be noted that mathematical ability, and therefore difficulty, is made up of a variety of different components, both at preschool (Dowker, 2008) and at the primary school level (Dowker, 1998; Jordan, Mulhern, & Wylie, 2009).

Few studies have attempted to distinguish those children with reading difficulty who may exhibit some selective mathematical impairments, albeit insufficiently severe for them to be classified as MD, from those children who have weak performance in reading and in most areas of mathematical achievement (MDRD). Indeed, when considering the latter, it is difficult to identify any possible impact of their reading difficulty on their mathematical ability, because their mathematical difficulties typically may be the result of a range of factors unrelated to their reading, such as the dysfunction of various executive abilities (Geary, Hoard, Nugent, & Byrd-Craven, 2007) or poor visuospatial ability (Rourke & Conway, 1997).

MDRD children have been found to be significantly weaker than TA children in virtually every area of mathematics (Geary, Hamson, & Hoard, 2000; Hanich et al., 2001; Jordan et al., 2003). RD children, on the other hand, despite having typical overall mathematics achievement, have been found to have weaknesses in particular areas of mathematics, such as fact retrieval, written multidigit calculation and use of place value (Geary et al., 2000; Hanich et al., 2001).

A longitudinal study by Jordan et al. (2003) reported that the influence of reading ability on mathematics achievement increased over time. They found that, although 7-year-olds with RD initially matched their TA counterparts on mathematics achievement, by age 9 years, they had fallen behind TA children. This pattern was consistent across all calculation and applied problems subtests of the Woodcock-Johnson Psychoeducational Battery (Woodcock & Johnson, 1990).

**Informal versus formal mathematics**

When predicting the development of mathematical literacy, particularly in younger children, it is important to bear in mind the distinction between formal and informal
mathematics. Some informal mathematical knowledge is acquired through a child's interaction with their environment, whereas other aspects, such as perception of more or less, are considered to be innate (Ginsburg & Baroody, 2003). Formal mathematical knowledge, by contrast, refers to the mathematics taught at school, such as knowledge of numerals and numerical operations, mathematical language, and the rules and procedures that underlie calculation. Overall, the language requirements of informal mathematics are less than those of formal mathematics (Dowker, 2005), which may explain why componential analyses suggest that RD children aged 7–9 years tend to perform on a par with TA children in many areas of informal mathematics, such as approximate arithmetic, calculation with the aid of fingers or counters (Hanich et al., 2001; Jordan et al., 2003), number comparison, and subitizing (Landerl, Bevan, & Butterworth, 2004). By contrast, RD children have been found to perform relatively poorly in formal areas of mathematics including fact retrieval, place value, and written multidigit calculation (Hanich et al., 2001; Jordan et al., 2003).

The picture, however, is not entirely uniform. Occasionally, RD children have been reported to demonstrate comparable performance to TA children on some formal aspects of mathematics. For example, Landerl et al. (2004) found that 8- to 9-year-olds with RD were unimpaired on a number naming task. This finding is surprising considering that we would expect RD children to have weaker representations of number names. However, it may be the case that young children with RD have initial difficulty on this task when they enter school, but then outgrow the difficulty.

A recent componential study by Andersson (2008) assessed children aged 10 on a broad range of mathematical tasks, but found no difference between TA and RD subtypes on any task. A possible explanation for this is that the mathematics screening test used to classify children as RD or MDRD was loaded with items assessing formal mathematics. Considering that formal mathematics is a particular problem area for those with reading difficulty, it is likely that Andersson (2008) would have misclassified those RD children with problems in formal mathematics as MDRD. Only those with little difficulty in formal mathematics would have been classified as RD and it would be unsurprising if those children had failed to show difficulties with formal tasks such as multidigit calculation and fact retrieval.

In order to avoid the problems of classification identified above, the present study used a standardized test, ‘Test of Early Mathematics Ability – 3rd edition’ (TEMA-3; Ginsburg & Baroody, 2003), to assess a broad range of both informal and formal mathematical skills. Thus, since upon entering school, children have little formal mathematics knowledge, the items aimed at children in their first year of schooling in the TEMA have an emphasis on informal mathematical knowledge. Over time, since children are exposed to increasing amounts of formal mathematics, the TEMA mirrors this change. Thus, as children get older, they are able to progress further through the test and are able to complete a greater number of formal items.

