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Hybrid Positioning through Extended Kalman Filter with Inertial Data Fusion

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Abstract—In wireless sensor networks (WSNs), hybrid algorithms are widely used in order to improve the final positioning accuracy. This paper presents a hybrid positioning algorithm which combines time of arrival (TOA) and received signal strength (RSS) measurements using two different radio technologies, ultra wide band (UWB) and ZigBee, respectively. The TOA measurements are used to estimate the distances between a mobile node and a set of anchor nodes. Both UWB-based distance estimates and RSS measurements based on ZigBee are simultaneously processed by an Extended Kalman Filter (EKF). Moreover, a low cost inertial device is also used to acquire acceleration measurements which proved to be useful in order to detect the motion of the mobile node. This information has also been integrated in the EKF algorithm accordingly. The performance of the final hybrid positioning algorithm is compared with the conventional EKF which uses a single type of range measurements, TOA or RSS. Simulation results based on a real measurements campaign, show that the hybrid algorithm significantly improves positioning accuracy. In addition, a further improvement has been achieved by applying the motion detection approach based on inertial measurements performed by the low cost acceleration sensor.

Index Terms—Hybrid Positioning, Extended Kalman Filter, TOA/RSS, Data fusion, Power measurement, Wireless Sensor Networks.

I. INTRODUCTION

A wide range of applications based on wireless sensor networks (WSNs) exist, where the knowledge of the sensors location is required e.g., indoor navigation, water level detection, vehicles detection, environment monitoring, machine health monitoring, surveillance and some military applications. The most common positioning systems, such as GPS and the ones based on cellular networks use time of arrival (TOA) measurements. Usually, these systems use a trilateration approach which is based on distance estimates performed from a fixed number of anchor nodes. In indoor environments, accurate localization becomes very difficult because range measurements are affected by errors due to diffraction from objects and multipath [1]. Positioning algorithms based only on a single type of range measurements either TOA or received signal strength (RSS) are not enough to achieve the desired level of accuracy required by some critical applications [2]. Thus, different techniques have been adopted in order to improve the localization accuracy such as hybrid positioning algorithms. Hybrid algorithms, compared to solutions based only on a single type of measurement, receive more range measurements. Consequently, this results in an increased of both position estimation availability and position accuracy. This paper proposes a hybrid positioning algorithm which combines both TOA measurements based on ultra wide band (UWB) and RSS measurements based on ZigBee. Furthermore, acceleration measurements acquired by a low cost inertial sensor are also processed by the algorithm.

In general, Extended Kalman Filter (EKF) has been heavily studied and adopted for tracking and position estimation in WSNs as it can better handle non-linear systems. Moreover, it offers a low computation complexity, thus well suited to be implemented on constrained devices like WSN nodes. The obtained experimental results show that the proposed hybrid positioning algorithm improves the localization accuracy. Moreover, performance further improves through the use of the motion detection technique based on inertial measurements.

The remainder of the paper is organized as follows. Section 2 presents EKF algorithm basics. Section 3 presents different EKF-based positioning techniques. Section 4 describes the improvement in localization accuracy through inertial data. Section 5 shows performance of the different positioning techniques. Finally, section 6 concludes the paper.

II. OVERVIEW OF THE EXTENDED KALMAN FILTER ALGORITHM

The Kalman filter (KF) has become integral part of many civilian and military applications since its introduction in 1960. It is a very efficient, versatile and effective method in combining noisy measurements from the sensors and inferring state estimation [3]. The KF algorithm uses a recursive approach which is composed of two main phases, namely ‘state prediction’ and ‘state update’. The KF provides the optimal solution only when the system is linear and measurement errors have Gaussian distribution.

