Full-Duplex Spectrum Sharing in Cooperative Single Carrier Systems

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Abstract—We propose cyclic prefix single carrier full-duplex transmission in amplify-and-forward cooperative spectrum sharing networks to achieve multipath diversity and full-duplex spectral efficiency. Integrating full-duplex transmission into cooperative spectrum sharing systems results in two intrinsic problems: 1) the residual loop interference occurs between the transmit and the receive antennas at the secondary relays and 2) the primary users simultaneously suffer interference from the secondary source (SS) and the secondary relays (SRs). Thus, examining the effects of residual loop interference under peak interference power constraint at the primary users and maximum transmit power constraints at the SS and the SRs is a particularly challenging problem in frequency selective fading channels. To do so, we derive and quantitatively compare the lower bounds on the outage probability and the corresponding asymptotic outage probability for max–min relay selection, partial relay selection, and maximum interference relay selection policies in frequency selective fading channels. To facilitate comparison, we provide the corresponding analysis for half-duplex. Our results show two complementary regions, named as the signal-to-noise ratio (SNR) dominant region and the residual loop interference dominant region, where the multipath diversity and spatial diversity can be achievable only in the SNR dominant region, however the diversity gain collapses to zero in the residual loop interference dominant region.

Index Terms—Cooperative transmission, cyclic prefix single carrier transmission, frequency selective fading, full-duplex transmission, residual loop interference, spectrum sharing.

I. INTRODUCTION

COGNITIVE radio (CR) has emerged as a revolutionary approach to ease the spectrum utilization inefficiency [2]. In underlay CR networks, the secondary users (SUs) are permitted to access the spectrum of the primary users (PUs), only when the peak interference power constraint at the PUs is satisfied [3]. One drawback of this approach is the constrained transmit power at the SU, which typically results in unstable transmission and restricted coverage [4], [5]. To overcome this challenge, cognitive relaying was proposed as a solution for reliable communication and coverage extension at the secondary network, and interference reduction at the primary network [6]–[12]. In [6] and [7], the generalized selection combining is proposed for spectrum sharing cooperative relay networks. In [8], the performance of cognitive relaying with max-min relay selection was evaluated. In [12], the partial relay selection was proposed in underlay CR networks.

Full-duplex transmission has been initiated as a new technology for the future Wireless Local Area Network (WLAN) [13], WiFi network [14], and the Full-Duplex Radios for Local Access (DUPLO) projects, which aims at developing new technology and system solutions for future generations of mobile data networks [15], 3GPP Long-Term Evolution (LTE), and Worldwide Interoperability for Microwave Access (WiMAX) systems [16]. Recent advances in radio frequency integrated circuit design and complementary metal oxide semiconductor processing have enabled the suppression of residual loop interference. For example, advanced time-domain interference cancellation [17], physical isolation between antennas [18], and antenna directivity [19] have been proposed in existing works. However, these techniques can not enable perfect isolation [20], [21]. Thus, the residual loop interference is still inevitable and significantly deteriorates the performance. Recent research and development on full-duplex relaying (FDR) without utilizing residual loop interference mitigation has attracted increasing attention, considering that FDR offers high spectral efficiency compared to half-duplex relaying (HDR) by transmitting and receiving signals simultaneously using the same channel [22]–[26]. In [25], FDR was first applied in underlay cognitive relay networks with single PU, the optimal power allocation is studied to minimize the outage probability.

The main objective of this paper is to consider the full-duplex spectrum sharing cooperative system with limited transmit power in the transmitter over frequency...
selective fading environment. We can convert the frequency selective fading channels into flat fading channels via Orthogonal Frequency-Division Multiplexing (OFDM) transmission. However, the peak-to-average power ratio (PAPR) is an intrinsic problem in the OFDM-based system. Also, in general, development of the channel equalizer is a big burden to the receiver of single carrier (SC) transmission [27] in the frequency selective fading channels. Thus, to jointly reduce PAPR and channel equalization burden in the practical system, we consider SC with the cyclic prefix (CP). Single carrier (SC) transmission [27] is currently under consideration for IEEE 802.11ad [28] and LTE [29], owing to the fact that SC can provide lower peak-to-average power ratio and power amplifier back-off [30], [31] compared to Orthogonal Frequency-Division Multiplexing (OFDM). In addition, by adding the cyclic prefix (CP) to the front of the transmission symbol block, the multipath diversity gain can be obtained [32].