When considering how children with RD may progress over time in terms of their overall level of mathematics ability, it is important to bear in mind that the most commonly highlighted problem areas for RD children, such as fact retrieval (e.g. Geary et al., 2000), are formal mathematical skills. It is possible that many RD children may achieve a typical score in mathematics initially, due to the heavy weighting of informal items. However, as the balance of test items starts to shift from informal to formal, their weaknesses in formal mathematics may have a negative impact on their overall achievement.
**The present study**

As noted above, much of the research on mathematics and reading-related difficulties has involved children who have received formal instruction in reading, including those studies which have focused on phonological processing. Although some of these phonological studies attempted to control for the effect of reading ability (e.g. Bull & Johnston, 1997; Hecht et al., 2001), it remains the case that phonological processing and reading remain somewhat conflated. The present study sought to address this point by initially investigating pre-reading children. In the Northern Ireland curriculum, children do not begin formal reading instruction at age 5 and often do not do so until significantly later. Using Jordan et al.'s (2003) subtyping approach, the present study identified three achievement subtypes – TA children, children with phonological difficulty (PD) and children with comorbid poor phonology and mathematical difficulty (PDMD).

The study also sought to build-upon the work of Jordan et al. (2003) in several respects. Firstly, we aimed to address the issue of individual differences in children’s development. Although Jordan et al. (2003) did not explicitly address the issue, it is possible that a number of their RD children at age 7 may have met the criteria for MDRD by age 9. More recently, Jordan et al. (2009) found that considerable individual differences in trajectories of mathematical achievement were evident in children with reading and mathematical difficulties, and even among TA children. Thus, it may be the case that not all RD children go on to develop mathematical difficulties and, if indeed some children do not develop mathematical difficulties, then what factors serve to protect them? Of course, it is also possible that many of Jordan et al.’s (2003) 7-year-olds currently classified as MDRD may not have been classified as such when they first started school at age 5. Thus, our aim was to see whether it may be possible to predict mathematical difficulty in children identified as having poor phonological ability at age 5.

The present study focused on a range of mathematical tasks emphasizing different components of mathematical ability. Since Jordan et al. (2003) did not perform more detailed componential analysis of performance on the standardized test of mathematics achievement, it remains unclear as to whether their RD subtype fell behind due to performance in mathematics that was globally weaker or component specific. Given that both cross-sectional and longitudinal research has suggested that RD children have uneven performance across mathematical tasks (e.g. Jordan et al., 2003) it is reasonable to suggest that RD children may fall behind in mathematics due to difficulties with particular components of a mathematics test rather than due to global difficulties. It is also possible that RD children’s profile of strength and weakness in mathematics may change over time. For example, Jordan et al. (2009) found that TA children often displayed a range of trajectory patterns across mathematical subtasks, for example, steady growth, persistent difficulty, initial difficulty followed by typical growth, or initially good achievement but weak growth. By adopting a componential and longitudinal approach we sought to identify initial areas of mathematical difficulty for those PD children who later became PDMD.

In the present study, four informal (numbering, number comparison, calculation, and concepts) and four formal (numeral literacy, number facts, calculation, and concepts) mathematical components were assessed. We addressed four main issues: (i) the stability over time of the mathematical achievement of PD children; (ii) whether PD children, in spite of their overall typical mathematical achievement, demonstrated weaknesses relative to TA children in specific components mathematical achievement and whether this profile changed over time; (iii) whether poor performance on a
specific mathematical component indicated which PD children were most likely to develop mathematical difficulty; and (iv) those factors contributing to the stability over time of the PD subtype’s mathematical achievement.

**Method**

**Materials**
For initial screening and subtype allocation, the following standardized tests were used.

**Phonological Abilities Test (PAT; Muter, Hulme, & Snowling, 1997)**. Rhyme Detection and Phoneme Deletion (beginning sounds) subtests measured children’s phonological ability.

**TEMA-3** (Ginsburg & Baroody, 2003). Administration of the TEMA is based on the method of entry points, basals and ceilings as outlined in the test manual. TEMA items assessed the following aspects of both informal and formal mathematical components.

**Informal**

**Numbering.** Assesses a child’s ability to recite the verbal counting sequence, and is considered a prerequisite for other skills such as number comparison and calculation.

Example: What number comes next; 24 and then comes …?

**Number comparison.** This is assessed by items which require children to judge which of two collections of objects is larger or which of two numbers is higher.

Example: Which is more 2 or 5?

**Calculation.** These include simple addition and subtraction problems. The earlier informal calculation test items (e.g. 3 + 2, 4 − 3) are represented by counters, whereas the later items are presented orally. Later items (e.g. 18 − 6, 14 + 13) are not represented by counters, nevertheless the problems are untimed so the child still has the opportunity to represent the problem with their fingers.

Example: How much is 4 and 8 more altogether?

**Concepts.** Understanding of concepts such as the cardinality rule (the last counting word used represents the total number of items in the collection) and equal partitioning (e.g. share 12 tokens between two people).

