PREDICTING AXIAL VELOCITY PROFILES WITHIN A DIFFUSING MARINE PROPELLER JET

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Word Count: 5847

ABSTRACT

A full understanding of the hydrodynamic processes within the jet produced by a manoeuvring ship's propeller is essential in the development and maintenance of ports, docks and harbours. In this investigation the predominant axial velocity component within a freely expanding wash was studied. The flow fields formed by four propellers, each operating at four power levels (speeds of rotation), were investigated under bollard pull conditions and in the absence of a rudder, within a large free surface tank using Laser Doppler Anemometry. The characteristics of these propellers extended the range over which high accuracy measurements have been previously attempted. Comparison were made to existing methodologies by which a prediction of the magnitudes of the axial velocity can be made, and where deficient modifications to the methodologies have been developed and presented. The jets were found to produce a maximum axial velocity along the initial efflux plane at a location near the blade mid-span. The position and magnitude of the axial velocity was seen to decrease as the jet entrained more flow and transitioned from the zone of flow establishment into the zone of established flow.

KEYWORDS

Propeller Jets, Scour, Ports, Dock and Harbours, Hydraulics & Hydrodynamics

31 NOTATION

A	(-)	Coefficient defined in Equation 23
B	(-)	Coefficient defined in Equation 23
C	(-)	Experimentally determined constant (σ/X_o)
C_t	(-)	Thrust coefficient of propeller ($T/\rho n^2 D_p^4$)
c	(m)	Chord length
D_h	(m)	Diameter of hub
D_o	(m)	Initial diameter of slipstream
D_p	(m)	Diameter of propeller
h_d	(m)	Helical distance from the blade section leading edge to rake datum line
h_t	(m)	Helical distance from the blade section leading edge to position of maximum thickness
L_m	(m)	Characteristic length
N	(-)	Number of propeller blades
n	(rpm)	Propeller rotational speed
P'	(-)	Propeller pitch to diameter ratio
p	(m)	Propeller blade pitch
Re_{flow}	(-)	Reynolds number of jet flow ($V_o D_p/v$)
Re_{prop}	(-)	Reynolds number of propeller ($nD_p L_m/ v$)
R_h	(m)	Radius of propeller hub ($D_p/2$)
R_m	(m)	Radial position of maximum axial velocity relative to the jet centreline at any section within the zone of flow establishment
R_{m0}	(m)	Radial distance from propeller axis to location of maximum axial velocity along efflux plane
R_p	(m)	Radius of propeller
R^2	(-)	Coefficient of determination
r	(m)	Radial distance across blade from propeller centreline
V_{max}	(m/s)	Maximum axial velocity

V_o	(m/s)	Maximum axial velocity along efflux plane
$V_{x,r}$	(m/s)	Axial velocity at position x, r
X	(m)	Cartesian co-ordinate measured laterally from face of propeller
X_o	(m)	Distance from propeller to end of zone of flow establishment
β		Blade Area Ratio
ν	(m^2/s)	Kinematic viscosity of fluid
π	(-)	Constant number pronounced pi (π =3.142)
σ	(m)	Standard deviation of velocity

1.0 INTRODUCTION

The problems within harbours and navigation channels associated with the close proximity of manoeuvring vessels, have been well discussed in a range of both case studies and research investigations, Fuehrer & Römisch (1977), Blaauw, H.G., and van de Kaa, E.J. (1978), Bergh & Cederwall (1981), Berger *et al.* (1981), Fuehrer et (1981), Verhey *et al.* (1987), Hamill (1987), Chait (1987), Stewart (1992), Hashmi (1993), Qurrain (1994), Froehlich & Shea (2000), Sumer & Fredsoe (2002), Hong *et al.* (2013), Geisenhainer & Aberle (2013) and Hamill *et al.* (2014). Guidelines for engineers have been developed (PIANC (2015), BAW (2010) and CIRIA (2007)) incorporating the influence of engineering surfaces, beds and slopes. In all cases these methodologies rely on an understanding of the fundamental process that control the formation and diffusion of the jets formed.

Studies that have concentrated on the formation and diffusion of the jets created by the manoeuvring vessels have been limited by the numbers of test propellers used in the studies, Lam *et al.* (2012), and while providing a useful insight have not been in a position to provide predictive methods that covered a meaningful range of operation as only one test propeller was used. The formation process, and subsequent diffusion, of a ship's propeller jet must be fully understood if an engineer is to be able to quantify any scouring damage that may occur, and, more importantly, size protection systems to be deployed to prevent further damage.