However, EKF has been designed for many practical systems that have non-linear state update and/or measurements equations [4]. In addition, the performance of the EKF heavily depends on how the system dynamics and measurements are modelled [5]. The state transition equation can be expressed as:

$$x_k = f(x_{k-1}, u_{k-1}) + w_k$$  

where $w_k \sim N(0, Q_k)$ represents the process noise and $x_k$ is the true state vector at time $t_k$. The non-linear state function
The observation vector $\mathbf{z}_k$ can be expressed as a function of the state through the measurement equation:

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k$$  \hspace{1cm} (2)

where $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k)$ is the observation noise vector. The observation function $h$ relates the measurements with the true state $\mathbf{x}_k$. As mentioned before the EKF algorithm is composed of two main phases: 'predict phase' and 'update phase' [4], [5] described as follows.

### A. Predict Phase

The predict phase is used to predict the a priori state vector $\hat{\mathbf{x}}_{k|k-1}$ on the basis of the previous a posteriori estimated state vector $\hat{\mathbf{x}}_{k-1|k-1}$. It can be expressed as follows:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \cdot \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \cdot \mathbf{u}_k$$  \hspace{1cm} (3)

where $\mathbf{F}_k$ represents the linearized state transition matrix, $\mathbf{B}_k$ represents the input matrix and $\mathbf{u}_k$ represents the input to the system. Considering a PV model for the state equation in 2-dimensional case, the state vector can be defined as $\mathbf{x}_k = [\dot{x}_k \ y_k \ \dot{y}_k]$ where $\dot{x}_k$ and $\dot{y}_k$ denote the speed of the mobile target node along the $x$ and $y$ axis, respectively. Accordingly, the matrix $\mathbf{F}_k$ can be defined as follows:

$$\mathbf{F}_k = \begin{bmatrix} 1 & 0 & \Delta t_k & 0 \\ 0 & 1 & 0 & \Delta t_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (4)

where $\Delta t_k$ represents the time elapsed between the previous estimation time $t_{k-1}$ and the current one $t_k$. The estimated covariance matrix $\mathbf{P}_{k|k-1}$ related to the current a priori state vector $\hat{\mathbf{x}}_{k|k-1}$ is evaluated from the previous a posteriori covariance matrix $\mathbf{P}_{k-1|k-1}$ and the process noise covariance matrix $\mathbf{Q}$.

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \cdot \mathbf{P}_{k-1|k-1} \cdot \mathbf{F}_k^T + \mathbf{Q}$$  \hspace{1cm} (5)

The $\mathbf{Q}$ matrix takes into account un-modeled factors of the system. For a PV model [5], it can be defined as follows:

$$\mathbf{Q} = \mathbf{A} \cdot \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \cdot \mathbf{A}^T$$  \hspace{1cm} (6)

where $\sigma_x^2$ and $\sigma_y^2$ denote the variances of the acceleration noise along the $x$ and $y$ axis, respectively. The matrix $\mathbf{A}$ can be defined as:

$$\mathbf{A} = \begin{bmatrix} \frac{\Delta t_k^2}{2} & \frac{\Delta t_k}{2} \\ \frac{\Delta t_k}{2} & \frac{\Delta t_k^2}{2} \end{bmatrix}$$

where $\mathbf{I}_2$ represents the identity matrix of dimensions 2. The covariance matrix $\mathbf{P}_0$ related to the initial state vector $\mathbf{x}_0$, for the PV model in 2-dimensional case can be defined as:

$$\mathbf{P}_0 = \begin{bmatrix} \sigma_{x_0}^2 & 0 & 0 & 0 \\ 0 & \sigma_{y_0}^2 & 0 & 0 \\ 0 & 0 & \sigma_{x_0}^2 & 0 \\ 0 & 0 & 0 & \sigma_{y_0}^2 \end{bmatrix}$$  \hspace{1cm} (7)

where $\sigma_{x_0}^2$, $\sigma_{y_0}^2$, $\sigma_{x_0}^2$ and $\sigma_{y_0}^2$ represent the initial variances of the state vector components.