**Formal**

**Numeral literacy.** Items measure a child’s ability to read and write single and multi digit numbers. To name multi digit numbers accurately a child must have a good understanding of place value.

**Number facts.** These items require children to be able to quickly give the answer to addition, subtraction, and multiplication problems that are presented visually. Many of the items are similar to those in the informal calculation task, except the problems are never represented with tokens, and in order to encourage children to retrieve the answer directly from memory rather than using their fingers a three second time limit is imposed.

**Calculation.** Includes both mental and written multi digit (35 + 28, 156 + 158) problems. The later items in this scale involve carrying. The written problems are presented in a vertical format which requires the child to be able to align the digits correctly.
Concepts. Items include correctly identifying if a number fact \((9 + 5)\) correctly represents a word problem (e.g. how much are 9 apples and 5 apples?). Understanding of grouping by ten and place value are also assessed by this scale.

Post-screening, the PAT and TEMA were used again at subsequent time points as detailed in the ‘Participants and procedure’ section. Additionally, although not used for screening, we administered the ‘Test of Early Reading Ability – 3rd edition’ (TERA-3; Reid, Hresko, & Hammill, 2001) at Times 1 and 3. This test assessed mastery of early developing reading skills and was standardized for children aged 3 years 6 months to 8 years 6 months. It comprised three subtests: knowledge of the alphabet and its uses, knowledge of the conventions of print, and construction of meaning from print. The relevance of the test is somewhat restricted given the focus of the present study on phonology, however, total TERA scores are presented as a general indication of early reading ability.

Also children’s verbal and non-verbal abilities were assessed at Time 3 using the ‘British Ability Scales – 2nd edition’ (BAS-2; Elliott, Smith, & McCulloch, 1997), verbal (word definitions and verbal similarities) and non-verbal clusters (quantitative reasoning and matrices).

Participants and procedure
Initially, 256 children with mean age 5;6 years were screened for phonological and mathematical difficulties using the PAT and the TEMA. Prior to screening, any children with other underlying impairments were excluded. All tests were individually administered with a typical testing session lasting 25–30 min. Those children who scored at or below the 35th percentile on either test were classified as having difficulty in the particular achievement area. To be considered TA, a child needed to score at or above the 40th percentile on the relevant test. The 35th percentile was chosen for consistency with Hanich et al. (2001) and Jordan et al. (2003) which were closely related to the present research, although these studies focused on reading in slightly older children rather than on phonological awareness. Other researchers have used different percentile cut-off points which present some difficulties for the comparability of different studies (Dirks et al., 2008). For example, Geary and colleagues (e.g. Geary, 1993; Geary et al., 1999, 2000, 2007) have variously used the 30th or 35th percentile depending on the age of the child. Like Geary et al. (2000) and Hanich et al. (2001), due to the young age of our children, we used the 35th percentile in order to ensure adequate sample sizes.

Following screening, a total of 97 children were allocated to one of the three subtypes (TA, PD, PDMD). Scores obtained during the initial screening thus constituted these children’s Time 1 data. The TEMA was administered on a further two occasions, at mean age 6;6 (Time 2) and 7;6 (Time 3) years, and the PAT on one further occasion (Time 3).

The BAS subscales were administered at Time 3 in order to check the average ability levels of the subtypes. Administration was on an individual basis and lasted 20–30 min depending on the ability level of the child. All testing was completed in accordance with the administration procedures outlined in the test manual.

Of the original 97 children, 15 failed to complete all measures and were excluded from analysis. Of these children, 4 were PDMD, 6 PD, and 5 TA. A total of 82 children completed all verbal, non-verbal, phonological and early reading tests (Time points 1 and 3), and the mathematics test (Time points 1–3). Table 1 presents the sample characteristics, including Time 3 BAS measures. At Time 1 (mean age 5;6 years), PD and
<table>
<thead>
<tr>
<th></th>
<th>Gender (f/m)</th>
<th>Time 1</th>
<th>Time 1</th>
<th>Time 3</th>
<th>Time 1</th>
<th>Time 3</th>
<th>Word definitions</th>
<th>Verbal similarities</th>
<th>Matrices</th>
<th>Quantitative reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>20</td>
<td>44.90 (16.34)</td>
<td>20.09 (7.94)</td>
<td>36.72 (18.45)</td>
<td>45.45 (36.13)</td>
<td>35.91 (3.62)</td>
<td>24.30 (14.69)</td>
<td>45.95 (23.99)</td>
<td>50.85 (24.45)</td>
<td>41.50 (27.22)</td>
</tr>
<tr>
<td>PDMD</td>
<td>31</td>
<td>22.29 (9.00)</td>
<td>18.61 (7.43)</td>
<td>30.36 (17.42)</td>
<td>31.48 (32.43)</td>
<td>32.10 (15.89)</td>
<td>17.77 (13.41)</td>
<td>35.87 (22.19)</td>
<td>37.06 (23.12)</td>
<td>35.81 (24.58)</td>
</tr>
<tr>
<td>TA</td>
<td>31</td>
<td>53.32 (15.59)</td>
<td>56.15 (12.76)</td>
<td>56.81 (11.41)</td>
<td>66.42 (29.35)</td>
<td>47.79 (10.54)</td>
<td>41.29 (21.30)</td>
<td>53.77 (20.93)</td>
<td>45.61 (23.15)</td>
<td>59.16 (23.62)</td>
</tr>
</tbody>
</table>
PDMD children demonstrated considerably weaker phonological ability than TA children (Table 1). PD and PDMD scores increased at Time 3, although both were at or around the 35th percentile and the variability of scores had also increased. PAT scores for TA children, on the other hand, remained very consistent across Times 1 and 3 and the variability remained similar. This suggests that the TA participants had improved in line with age and that, unlike the other two groups who were starting from a very low base, the scope for an increase in mean percentile score was reduced.