The flow field produced by the action of rotating propeller blades is complex in nature. Near to the propeller, the passing blades and rotating hub influence the characteristics of the flow. As the jet diffuses downstream, the velocity characteristics become similar to a submerged three-dimensional jet, Albertson *et al.* (1950).

Under normal operation the propeller flow is influenced by external characteristics such as the hull of the ship or the presence of a rudder for directional purposes. While manoeuvring or near to bollard pull conditions it has been found that such hull effects are negligible, Prosser (1986). The jet produced by a rotating propeller under such conditions is a complex three-dimensional flow with axial, radial and rotational velocity components, Hamill *et al.* (2003).

The axial velocity is the most significant component and is found along the propeller axis of rotation. This component is used to impart a forward thrust to propel the ship in the direction of movement. From the early work of Blauuw and van de Kaa (1978) to the recent PIANC (2015) report, it has been cited that as the axial component is in the order of 10 times the magnitude other components of velocity within the jet, those components “do not need to be considered in the flow analysis of propeller or thruster jets” (PIANC 2015).

Experimental investigations by naval architects into the velocity fields produced by rotating propeller blades have been focussed on the vicinity of the propeller: Min (1978), Cenedese *et al.* (1988) and Felli *et al.*(2006). In contrast, most civil engineering designs of structures and scour prevention systems require the downstream evolution characteristics of turbulent propeller jets in order to determine the magnitude and position of propeller-induced scour.

This paper presents the findings from an extensive experimental investigation which tested four propellers which were allowed to freely expand and whose characteristics covered a wide range typical propeller types, with each propeller being tested at four speeds of rotation (power settings) with velocity measurements of the time averaged components of velocity being taken using Laser Doppler Anemometry (LDA).

2.0 EXPERIMENTAL SETUP

The propellers used in this investigation varied in size (D_p), numbers of blades (N), pitch to diameter ratios (P’), thrust coefficients (C_t), rake and blade area ratios (β), as shown in **Table 1**. The number of propeller blades varied from three to six. The pitch to diameter ratio ranged from a minimum of 0.735 up to a maximum of 1.0. The thrust coefficient, at zero advance speeds, ranged from 0.2908 up to 0.558. The blade area ratios varied from of 0.4525 to 0.922. The blades of propeller 1, 3 and 4 had no forward inclination i.e. all blades are at 90^o angles to the hub while the blades of propeller 2 were inclined by a further 10^o. In selecting these differing propellers it was intended to test over a large practical variation of characteristics typical of sea going vessels.

Froudian scaling was used to determine the speeds of rotation tested. It has been established by Blaauw & van de Kaa (1978) that scale effects due to viscosity can be ignored if the Reynolds number for the propeller exceeded 7 x 10^4 and the Reynolds number for the propeller flow was greater than 3 x 10^3. The Reynolds number for the jet flow is given by:

$$R_{e_{flow}} = \frac{V_0 D_p}{\upsilon} \quad \text{Equation 1}$$

The Reynolds number for the propeller is given by:

$$R_{e_{prop}} = \frac{n\, D_p\, L_m}{\upsilon} \quad \text{Equation 2}$$

The characteristic length, L_m depends on the blade area ratio, propeller and hub diameters as well as the number of blades. Blaauw & van de Kaa (1978) defined this length term as follows:

$$L_m = (\beta) D_p\, \pi \left(2N\left(1 - \frac{D_h}{D_p}\right)\right)^{-1} \quad \text{Equation 3}$$

The rotational speeds used in the programme of work were based on standard Froudian scale of the efflux velocity within the jet and were based on calculations for a generic propeller determined by Qurrain (1994) in a survey of typical ro-ro vessel operating from British ports. This propeller had a diameter of 2.5m, power levels while manoeuvring gave rotations of 200 rpm and a typical thrust coefficient of 0.35 at bollard pull. The efflux velocity, calculated using the equation given by Fuehrer and Römisch (1997), gave a value of V_0=7.3m/s. The corresponding efflux velocity for each propeller was then scaled from this value and used to back calculate the corresponding speed of rotation required to match this providing target speeds for the experimental propellers (1 – 4) of 990, 1056, 865 and 640 rpm respectively. The propellers were operated across a range of speeds that bounded these target values, and these are listed in full in **Table 2**.