### B. Update Phase

The update phase, also called correction phase, further refines the a priori position estimate by using the observation vector $\mathbf{z}_k$. First of all, the innovation vector $\tilde{\mathbf{y}}_k$ is calculated as the residual between the observed measurement $\mathbf{z}_k$ and the expected measurement $h(\hat{\mathbf{x}}_k)$:

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - h(\hat{\mathbf{x}}_k)$$  \hspace{1cm} (8)

The covariance matrix $\mathbf{S}_k$ of the innovation vector can be computed as follows:

$$\mathbf{S}_k = \mathbf{H}_k \cdot \mathbf{P}_{k|k-1} \cdot \mathbf{H}_k^T + \mathbf{R}_k$$  \hspace{1cm} (9)

where $\mathbf{R}_k$ represents the covariance matrix related to the observation vector and $\mathbf{H}_k$ represents Jacobian matrix related to expected measurements. The a posteriori state estimate $\hat{\mathbf{x}}_{k|k}$ is computed by correcting the a priori state estimate $\hat{\mathbf{x}}_{k|k-1}$ as follows:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \cdot \tilde{\mathbf{y}}_k$$  \hspace{1cm} (10)

where $\mathbf{K}_k$ is the optimal Kalman gain which can be expressed as:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \cdot \mathbf{H}_k^T \cdot \mathbf{S}_k^{-1}$$  \hspace{1cm} (11)

Finally, the a posteriori state covariance matrix $\mathbf{P}_{k|k}$ is computed by correcting the a priori state covariance matrix $\mathbf{P}_{k|k-1}$ as follows:

$$\mathbf{P}_{k|k} = (\mathbf{I}_n - \mathbf{K}_k \cdot \mathbf{H}_k) \cdot \mathbf{P}_{k|k-1}$$  \hspace{1cm} (12)

### III. POSITIONING TECHNIQUES

Position estimation of a target node in WSN can be obtained by performing range measurements between the mobile node and a set of anchor nodes [6]. Depending on the WSN technology, different types of range measurements can be processed by the EKF. These different type of measurements have different accuracies and different limitations. The RSS-based distance estimation, for instance, highly depend on the actual distance between the mobile node and anchor nodes. In fact, range measurement error increases with the distance. Thus when RSS measurements
are available, anchor nodes are usually densely deployed close to the mobile node trajectory [1]. Moreover, RSS-based range estimation depends also on the environment [5].

A. Positioning through Distance Measurements

Indoor positioning systems are mainly based on UWB technology as UWB signals provide very accurate distance estimation [7]. Typically, UWB nodes perform Two-Way TOA (TW-TOA) [8]. Consequently, the distance can be estimated as follows:

\[ d = c \cdot \frac{\tau_{\text{TW-TOA}}}{2} \]

where \( c \) is the speed of the light. The EKF estimates the position of the mobile node by using the distance measurements between the mobile node and a set of anchor nodes [5]. Considering the 2-dimension case, the generic anchor node \( A \) has known coordinates \( [x_{A_i}, y_{A_i}] \) for \( i = 1, \ldots, L \), where \( L \) represents the total number of UWB anchor nodes deployed in the environment. Let \( \mathbf{x}_M = [\vec{x}_M, \vec{y}_M]^T \) represents coordinates of mobile node at time instant \( t_k \). The observation vector \( \mathbf{z}_{\text{dist},k} \) used by the EKF can be defined as:

\[
\mathbf{z}_{\text{dist},k} = \begin{bmatrix} \bar{d}_{A_1,k} & \bar{d}_{A_2,k} & \cdots & \bar{d}_{A_L,k} \end{bmatrix}
\]