On the mathematics test (TEMA) the PDMD group scored poorly, while both PD and TA scored in the TA range, thus confirming the subtyping in terms of mathematical ability. TA and PD children were also in the typical range on non-verbal ability. PD children did, however, score relatively poorly on word definitions, but within the typical range on verbal similarities. It is important to note that we did not attempt to match TA and PD groups on verbal and non-verbal ability and, due to the potential for comorbidity, though by definition not with mathematical difficulty in PD participants, the likelihood of finding children with below average skills in some cognitive domains is high, and the scope for precise matching low.

TERA percentile scores, based on total test performance, show that the TA group performed well above average on early reading at Time 1, although the variability of scores suggests that some of the TA children were likely to have overlapped with children in the other two subtypes. At Time 3, however, TA children’s TERA scores were broadly consistent with our classifications based on the PAT measures, with PD and PDMD children averaging around the 35th percentile, and TA children close to the 50th. The fall-off in TA children’s mean TERA scores from the 66th to the 48th percentile between age 5 and 7 years may reflect some issues with the standardization of the test, although the much reduced variability at Time 3 suggests that the distribution of scores is in line with that indicated in the test manual.

Results

Stability of overall mathematics achievement over time for different subtypes

Averaging across all eight component tasks, a significant interaction between subtype and time was observed, $F(4, 158) = 3.354$, $p = .012$, $\eta^2_p = .08$. Bonferroni adjusted pairwise comparisons indicated that for PD mathematics performance was stable between Times 1 and 2, but dropped significantly between Times 2 and 3 (Table 2). In fact, by Time 3, 50% of the PD group scored at or below 35th percentile on the standardized test of mathematics achievement.

<table>
<thead>
<tr>
<th>Mathematics (TEMA)</th>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>44.90 (16.34)</td>
<td>53.30 (13.64)</td>
<td>35.15 (16.30)</td>
</tr>
<tr>
<td>PDMD</td>
<td>22.29 (9.00)</td>
<td>21.97 (11.80)</td>
<td>17.94 (9.87)</td>
</tr>
<tr>
<td>TA</td>
<td>53.32 (15.59)</td>
<td>59.71 (15.04)</td>
<td>56.84 (21.31)</td>
</tr>
</tbody>
</table>

Profiles of mathematics achievement over time for PD children

For each time point, proportion scores were calculated for each of the eight TEMA components. Proportion correct for each component was calculated based on the
furthest item in the scale reached by the best child at that particular time point. Table 3 presents the performance of each subtype on each of the eight components at the three time points. Separate two-way ANOVAs were carried out at each time point. Three-way ANOVA with Time as a third factor was not undertaken due to the fact that different items contributed to the component scores at the different time points.

**Time 1**
Analysis revealed a significant interaction between subtype and task at Time 1, $F(14, 553) = 5.42$, $p < .001$, $\eta_p^2 = .12$. TA children outperformed PDMD on all informal aspects of mathematics, namely, numbering, number comparisons, calculation, and concepts. Numeral literacy was the only aspect of formal mathematics on which TA scored significantly higher than PDMD. The PD group scored higher than PDMD on one formal (concepts) and two informal (numbering and number comparisons) mathematical tasks. The PD group scored significantly lower than TA on informal calculation.

**Time 2**
There was a significant interaction between subtype and task, $F(14, 553) = 8.44$, $p < .001$, $\eta_p^2 = .18$. The PDMD subtype continued to perform worse than TA on all aspects of informal mathematics, and on two formal tasks (numeral literacy and number facts). Children with PD also performed better than PDMD on all informal mathematical tasks and also on one formal task (numeral literacy). No significant differences were observed between the PD and TA subtypes at this time point.