The Reynolds numbers for the propellers operating at these rotational speeds ranged from 1.4 x 10^4 to 7.7 x 10^4, while the Reynolds numbers for the propeller jet ranged from 5.3 x 10^4 to 30 x 10^4, **Table 2**. The Reynolds numbers for the propellers were, in some cases, slightly less than 7 x 10^4 however, Blaauw & van de Kaa (1978) and Verhey *et al.* (1987) proposed these scale effects would be insignificant. The Reynolds numbers for the jets were all greater than 3 x 10^3 for the speeds of rotation investigated satisfying the criteria for Froudian scaling. All experiments were carried out in a free-surface tank 7.5 x 4.4 x 1 m in size, partitioned to allow the unhindered expansion of the propeller jets to be investigated (Qurrain 1994).

Velocity was measured using Laser Doppler Anemometry (LDA), which is a well-established non-intrusive technique developed by Yeh & Cummins (1964). The 3D LDA adopted in this research was a Dantec Dynamics three-component backscatter system with a water-cooled Stabilite 2017 5W Argon-Ion laser manufactured by Spectra Physics as the illuminating light source. Frequency shifting of 40 MHz using a Bragg cell was used to remove directional ambiguity in the velocity measurements.

The optical probe was mounted on an automatic Dantec Dynamics 3D-traverse with measurement accuracies within ± 0.05 mm in three orthogonal directions. The measurement volume was located at a distance of 240 mm from the LDA probe. Three-dimensional LDA configurations required the transformation of measurements made in a non-orthogonal coordinate system into a Cartesian system. The transformation of measurements was carried out each time the laser was set-up.

The LDA technique indirectly measured the velocity of the flow by measuring the speed of the (seeding) particles suspended in the flow. The seeding material used in this study was non-spherically shaped polyamide particles having a mean particle size of 20 μm and density of 1.03 g/cm^3. All measurements were made in fully coincident mode i.e. all three processors had to recognise a valid data point before accepting the data. The maximum data rates were determined by the rates obtained with the lowest power channel. Data rates ranged between a minimum of 30 and a maximum of 1000 particles per second.

An experimental measurement grid was established at which velocity readings were taken in sections across the face of the propeller. The centre of the propeller hub, at the cutting edge of the propeller blades, was taken as the zero location and measurements were taken on a Y (horizontal), Z (vertical) grid in 2 – 5mm steps. The sections were repeated at 20mm intervals moving away from the propeller in a horizontal plane, X.

3.0 TIME-AVERAGED ANALYSIS OF THE AXIAL VELOCITY COMPONENT

3.1 Zone of Flow Establishment

The maximum velocity, located on the initial plane of the jet, is termed the efflux velocity: V_0. Hamill *et al.* (2014) discuss the 3D nature of this velocity and concluded that for the axial component, the magnitude could be obtained from:

$$V_0 = 1.22\, n^{1.01}\, D_p^{0.84}\, C_t^{0.62} \qquad \text{Equation 4}$$

This equation presents an alternative means of calculating V_0, which although still based on the form of equation developed from the traditional actuator disc theory used in current design guideline such as PIANC (2015), it attempts to provide corrections to the limiting assumptions used in that theory which tend to overestimate the V_0 value. This deviation in predicted values of V_0 is clearer for larger propellers.

All subsequent velocity values, at any location within the diffusing jet, have been shown to be dependent on the magnitude of this initial value V_o. The formation and diffusion process that occur within the jet are also accepted to occur within two regions of transition as shown in **Figure 1**. The first, where the jet forms and becomes established, is called the Zone of Flow Establishment (ZFE). The second, where the jet subsequently decays to merge with any background flow, is called the Zone of Established Flow (ZEF), Albertson *et al.* (1950). In propeller jets the flow is said to be fully established when the maximum velocity location moves from across the blade to act along the line of the propeller shaft axis. The differing mechanisms that operate within these zones has resulted in previous researchers trying to establish the location of the changeover so that different analytical techniques can be applied to each zone.

Fuehrer & Römisch (1977) and Blaauw & van de Kaa (1978) found the end of the “ZFE” occurred at a relative distance of X_o/D_p = 2.6. Verhey *et al.* (1987) suggested the zone length was $X_o/D_p = 2.77$, while Stewart (1992) proposed the zone extended to approximately $X_o/D_p = 3.25$ from the initial efflux plane.