Different from the aforementioned works, we introduce FDR and amplify and forward (AF) relay selection in SC spectrum sharing systems to obtain spatial diversity and spectral efficiency. The full-duplex relaying proposed in this paper is a promising approach to prevent capacity degradation due to additional use of time slots, even though additional design innovations are needed before it is used in operational networks. We consider three relay selection policies, namely max-min relay selection (MM), partial relay selection (PS), and maximum interference relay selection (MI), each with a different channel state information (CSI) requirement. We consider a realistic scenario where transmissions from the secondary source (SS) and the selected secondary relay (SR) are conducted simultaneously in the presence of multiple PU receivers. Unlike the cognitive half-duplex relay network (CogHRN), in the cognitive full-duplex relay network (CogFRN) the concurrent reception and transmission entails two intrinsic problems: 1) the peak interference power constraint at the PUs are concurrently inflicted on the transmit power at the SS and the SRs; and 2) the residual loop interference due to signal leakage is introduced between the transmit and the receive antennas at each SR. Against this background, the preeminent objective of this paper is to characterize the feasibility of full-duplex relaying in the presence of residual loop interference by comparing with half-duplex systems. The impact of frequency selectivity in fading channels is another important dimension far from trivial. For purpose of comparison, we provide the corresponding analysis for cooperative CP-SC CogHRN.

Our main contributions are summarized as follows.

1) Taking into account the residual loop interference, we derive new expressions for the probability density function (PDF) and the cumulative distribution function (CDF) of the signal-to-noise ratio (SNR) of the SS to the kth SR link under frequency selective fading channels.

2) We then derive the expressions for the lower bound on the outage probability. We establish that outage probability floors occur in the residual loop interference dominant region with high SNRs for all the policies in CogFDR. We show that irrespective of the SNR, the MM policy outperforms the PS and the MI policies. We also show that the PS policy outperforms the MI policy.

3) To understand the impact of the system parameters, we derive the asymptotic outage probability and characterize the diversity gain. For FDR, in the residual loop interference dominant region, we see that the asymptotic diversity gain is zero regardless of the spatial diversity might be offered by the relay selection policy, and the multipath diversity might be offered by the single carrier system. However, the full diversity gain of HDR is achievable.

4) We verify our new expressions for lower bound on the outage probabilities and their corresponding asymptotic diversity gains via simulations. We showcase the impact of the number of SRs and the number of PUs on the outage probability. We conclude that the outage probability of CogFDR decreases with increasing number of SRs, and increases with increasing the number of PUs. Interestingly, we notice that the outage probability of CogFDR decreases as the ratio of the maximum transmit power constraint at the SR to the maximum transmit power at the SS decreases.

5) We compare the outage performance between CogHDR with the target data rate 2RT and CogFDR with the target data rate RT, considering that the SS and the SRs transmit using two different channels in CogHDR, while the transmission in CogFDR only require one channel. We conclude that CogFDR is a good solution for the systems that operate at low to medium SNRs, while CogHDR is more favorable to those operate in the high SNRs.

The rest of the paper is organized as follows. In Section II, we present the system and the channel model for cooperative CP-SC CogFRN and cooperative CP-SC CogHRN with AF relaying. Distributions of the SNRs are derived in Section III. The asymptotic description is given in Section IV. The outage probability and the corresponding asymptotic outage probability of CogFRN and CogHRN with several relay selection policies are derived in Sections V and VI, respectively. Simulation results are provided in Section VII. Conclusions are drawn in Section VIII.

**Notations:** The superscript $\cdot^H$ denotes complex conjugate transposition, $E(\cdot)$ denotes expectation, and $CN(\mu, \sigma^2)$ denotes the complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$. The $\mathbb{E}_\varphi(\cdot)$ and $f_\varphi(\cdot)$ denote the CDF of the random variable (RV) $\varphi$ for FDR and HDR, respectively. Also, $\bar{f}_\varphi(\cdot)$ and $f_\varphi(\cdot)$ denote the PDF of $\varphi$ for FDR and HDR, respectively. The binomial coefficient is denoted by $\binom{n}{k} \triangleq \frac{n!}{(n-k)! k!}$.

**II. SYSTEM AND CHANNEL MODEL**

We consider a cooperative spectrum sharing network consisting of $L$ PU-receivers ($PU_1, \ldots, PU_L$), a single SS, a single secondary destination (SD), and a cluster of $K$ SRs ($SR_1, \ldots, SR_K$) as shown in Fig. 1, where the solid and the dashed lines represent the secondary channel and the interference channel, respectively. The CP-SC transmission is used in this network. Among the $K$ SRs, the best SR which fulfills
the relay selection criterion is selected to forward the transmission to the SD using the AF relaying protocol. Similar to the model used in [8], [33], and [34], we focus on the coexistence of long-range primary system such as IEEE 802.22, and short range CR networks, such as WLANs, D2D networks and sensor networks. In this case, the primary to secondary link is severely attenuated to neglect the interference from the PU transmitters to the SU receivers. We also assume there is no direct link between the SS and the SRs due to long distance and deep fades. In this network, we make the following assumptions for the channel models, which are practically valid in cooperative spectrum sharing networks.