**Time 3**
A significant interaction between subtype and task was also found at this time point, $F(14, 553) = 4.34$, $p < .001$, $\eta_p^2 = .10$. *Post hoc* analysis revealed that TA outperformed PDMD on all informal and formal tasks. On the other hand, PD outperformed PDMD on two informal (numbering and number comparisons) and on one formal task (numeral literacy). TA outperformed PD on informal calculation and three formal (number facts, calculation, and concepts) mathematical tasks.

**Does PD children’s mathematics performance at earlier time points predict later mathematical difficulty?**
PD children classified as having typical mathematics achievement at Time 1 were divided into two subgroups based on their mathematics achievement at Time 3.

Those who continued to score within the normal range on the standardized test of mathematics achievement at Time 3 were classified as ‘stable’ PD (PD-SM). Those who were found to have moved below the 35th percentile at Time 3 were classified as ‘unstable’ PD (PD-UM).

The mathematical profiles of PD-UM, PD-SM, PDMD, and TA were then compared at Times 1–3 (see Table 4).
Table 3. Mean proportion correct (and SDs) on components of the TEMA by subtype

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<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td><strong>Time 1</strong></td>
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<tr>
<td>PD</td>
<td>54.77 (9.50)</td>
<td>59.00 (19.97)</td>
<td>43.75 (24.16)</td>
<td>52.50 (7.69)</td>
<td>35.00 (18.23)</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
<td>33.33 (0.00)</td>
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<td>MDPD</td>
<td>45.75 (8.86)</td>
<td>41.94 (20.24)</td>
<td>38.71 (21.25)</td>
<td>48.39 (8.98)</td>
<td>23.04 (15.94)</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
<td>23.66 (15.38)</td>
</tr>
<tr>
<td>TA</td>
<td>62.46 (14.42)</td>
<td>69.68 (23.02)</td>
<td>60.48 (25.64)</td>
<td>57.26 (11.54)</td>
<td>41.47 (20.65)</td>
<td>2.15 (8.32)</td>
<td>0.00 (0.00)</td>
<td>27.96 (12.46)</td>
</tr>
<tr>
<td><strong>Time 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>81.59 (8.01)</td>
<td>84.00 (16.67)</td>
<td>78.75 (14.68)</td>
<td>73.75 (9.85)</td>
<td>66.43 (11.61)</td>
<td>4.29 (9.38)</td>
<td>0.00 (0.00)</td>
<td>25.00 (0.00)</td>
</tr>
<tr>
<td>MDPD</td>
<td>61.73 (12.79)</td>
<td>67.10 (20.36)</td>
<td>63.71 (22.21)</td>
<td>60.48 (18.00)</td>
<td>47.93 (19.36)</td>
<td>1.84 (6.11)</td>
<td>0.81 (4.50)</td>
<td>22.58 (7.51)</td>
</tr>
<tr>
<td>TA</td>
<td>84.31 (11.06)</td>
<td>90.97 (12.48)</td>
<td>89.52 (12.54)</td>
<td>75.81 (7.86)</td>
<td>69.59 (13.67)</td>
<td>7.83 (10.32)</td>
<td>1.61 (6.24)</td>
<td>25.81 (4.49)</td>
</tr>
<tr>
<td><strong>Time 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>89.57 (6.68)</td>
<td>77.50 (16.47)</td>
<td>54.29 (8.80)</td>
<td>97.50 (7.70)</td>
<td>73.13 (11.67)</td>
<td>20.00 (21.81)</td>
<td>7.00 (12.61)</td>
<td>30.00 (16.54)</td>
</tr>
<tr>
<td>MDPD</td>
<td>82.33 (10.71)</td>
<td>58.06 (18.19)</td>
<td>47.93 (11.98)</td>
<td>90.32 (17.89)</td>
<td>62.90 (12.70)</td>
<td>11.83 (17.67)</td>
<td>2.58 (5.75)</td>
<td>23.87 (9.55)</td>
</tr>
<tr>
<td>TA</td>
<td>92.29 (4.86)</td>
<td>84.41 (14.23)</td>
<td>62.67 (12.60)</td>
<td>99.19 (4.49)</td>
<td>79.44 (12.73)</td>
<td>38.00 (21.04)</td>
<td>28.06 (24.00)</td>
<td>41.29 (16.28)</td>
</tr>
</tbody>
</table>

Note. 1, informal numbering; 2, informal number comparisons; 3, informal calculation; 4, informal concepts; 5, formal numeral literacy; 6, formal number facts; 7, formal calculation; 8, formal concepts.
Table 4. Mean proportion correct (and SDs) on components of the TEMA for PD-UM and PD-SM