Figure 2 shows the measured velocity distributions obtained for propeller 2, at a test rotational speed of 1000rpm. This profile is typical of all the tests conducted, for all the propellers tested. The axial velocity distribution at $2D_p$ consisted of a low velocity core with the maximum peak velocities located either side of the jet centreline. By $3D_p$, further entrainment of surrounding fluid caused a decrease in the magnitude of the axial velocity distribution. The locations of the peak velocities were still evident at positions along the propeller blades. However by $4D_p$, the profiles have taken on the uniform normal distribution shape associated with the zone of established flow. The central core was fully entrained and the maximum velocity reverting to the centreline of the jet.

Investigations of the axial velocity profiles between $2D_p$ and $4D_p$, at 20 mm intervals, showed that the transition location from the “ZFE” to the “ZEF” occurred at $X_o/D_p = 3.15$, 3.26, 3.49 and 2.9 for propellers 1, 2, 3 and 4. Over the range of propeller characteristics tested in this study it is suggested that the extent of the initial zone can be approximated to be between $3 \le X_o/D_p \le 3.5$, indicating significant difference from some of the earlier published work. Stewart (1992) confirmed the extent of the zone of flow establishment occurred when the maximum axial velocity was located along the propeller centreline at approximately $X_o/D_p = 3.25$. This compares favourably with the results of this investigation.

3.1.1 Magnitude of the Maximum Axial Velocity

Albertson *et al.* (1950) assumed there was no decay of the maximum axial velocity in the zone of flow establishment as distance from the jet source increased. Blaauw & van de Kaa (1978), Verhey (1983) and Fuehrer & Römisch (1977), working with propeller jets, also agreed with this statement. Hamill (1987) however, found this hypothesis only held true up to a short distance of approximately $X/D_p = 0.35$ behind the propeller. Beyond this distance, through direct measurements, Hamill (1987) concluded the maximum axial velocities within the propeller jet decreased with distance from the propeller as a result of lateral mixing i.e. the jets expansion and its entrainment of ambient fluid, and was influenced by the blade area ratio (β) as shown in equation 5:

$$\frac{V_{max}}{V_0} = 0.87 \left(\frac{X}{D_p}\right)^{-\frac{\beta}{4}} \qquad \text{Equation 5}$$

Stewart (1992) stated the application of equation 5 could not be generalised to any propeller and developed the following linear decay equation:

$$\frac{V_{max}}{V_0} = 1.0172 - 0.1835 \left(\frac{X}{D_p}\right) \qquad \text{Equation 6}$$

The predictive solutions from the methods proposed by Albertson *et al.* (1950), Hamill (1987) and Stewart (1992) were compared with the measured results from this investigation. **Figure 3** shows an exemplar of the comparison found between the current predictive methodologies and the measurements taken. Decay in magnitude of the velocity with distance from the propeller was found in all cases demonstrating that the suggestions based on the work by Albertson *et al.* (1950) are invalid. Equation 7, proposed by Hamill (1987), was found to overestimate the decay of the maximum axial velocity for propellers 2 and 4, with limited fit being found form short regions with propellers 1 and 3. In the remainder of the zone, the equation did not adequately determine the measured data. Equation 6 was developed from tests conducted using propellers 1 and 4, which were also used in this investigation so it was expected that the solutions of equation 6 would adequately predict the axial velocity decay trends for those propellers. However, equation 6 was found to underestimate the axial velocity decay trends, by up to 25%, for propellers 2 and 3 and therefore insufficiently extrapolated outside the test range from which it was derived. Over all none of the current methods provide an adequate method by which the maximum velocity at any axial distance within the ZFE could be determined.