(13)

where \( \bar{d}_{A_i,k} \) represents the estimated distance between the mobile node and the \( i \)-th UWB anchor node at the current estimation time \( t_k \). The \( \mathbf{h}_{\text{dist}}(\mathbf{x}_{k|k-1}) \) is a vector function composed of the Euclidean distances between the mobile node and all anchor nodes at the estimation time \( t_k \), expressed as follows:

\[
\mathbf{h}_{\text{dist}}(\mathbf{x}_{k|k-1}) = \begin{bmatrix} \text{dist}((\vec{x}_M, \vec{x}_{A_1}) \text{dist}((\vec{x}_M, \vec{x}_{A_2}) \cdots \text{dist}((\vec{x}_M, \vec{x}_{A_L})) \end{bmatrix}
\]

(14)

where \( \text{dist}(...) \) is the Euclidean distance operator. For the \( i \)-th anchor node,

\[
\text{dist}(\vec{x}_M, \vec{x}_{A_i}) = \sqrt{(\vec{x}_M - x_{A_i})^2 + (\vec{y}_M - y_{A_i})^2}
\]

The Jacobian \( \mathbf{H}_{\text{dist},k} \) matrix of the expected measurement vector \( \mathbf{h}_{\text{dist}}(\mathbf{x}_{k|k-1}) \) needs to be computed around the a priori state vector \( \mathbf{x}_{k|k-1} \). Thus, it can be defined as:

\[
\mathbf{H}_{\text{dist},k} = \begin{bmatrix} \frac{\vec{x}_M - x_{A_1}}{\text{dist}(\vec{x}_M, \vec{x}_{A_1})} & \frac{\vec{y}_M - y_{A_1}}{\text{dist}(\vec{x}_M, \vec{x}_{A_1})} & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\vec{x}_M - x_{A_L}}{\text{dist}(\vec{x}_M, \vec{x}_{A_L})} & \frac{\vec{y}_M - y_{A_L}}{\text{dist}(\vec{x}_M, \vec{x}_{A_L})} & 0 & 0 
\end{bmatrix}
\]

(15)

The covariance matrix \( \mathbf{R}_{\text{dist},k} \) of the observation vector for the distance measurements can be defined as:

\[
\mathbf{R}_{\text{dist},k} = \text{diag}(\sigma_{\text{dist},A_1,k}^2, \cdots, \sigma_{\text{dist},A_L,k}^2)
\]

(16)

where \( \sigma_{\text{dist},A_i,k}^2 \) represents the initial variance of distance measurement for the \( i \)-th anchor node.

B. Positioning through RSS Measurements

The RSS measurement capability is relatively cheap to be implemented in hardware. However, distances based on RSS are less accurate than TOA measurements [5]. The observation vector \( \mathbf{z}_{\text{RSS},k} \) for RSS measurements at the estimation time \( t_k \) can be defined as:

\[
\mathbf{z}_{\text{RSS},k} = \begin{bmatrix} \hat{P}_{A_1,k} & \hat{P}_{A_2,k} & \cdots & \hat{P}_{A_M,k} \end{bmatrix}^T
\]

(17)

where \( \hat{P}_{A_i,k} \) represents the RSS measurement between the mobile node and the \( i \)-th ZigBee anchor node and \( M \) represents total number of ZigBee anchor nodes. The expected measurements vector \( \mathbf{h}_{\text{RSS}}(\mathbf{x}_{k|k-1}) \) contains expected RSS measurements between the mobile node and all anchor nodes at the estimation time \( t_k \) and can be expressed as follows:

\[
\mathbf{h}_{\text{RSS}}(\mathbf{x}_{k|k-1}) = \begin{bmatrix} P_{A_1}(\mathbf{x}_{k|k-1}) \\
\vdots \\
P_{A_M}(\mathbf{x}_{k|k-1}) \end{bmatrix}
\]