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD-SM</td>
<td>53.63 (10.23)</td>
<td>58.00 (17.51)</td>
<td>45.00 (25.82)</td>
<td>52.50 (7.91)</td>
<td>40.00 (22.13)</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
<td>33.33 (0.00)</td>
</tr>
<tr>
<td>PD-UM</td>
<td>55.91 (9.10)</td>
<td>60.00 (23.09)</td>
<td>42.50 (23.72)</td>
<td>52.50 (7.91)</td>
<td>30.00 (12.51)</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
<td>33.33 (0.00)</td>
</tr>
<tr>
<td>Time 2</td>
<td>81.36 (7.25)</td>
<td>80.00 (18.86)</td>
<td>80.00 (15.81)</td>
<td>72.50 (7.91)</td>
<td>68.57 (13.13)</td>
<td>7.14 (12.14)</td>
<td>0.00 (0.00)</td>
<td>25.00 (0.00)</td>
</tr>
<tr>
<td>PD-SM</td>
<td>81.81 (9.09)</td>
<td>88.00 (13.98)</td>
<td>77.50 (14.19)</td>
<td>75.00 (11.79)</td>
<td>64.29 (10.10)</td>
<td>1.43 (4.52)</td>
<td>0.00 (0.00)</td>
<td>25.00 (0.00)</td>
</tr>
<tr>
<td>PD-UM</td>
<td>88.70 (6.22)</td>
<td>83.33 (11.11)</td>
<td>51.43 (10.00)</td>
<td>97.50 (7.91)</td>
<td>72.50 (9.86)</td>
<td>12.22 (14.30)</td>
<td>2.00 (4.22)</td>
<td>24.00 (8.43)</td>
</tr>
<tr>
<td>Time 3</td>
<td>90.43 (7.33)</td>
<td>71.67 (19.33)</td>
<td>57.14 (6.73)</td>
<td>97.50 (7.91)</td>
<td>73.75 (13.76)</td>
<td>27.78 (25.80)</td>
<td>12.00 (16.19)</td>
<td>36.00 (20.66)</td>
</tr>
<tr>
<td>PD-SM</td>
<td>88.70 (6.22)</td>
<td>83.33 (11.11)</td>
<td>51.43 (10.00)</td>
<td>97.50 (7.91)</td>
<td>72.50 (9.86)</td>
<td>12.22 (14.30)</td>
<td>2.00 (4.22)</td>
<td>24.00 (8.43)</td>
</tr>
<tr>
<td>PD-UM</td>
<td>88.70 (6.22)</td>
<td>83.33 (11.11)</td>
<td>51.43 (10.00)</td>
<td>97.50 (7.91)</td>
<td>72.50 (9.86)</td>
<td>12.22 (14.30)</td>
<td>2.00 (4.22)</td>
<td>24.00 (8.43)</td>
</tr>
</tbody>
</table>

Note. 1, informal numbering; 2, informal number comparisons; 3, informal calculation; 4, informal concepts; 5, formal numeral literacy; 6, formal number facts; 7, formal calculation; 8, formal concepts.
Time 1
There was a significant interaction between group and task at Time 1, $F(21, 546) = 3.74, p < .001$, $\eta_p^2 = .13$. Multiple comparisons indicated that PD-UM and PD-SM had similar achievement on the eight mathematical tasks.

Time 2
There was a significant interaction between group and task at Time 2, $F(21, 546), = 5.90, p < .001$, $\eta_p^2 = .19$. Both PD subtypes scored higher than PDMD on informal numbering and formal numeral literacy. PD-UM scored significantly higher than PDMD on informal concepts and the TA group performed better than PD-UM on the informal number comparison task.

Time 3
A significant interaction between group and task was also observed at Time 3, $F(21, 546) = 3.93, p < .001$, $\eta_p^2 = .13$. On informal numbering PD-SM outperformed PDMD, while PD-UM outperformed PDMD on informal number comparison. TA children scored higher than PD-SM on formal calculation, and better than PD-UM on informal calculation and formal number facts, calculation, and concepts.

Possible factors influencing the mathematical development of PD children
Further comparisons of PD-UM, PD-SM, PDMD, and TA were conducted to explore factors associated with unstable mathematics ability in those children who were initially classified as having PD only (Table 5).

Verbal and non-verbal ability
Two-way ANOVA was conducted to compare the profiles of each subtype on verbal (word definitions and verbal similarities) and non-verbal (matrices and quantitative reasoning) ability measures. There was no significant interaction between ability measure and subtype. However, there was a significant main effect of subtype on performance, $F(3, 78) = 8.45, p < .001$, $\eta_p^2 = .25$. Multiple comparisons indicated that, in terms of overall ability, TA scored higher than both PDMD and PD-UM, and PD-SM outperformed PD-UM.