It was apparent from examining the measured data that the decay trends of the maximum axial velocity follows a linear profile as was suggested by Stewart (1992). Based on a stepwise variable selection process, of all available data for the four propellers tested at four speeds of rotation, analysis determined that the variables that most influenced maximum axial velocity (V_{max}) were the non-dimensionalised distance from the propeller source (X/D_p) and the propeller pitch to diameter ratio (P'). The following equation having a high coefficient of determination ($R^2 = 0.964$) was derived:

$$\frac{V_{max}}{V_0} = 1.51 - 0.175\left(\frac{X}{D_p}\right) - 0.46\,P' \qquad \text{Equation 7}$$

The output solutions of equation 7 were compared with the results of the empirical investigation and in all cases, the output solutions of this equation adequately predicted the decay trends of the maximum axial velocity from $X/D_p = 0.35$ to the end of the initial zone of flow establishment, **Figure 4**. It is therefore suggested for distances up to $X/D_p = 0.35$ no decay of the efflux velocity occurs as suggested by Hamill (1987) and that the maximum velocity with distance is equal to that found on the efflux plane. After this, the maximum axial velocity decays linearly throughout the remainder of the zone of flow establishment and can be determined using equation 7, given the efflux velocity (V_o), distance from the propeller (X), propeller diameter (D_p) and pitch to diameter ratio (P') as input variables.

3.1.2 Axial Velocity Distributions within the Zone of Flow Establishment

Along the initial efflux plane, and throughout the zone of flow establishment, the distribution of the axial velocity component was found to increase from the jet centreline towards a maximum value before then decreasing rapidly towards the tip of the blade, Hamill (1987). McGarvey (1996) derived an equation based on the physical properties of propeller blades to determine the distribution of the axial velocity component along the efflux plane:

$$\frac{V_{x,r}}{Nnr} = 1.261 - 0.974\left(\frac{p}{r}\right) + 0.733\left(\frac{c}{r}\right) + 18.53\left(\frac{t}{r}\right) + 5.028\left(\frac{h_d}{r}\right) + 0.106\left(\frac{p}{r}\right)^2 - 7.277\left(\frac{h_d}{r}\right)^2 - 4.093\left(\frac{h_t}{c}\right)^2$$

Equation 8

Albertson *et al.* (1950) found the velocity distribution at any section within a submerged jet to follow the general trend of the Gaussian normal probability function. Hamill (1987) made changes to the normal probability function and produced the following equation:

$$\frac{V_{x,r}}{V_{max}} = EXP\left(-\frac{1}{2}\frac{(r - R_{m0})^2}{\sigma^2}\right) \qquad \text{Equation 9}$$

Hamill (1987) measured the standard deviation, σ, as constant and equal to $0.5R_{m0}$ up to a downstream distance of $X/D_p = 0.5$:

$$\sigma = \frac{1}{2}R_{m0} \qquad \text{for } X/D_p < 0.5 \qquad \text{Equation 10}$$

Beyond $X/D_p = 0.5$, to the end of the zone of flow establishment, the standard deviation was defined as:

$$\sigma = \frac{1}{2}R_{m0} + 0.075\left(X - \frac{D_p}{2}\right) \qquad \text{for } X/D_p > 0.5 \qquad \text{Equation 11}$$

The output results of equation 8, proposed by McGarvey (1996), were compared with the experimental results in this study and **Figure 5** is a typical representation of the findings. While the shape of the profile predicted does follow that expected the only propeller that gave good agreement was propeller 1 (upon which the equation was developed). The method is overly cumbersome and can be difficult to apply. This equation has therefore poor generalisation capabilities when applied to any propeller.

Axial velocity distributions within the zone of flow establishment were measured and compared with the output results of equations 9, 7 and 4 using non-dimensionalised values of $V_{x,r}/V_{max}$ versus X/D_p. **Figures 6** shows a typical comparison, with good agreement being predicted both in terms of magnitudes and profile shape. The use of equation 9, in conjunction with equations 11, 10, 7 and 4, adequately determined the axial velocity distributions within the Zone of Flow Establishment in the jets produced by each of the experimental propellers tested, and removes the need to establish refined methods of analysis, and is recommended for use in predicting the velocity distributions of the axial velocity within the zone.

3.2 Zone of Established Flow

3.2.1 Magnitude of the Maximum Axial Velocity Decay within the Zone of Established Flow

Differences exist in the decay between the zone of flow establishment and the zone of established flow. This can be explained by the differences in the diffusion processes in these two zones. In the first zone, diffusion is occurring both internally and externally. The jet is entraining its low velocity core as well as the ambient fluid. The decay of maximum velocity is therefore much more rapid than in the zone of established flow were the central core has already been entrained and only the external entrainment of the surrounding fluid is taking place, Stewart (1992).