(18)

where the received power from the \( i \)-th ZigBee anchor node \( P_{A_i}(\mathbf{x}_{k|k-1}) \), expressed in dBm, is modeled by the log-normal shadowing path loss model [5] and can be defined as:

\[
P_{A_i}(\mathbf{x}_{k|k-1}) = P_0 - 10\alpha \log_{10}(\frac{(\vec{x}_M - x_{A_i})^2 + (\vec{y}_M - y_{A_i})^2}{d_0})
\]

where \( P_0 \) represents the received power at the distance \( d_0 \) and \( \alpha \) represents the path loss exponent. For the PV model in 2-dimension case, the Jacobian matrix \( \mathbf{H}_{\text{RSS},k} \) can be defined as:

\[
\mathbf{H}_{\text{RSS},k} = \frac{-10\alpha}{\ln(10)} \begin{bmatrix} \frac{x_{A_1} - x_{A_{k}}}{\text{dist}^2(\vec{x}_M, \vec{x}_{A_1})} & \cdots & \frac{y_{A_1} - y_{A_{k}}}{\text{dist}^2(\vec{x}_M, \vec{x}_{A_1})} \\
\vdots & \ddots & \vdots \\
\frac{x_{A_M} - x_{A_{k}}}{\text{dist}^2(\vec{x}_M, \vec{x}_{A_M})} & \cdots & \frac{y_{A_M} - y_{A_{k}}}{\text{dist}^2(\vec{x}_M, \vec{x}_{A_M})} \end{bmatrix}
\]

(19)

The covariance matrix \( \mathbf{R}_{\text{RSS},k} \) of the observation vector for RSS measurements can be defined as:

\[
\mathbf{R}_{\text{RSS},k} = \text{diag}(\sigma_{\text{RSS},A_1,k}^2, \cdots, \sigma_{\text{RSS},A_M,k}^2)
\]

(20)

where \( \sigma_{\text{RSS},A_i,k}^2 \) represents the initial variance of the shadowing for the \( i \)-th ZigBee anchor node.
C. Hybrid Positioning Algorithm

Higher positioning accuracy can be achieved by mean of hybrid techniques which take advantage of smart combination of different types of measurements based on different technologies [9]. The hybrid technique proposed in this paper combines distances based on UWB measurements and RSS based on ZigBee measurements [10]. In general, hybrid techniques, compared to positioning algorithms based only on a single type of range measurement, have an increased number of measurements in the observation vector as the number of anchor nodes is increased. Consequently, both position estimation availability and position accuracy improves [2].

Let consider $L$ UWB and $M$ ZigBee anchor nodes. Starting from (13) and (17), the hybrid observation vector $z_k$ which feeds the hybrid EKF can be defined as follows:

$$ z_k = \begin{bmatrix} \text{dist}_{k|k-1} \\ \text{RSS}_{k|k-1} \end{bmatrix} $$ (21)

The vector $h(\hat{x}_{k|k-1})$ for the hybrid case can be defined using (14) and (18) as follows:

$$ h(\hat{x}_{k|k-1}) = \begin{bmatrix} h_{\text{dist}}(\hat{x}_{k|k-1}) \\ h_{\text{RSS}}(\hat{x}_{k|k-1}) \end{bmatrix} $$

The hybrid Jacobian matrix $H_k$ can be defined using (15) and (19) as follows:

$$ H_k = \begin{bmatrix} H_{\text{dist},k} \\ H_{\text{RSS},k} \end{bmatrix} $$ (22)

The hybrid covariance matrix $R_k$ of the observation vector can be defined starting from (16) and (20) as follows:

$$ R_k = \begin{bmatrix} R_{\text{dist},k} & 0_{L \times M} \\ 0_{M \times L} & R_{\text{RSS},k} \end{bmatrix} $$ (23)

where $0_{L \times M}$ and $0_{M \times L}$ represent zero matrices of size $L \times M$ and $M \times L$, respectively.