Mathematical ability
There was a significant interaction between time and group, $F(6, 156) = 3.41, p = .003$, $\eta_p^2 = .116$. Multiple comparisons showed that PD-UM, PD-SM, and TA outperformed PDMD in mathematics at Times 1 and 2. A different pattern was evident at Time 3, where TA and PD-SM outperformed PD-UM and PDMD.

Phonological ability
A significant interaction between group and time was found, $F(3, 78) = 4.94, p = .003$, $\eta_p^2 = .16$. Paired sample $t$ tests indicated that phonological ability percentile score increased significantly between Times 1 and 3 for PDMD ($t = 4.49, df = 30, p < .001$), PD-UM ($t = 2.85, df = 9, p = .019$), PD-SM ($t = 2.56, df = 9, p = .05$), but not for TA.
Table 5. Mean percentile scores (and SDs) for PD-UM and PD-SM children on ability measures

<table>
<thead>
<tr>
<th></th>
<th>Maths (TEMA)</th>
<th>Phonological ability (PAT)</th>
<th>Verbal ability (BAS)</th>
<th>Non-Verbal ability (BAS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Time 1</td>
<td>Time 2</td>
<td>Time 3</td>
</tr>
<tr>
<td>PD-SM</td>
<td>10</td>
<td>48.40 (16.79)</td>
<td>54.60 (16.02)</td>
<td>46.40 (15.45)</td>
</tr>
<tr>
<td>PD-UM</td>
<td>10</td>
<td>41.40 (15.95)</td>
<td>52.00 (11.51)</td>
<td>23.90 (6.42)</td>
</tr>
</tbody>
</table>
Discussion

The findings of the present study indicate that there are particular issues with respect to children with phonological difficulties and their mathematical achievement. We addressed several questions regarding PD children’s mathematical development between ages 5 and 7 years. Firstly, we investigated the stability, or otherwise, of PD children’s classification as TA in mathematics over time. We predicted that PD children, although initially matched with TA children, would not make as much progress as TA children due to the shift in balance of TEMA items from informal towards formal over time. Indeed, more generally, we suggested that previous research may have misclassified PD children as PDMD due to an over-emphasis on formal mathematics in tests used.

Previously, Jordan et al. (2003) reported that children with reading difficulty aged 7–9 years made less progress in mathematics than TA children. Inasmuch that PD may be considered to be indicative of later reading difficulty in some children, the results of the present study involving younger children aged 5–7 years are consistent with this finding. Relative to TA children, we found that PD children made similar progress in mathematics between Times 1 and 2, but much less progress between Times 2 and 3. The overall similarity of the present findings to those of earlier studies of reading and mathematical difficulties lends support to the view that individual differences in reading and mathematics may be due to common core phonological processing abilities influencing both domains (e.g. Ackerman & Dykman, 1995; Geary, 1993; Rourke & Conway, 1997).

Our study also considered whether PD children, despite demonstrating overall typical achievement in mathematics, showed evidence of selective impairments in some components of mathematical achievement. Our componential analysis suggested that this was the case, with the decline in mathematical performance experienced by PD children between Times 2 and 3 occurring as a result of difficulties in a few specific areas rather than globally. Initially, at Times 1 and 2, PD children had no significant impairments relative to the TA subtype, with the exception of informal calculation at Time 1. Like the PD and PDMD subtypes, TA children achieved very low accuracy on formal mathematics tasks at Times 1 and 2, which explains why no significant difference between TA and PD was apparent at these time points. However, by Time 3, when TA children showed a marked improvement in formal mathematics, TA outperformed PD on three of the four formal mathematics tasks (number facts, calculation, and concepts). The finding that PD children had difficulty with number fact retrieval and formal calculation is consistent with the results of previous research on reading difficulties in older children (Geary et al., 2000; Hanich et al., 2001).

No significant impairment was observed for PD children on one of the formal tasks, namely numeral literacy. Although this finding is consistent with some previous research (Landerl et al., 2004), given the role of phonological processing in number representation, we had predicted that younger PD children would have had difficulty with this task. It is possible that PD children did not show impairment on this task because, although phonological, it had lower verbal demands than the other formal tasks, and so may not have exceeded the PD children’s available phonological resources. More generally, the detrimental effects of PD on mathematics performance over time due to increasing language demands in mathematics may help to explain the trend for mathematical difficulties becoming more prevalent (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005).