Albertson *et al.* (1950) stated that for all jets, including propeller jets, the decay of velocity was proportional to the distance from the source could be found using:

$$\frac{V_{max}}{V_0} = \frac{1}{2C}\left(\frac{X}{D_p}\right)^{-1} \qquad \text{Equation 12}$$

where the constant C is the variation of the standard deviation of velocity with distance.

Other researchers also adopted the general form of equation 12: Fuehrer & Römisch (1977), Blaauw & van de Kaa (1978), Berger *et al.* (1981) and Verhey (1983). These equations are as follows for each author respectively:

$$\frac{V_{max}}{V_0} = 2.6\left(\frac{X}{D_p}\right)^{-1} \qquad \text{Equation 13}$$

$$\frac{V_{max}}{V_0} = 2.8\left(\frac{X}{D_p}\right)^{-1} \qquad \text{Equation 14}$$

$$\frac{V_{max}}{V_0} = 1.025\left(\frac{X}{D_p}\right)^{-0.6} \qquad \text{Equation 15}$$

$$\frac{V_{max}}{V_0} = 1.275\left(\frac{X}{D_p}\right)^{-0.7} \qquad \text{Equation 16}$$

Through direct experimental measurements Hamill (1987) suggested the decay of the maximum velocity can be described using the following equation, taking into account the propeller geometry:

$$\frac{V_{max}}{V_0} = A\left(\frac{X}{D_p}\right)^{B} \qquad \text{Equation 17}$$

where:

$$A = -11.4\ C_t + 6.65\ \beta + 2.16\ P'$$

$$B = -\ C_t^{0.216}.\ \beta^{1.024}.P'^{-1.87}$$

Stewart (1992) reported the decay of the maximum axial velocity was independent of the speed of rotation and propeller type used. A straight-line equation was proposed to determine the decay within the zone of established flow:

$$\frac{V_{max}}{V_0} = 0.543 - 0.0281 \left(\frac{X}{D_p}\right) \quad \text{Equation 18}$$

Hashmi (1993) found the maximum velocity in the wash was still measurable up to $X/D_p = 16$ downstream from the propeller. Hashmi (1993) therefore proposed the following equation in exponential form to predict the decrease in V_{max}:

$$\frac{V_{max}}{V_0} = 0.638\, e^{-0.097\left(\frac{X}{D_p}\right)} \quad \text{Equation 19}$$

Large differences therefore exist in the extensive range of semi-empirical equations available to determine the decay of the maximum axial velocity within the zone of established flow. The decay trends of the maximum axial velocity were therefore measured for each of the experimental propellers tested to allow a comparison to be made between the measured and predicted output solutions of the existing semi-empirical equations.

Equations 13 and 14 proposed by Fuehrer & Römisch (1977) and Blaauw & van de Kaa (1978) overestimated the measured decay trends, **Figure 7**. Equations 15 and 16 suggested by Berger *et al.* (1981) and Verhey (1983) produced similar decay trends throughout the zone of established flow but showed under predictions of propeller 2 (and over predictions of propeller 4) by some 20%, **Figure 8**. The linear equation 18 proposed by Stewart (1992) adequately predicted the decay of propellers 1 and 4 from which it was derived, **Figure 9.** However, the output solutions of equation 18 underestimated the decay trends of propellers 3 and 4, **Figure 9**. The generalisation capabilities of equation 18 were reduced when used to predict the decay trends of propellers outside the test range of which it was derived. The exponential form of equation 19 proposed by Hashmi (1993) also underestimated the decay of all propellers at the beginning of this zone, **Figure 9**. It is obvious from these comparisons that the simplified decay expressed by these equations is not sufficient to account for the variations measured.

The power trend equation 17 suggested by Hamill (1987) is based on the main propeller characteristics: propeller pitch to diameter ratio, blade area ratio and thrust coefficient. The output solutions of equation 17 were found to adequately determine the experimental results of propellers 1 and 2, giving low percentage differences of 20%, **Figures 10 a and b**. Equation 17 was also used to determine the maximum axial velocity within the ZEF of propellers 3 and 4. However, this equation overestimated the maximum axial velocity, **Figures 10 c and d**. It does however, show that the variations can be better described by including the aspects of the propeller geometry within the prediction.