IV. MOTION DETECTION APPROACH:

This section proves that the localization accuracy can be further improved by fusing inertial and radio measurements. During the measurement campaign, a 3-axis wireless accelerometer sensor was used in order to acquire acceleration data. Since the accelerometer sensor is a low cost device, it is not very accurate. Moreover, the acceleration measurements are also affected due to robot (mobile node) vibrations while moving [11]. Due to above reasons, acceleration data cannot be used as input for positioning, but it can help to detect the motion of mobile node. When the mobile node is detected in motion then both ‘predict’ and ‘update’ phases of EKF are used to estimate the position, but if the averaged acceleration value goes below a pre-defined threshold, then it means that the mobile node is detected static. Consequently, only the update phase of the EKF is executed to correct the previous predicted position. The described motion detection technique through acceleration measurements improves the positioning accuracy as reported in the experimental results in section 5.

V. EXPERIMENTAL RESULTS

In order to evaluate the positioning performance, we used experimental measurements reported in [11]. In particular, a small lego robot, equipped with a UWB device, a ZigBee device and a 3-axis accelerometer sensor, is used as mobile node shown in Fig. 1. A rail, stuck on the floor, was used in order to guide the movement of the robot on a pre-defined path. The robot speed is controlled by using a Lego embedded CPU. In the middle of the rail, a meter tape is attached in order to obtain the actual mobile position and the total distance traveled. Also a video camera is used to record the video of the mobile node motion as shown in Fig. 1.

![Fig. 1. Simulation of hybrid algorithm.](image)

The experiment was performed with a total of 12 UWB anchor nodes and 21 ZigBee anchor nodes that were placed at different locations in an indoor office environment. The UWB anchor nodes perform TOA measurements, while ZigBee anchor nodes perform RSS measurements. The mobile node sends a one-hop broadcast message to all anchor nodes every 50 ms which upon reception by ZigBee anchor nodes, perform RSS measurements and reply immediately to the mobile node along with anchor node ID. The mobile node sends a global RSS message containing responses from anchor nodes to the gateway (GW) node which is connected to a laptop through a serial cable. On the laptop, the information is stored in a log file with time stamp [11]. The UWB measurements are performed at every 500 ms and acceleration at every 2 ms. The estimation period $\Delta t_k$ for the EKF algorithm was set equal to 500 ms. During every estimation period, the mobile robot is in the range of only few UWB and few ZigBee anchor nodes. In Fig. 1, UWB anchor nodes are shown by red mark and ZigBee anchor nodes are shown by green mark.

The performance is evaluated for both constant speed and variable speed scenarios. In the variable speed scenario, the mobile node follows the same pre-defined path with the speed of the mobile node changing along the trajectory. The
Tracking performances are shown in Table I in terms of root mean square error (RMSE). It has been proved from simulation that the performance of the EKF algorithm is better for constant speed scenario compared to the variable speed scenario.

Positioning performance was evaluated also using the acceleration based motion detection approach. The tracking performances using acceleration motion detection approach, are shown in Table II. By comparing it with Tables I, it has been concluded that such motion detection using acceleration measurements further improves the final positioning accuracy. Moreover, as it can be observed from Tables I and II, the position estimation accuracy obtained with the EKF based only on UWB distance measurements is higher than the one achieved by the EKF based only on RSS measurements. The performance is further improved using the hybrid (distance+RSS) EKF algorithm.

### VI. CONCLUSIONS

The experimental results show that the EKF based on TOA based distance measurements provide higher accuracy compared to EKF based on RSS measurements. The performance of the hybrid positioning algorithm using both TOA-based distance and RSS measurements is better than the EKF based only on a single type of range measurement. Notice that at each estimation time, the hybrid algorithm uses more anchor nodes (UWB+ZigBee) than the EKF algorithm based only on either ZigBee or UWB measurements. Thus, positioning accuracy increases with the increase in number of anchor nodes because more range measurements are available at each estimation time. Moreover, the EKF algorithm provides better position estimation accuracy when the mobile node has constant speed along the whole trajectory, compared to the variable speed scenario. Finally, using acceleration measurements to detect the motion of the mobile node leads to a significant improvement in the final localization accuracy.

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