As predicted, we found the informal skills of the PD subtype to be broadly comparable to the TA subtype, with the exception of informal calculation. Indeed,
consistent with Landerl et al. (2004), PD performed as well as TA on number comparison. However, based on previous research it was difficult to predict whether young children with PD would have had difficulty on the informal counting task. Hanich et al. (2001) previously reported that neither MDRD nor RD had obvious difficulties with counting, suggesting that maybe the task was not difficult enough for the children aged 7–9 years in their sample, and may have lacked the level of sensitivity needed to detect weaknesses. In the present study, PDMD children had significant difficulties on this task at all three time points, but PD children did not. This would seem to indicate that counting is not a problem area for young children with PD. However, the picture is not entirely clear. Firstly, the difference between TA and PD on this task approached significance at Time 1. Secondly, PD had difficulty with informal calculation, and these problems are often solved through counting. It may be the case that counting only becomes problematic for PD children on tasks such as informal calculation, which may involve counting and other verbal requirements, such as retaining number names. Therefore, it is possible that PD children had difficulty on informal calculation because the combined verbal demands exceeded their capacity for verbal information (Geary, 1993). It is notable that PDMD children had considerable difficulty with all components both informal and formal mathematics. Since such global deficits cannot plausibly be explained only by phonological difficulties, this would suggest that PDMD and PD are qualitatively different subtypes.

A further focus of the study was the question of whether so-called ‘unstable’ PD children, that is, those who were reclassified as having mathematical difficulty at Time 3, could be identified on the basis of earlier achievement profiles. Our analysis of individual differences suggested that the tendency of PD children to become PDMD by Time 3 was not evident for all children. Rather, approximately half of the PD group had stable overall mathematical achievement across the three time points, while the other half met the criteria for mathematical difficulty by Time 3.

Given the high percentage of children who became PDMD, we considered whether, based on their mathematical profile of strength and weakness at Time 1 or 2, it was possible to distinguish between ‘stable’ and ‘unstable’ PD children. Our findings in this regard were that at no stage did PD-UM and PD-SM differ significantly on any formal or informal components. Thus, at least based solely on early performance on the TEMA, it would be difficult to predict which PD children were more likely to become PDMD at Time 3.

Finally, somewhat related to that previous question, we considered what factors, if any, might contribute to the stability of the mathematical profile of the PD subtype over time. A number of possible factors that may have been expected to predict which PD children would develop more severe difficulties in mathematics were explored. There was no evidence to suggest that PD-SM and PD-UM differed in terms of phonological ability (Times 1 and 3), nor overall mathematical ability at Times 1 and 2. On the other hand, PD-SM did perform better on both verbal and both non-verbal ability measures than PD-UM (see Table 5). There are a number of ways in which verbal and non-verbal ability could explain the difference in mathematical performance between PD-UM and PD-SM. There is some controversy in the literature regarding the possible impact of general ability on mathematical achievement in children with poor phonology and/or reading (see Hecht et al., 2001). It may be argued that all PD children had weaker performance because of their PD, but that PD-SM did not have the same level of mathematical impairment at Time 3 because they had the capacity to develop compensatory strategies. However, the fact that the PD children with typical non-verbal
ability in this sample still had significant difficulty in formal calculation, and lower performance on number facts and formal concepts, suggests that PD may influence performance in some areas of mathematics independently of general ability.

The instability of mathematics achievement over time in children with PD poses a major problem for developmentalists and educationists. If children were assessed at only one time point, it is difficult to know which PD children might be at risk of developing mathematical difficulty. A related issue is that the age at which children with phonological and/or reading difficulties are screened for mathematical difficulty will influence conclusions regarding the mathematical profiles of those children. As the present study has highlighted, when classification took place at age 5, those PD children with comorbid mathematical difficulty tended to have global difficulties in mathematics. By age 7, a number of other PD children also met the criteria for mathematical difficulty. However, the problems of these children were concentrated more in formal than in informal mathematics. If screening had taken place at this age, a much more heterogeneous group would have been classified as PDMD.

In terms of intervention it may be more useful to distinguish those PD children who have mathematical difficulty upon entering school from those who develop it later. Interventions to improve the phonological and more general reading skills of the latter group, whose difficulties in mathematics are more likely to be due largely to their phonological or reading difficulties, may also be effective in alleviating their mathematical difficulties.

Another issue for educationists is that, due the uneven achievement profiles of many PD children in mathematics, it is easy to miss those PD children who have difficulty in a few specific areas but with an overall typical level of achievement. The present research suggests that nearly all PD children have at least some difficulty in mathematics, regardless of overall level of ability, and therefore most should benefit from intervention. We would thus recommend assessing PD children, despite their overall classification as typical mathematics achievement on specific mathematics components, especially formal aspects, in order to have a better understanding of their likely future needs.

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References


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