In a manner similar to that adopted for the Zone of Flow Establishment, a stepwise variable selection process was tested and it was found that the variables which most influenced the determination of the maximum axial velocity (V_{max}) were the same, i.e. the non-dimensionalised distance from the propeller source (X/D_p) and propeller pitch to diameter ratio (P'). An equation having a high coefficient of determination ($R^2 = 0.924$) was derived.

$$\frac{V_{max}}{V_0} = 0.964 - 0.039\left(\frac{X}{D_p}\right) - 0.344\,P' \qquad \text{Equation 20}$$

Figure 11 shows an exemplar comparison of the output from equation 20 with the data obtained from the tests using propeller 4. The measured decay trends were adequately predicted using the distance from the initial efflux plane and pitch to diameter ratio as input variables. Overall, equation 20 performs well in predicting the decay of the maximum axial velocity within the zone of established flow, and it is recommended that it should be used in place of the existing methodologies.

3.2.2 Axial Velocity Distributions within the Zone of Established Flow

Hamill (1987) investigated the methods available to determine the axial velocity distributions within the zone of established flow proposed by Blaauw & van de Kaa (1978), Berger *et al.* (1981), Verhey (1983) Fuehrer & Römisch (1977). The equations proposed by Berger *et al.* (1981) and Verhey (1983) were found to be limited when applied to propeller jet flow. The solutions proposed by Blaauw & van de Kaa (1978) and Fuehrer & Römisch (1977) are respectively as follows:

$$\frac{V_{x,r}}{V_{max}} = EXP\left[-15.4\left(\frac{r}{x}\right)^2\right] \qquad \text{Equation 21}$$

$$\frac{V_{x,r}}{V_{max}} = EXP\left[-22.2\left(\frac{r}{x}\right)^2\right] \qquad \text{Equation 22}$$

The output solutions of equations 21 and 22, when calculated using equations 4 and 20, were compared with measured non-dimensionalised axial velocity profiles for all the tested propellers. Comparisons were made at downstream distances of X/D_p = 4, 5 and 6 within the zone of established flow, and an example of the typical output obtained is shown in **Figure 12**. The output solutions of equation 22 proposed by Fuehrer & Römisch (1977) were found to adequately predict the axial velocity distributions within the zone of established flow, consistently, for all four experimental propellers investigated. These results agree with those of Hamill (1987) and Stewart (1992), in that, equation 22 proposed by Fuehrer & Römisch (1977) adequately predicts the axial velocity distributions within the zone of established flow. It is suggested equation 22 needs no further modification and should be used in future analysis.

4.0 SUMMARY AND CONCLUSIONS

A range of experimental propellers was tested at zero advance speeds, simulating the manoeuvring operation when a ship departs from a port. The experiments simulated a freely expanding jet, with no interference from any harbour configuration or the presence of any rudder effect. The time-averaged (mean) velocity of these jets were investigated. This time-averaged analysis can be used to assist engineers in designing suitable scour protection systems to prevent damage of erodible seabed materials by expanding the envelope of information available upon which engineering decisions can be based.

Semi-empirical equations have been derived, based on the main propeller characteristics and rotational speed, to determine the location, magnitude and distribution of the axial velocity within the freely expanding propeller jet produced by an un-ducted propeller.

When used in conjunction with Equation 4 for prediction the efflux velocity V_0, (Hamill (2014)) the maximum axial velocity decayed linearly throughout the zone of flow establishment after an initial distance of X/D_p = 0.35. The variables which most influenced the decay of the maximum axial velocity were: the efflux velocity (V_o), distance from the propeller (X), propeller diameter (D_p) and propeller pitch to diameter ratio (P'):

$$\frac{V_{max}}{V_0} = 1.51 - 0.175\left(\frac{X}{D_p}\right) - 0.46\,P'$$

Similarly, within the zone of established flow a semi-empirical equation based on the propeller characteristics determined the magnitude of the maximum axial velocity:

$$\frac{V_{max}}{V_0} = 0.964 - 0.039\left(\frac{X}{D_p}\right) - 0.344\,P'$$

When used with equations 4, 7 and 20 the distribution of axial velocity within the Zone of Flow Establishment was found to be adequately described by the equations developed by Hamill (1987), while for distributions within the Zone of Established Flow the method reported by Fuehrer (1977) is recommended.

The suite of equations presented and discussed within this paper relate to a free expanding propeller jet and bring together the current knowledge available to the engineer. The testing conducted, using state of the art LDA measurement in an expansive experimental, has allowed knowledge gaps to be filled and an integrated axial velocity predictive method published.

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