Assumption 1: For the secondary channel, the instantaneous sets of channel impulse responses (CIRs) from the SS to the $k$th SR and from the $k$th SR to the SD composing of $N_{1,k}$ and $N_{2,k}$ multipath channels, are denoted as $\mathbf{g}_{N_{1,k}}^{s,k} = \left[ g_{0}^{s,k}, \ldots, g_{N_{1,k}-1}^{s,k} \right]^T \in \mathbb{C}^{N_{1,k} \times 1}$ and $\mathbf{g}_{N_{2,k}}^{k,d} = \left[ g_{0}^{k,d}, \ldots, g_{N_{2,k}-1}^{k,d} \right]^T \in \mathbb{C}^{N_{2,k} \times 1}$, respectively. For the primary channel, we assume perfect CSI from the SS to the $k$th PU link and from the $k$th SR to the $k$th PU link, which can be obtained through direct feedback from the PU [35], indirect feedback from a third party, and periodic sensing of pilot signal from the PU [36]. The instantaneous sets of CIRs from the SS to the $k$th PU (PU$_h$) and from the $k$th SR to the $k$th PU composing of $N_{3,l}$ and $N_{4,k,l}$ multipath channels, are denoted as $\mathbf{f}_{N_{3,l}}^{s,l} = \left[ f_{0}^{s,l}, \ldots, f_{N_{3,l}-1}^{s,l} \right]^T \in \mathbb{C}^{N_{3,l} \times 1}$ and $\mathbf{f}_{N_{4,k,l}}^{l,l} = \left[ f_{0}^{l,l}, \ldots, f_{N_{4,k,l}-1}^{l,l} \right]^T \in \mathbb{C}^{N_{4,k,l} \times 1}$, respectively. All channels are composed of independent and identically distributed (i.i.d.) complex Gaussian RVs with zero means and unit variances. The maximum channel length $N_{\text{max}} = \max\{N_{1,k}, N_{2,k}, N_{3,l}, N_{4,k,l}\}$ is assumed to be shorter than the CP length, denoted by $N_{\text{CP}}$, to restrain the interblock symbol interference (IBSI) and intersymbol interference (ISI) in single carrier transmission [31]. Accordingly, the path loss components from the SS to the $k$th SR, from the $k$th SR to the SD, from the SS to the PU$_h$, and from the $k$th SR to the PU$_l$ are defined as $\alpha_{1,k}, \alpha_{2,k}, \alpha_{3,l},$ and $\alpha_{4,k,l}$, respectively.

Assumption 2: For underlay spectrum sharing, the peak interference power constraint at the $k$th PU is denoted as $I_{\text{th}}$.

Also due to hardware limitations, the transmit power at the SS and the SRs are restricted by the maximum transmit power constraints $P_T$ and $P_R$, respectively.

A. CogFRN

In the full-duplex mode, each SR is equipped with a single transmit and a single receive antenna, which enable full-duplex transmission in the same frequency band at the expense of introducing residual loop interference. The SS and the SR transmit to the SD in the same time slot. As such, the PUs suffer interference from the SS and the SRs concurrently. Similar as [25], we simply assume that the maximum interference inflicted on the PUs by the SS or the SRs are set to be a half of the total peak interference power constraint at the PUs ($\frac{I_{\text{th}}}{2} = Q$), where $Q$ is the peak interference constraint. Therefore, the transmit power at the SS and the $k$th SR are given by

$$P_{S}^{F} = \min \left( \frac{Q}{Y_1}, P_T \right),$$

$$P_{R,k}^{F} = \min \left( \frac{Q}{Y_k}, P_R \right),$$

where

$$Y_1 \triangleq \max_{l=1,\ldots,k} \left\{ \alpha_{3,l} \| \mathbf{f}_{N_{3,l}}^{l,l} \|_2^2 \right\},$$

and

$$Y_k \triangleq \max_{l=1,\ldots,L} \left\{ \alpha_{4,k,l} \| \mathbf{f}_{N_{4,k,l}}^{l,l} \|_2^2 \right\}.$$
relay is modeled as a complex Gaussian random variable with zero mean and variance \( \sigma_n^2 \), i.e., \( n_{x,k} \sim \mathcal{C}\mathcal{N}(0, \sigma_n^2) \).

In AF relaying, the SRs are unable to distinguish between the signal from the SS and the residual loop interference signals at the SRs. Thus, both signals are amplified and forwarded to the SD. The received signal at the SD via the \( k \)th SR is given by

\[
y_{r,d} = \sqrt{\alpha_{x,k}} G_{N,k}^{k,k} G_k y_k + n_{r,d},
\]

where \( G_{N,k}^{k,k} \) is the right circulant matrix formed by \( \{g_{N,k}^{k,k} \}^T, \{0_{1 \times (N_S - N_k)} \} \}^T \in \mathbb{C}^{N \times 1} \). The relay gain matrix for the \( k \)th SR is denoted by \( G_k = \mathbb{E}[N_k] \). The relay gain \( g_k \) is given by

\[
g_k = \left[ \frac{P_R}{\sigma_n^2} \right]^{1/2} \left[ \frac{\alpha_{x,k}}{\|G_{N,k}^{k,k}\|^2} + \frac{P_R}{\sigma_n^2} \right],
\]

where \( h_k = \{h_{k,n} N_{n=1} \} \).

Inserting (5) and (7) into (6), the end-to-end SINR (e2e-SINR) at the SD is derived as

\[
\gamma_{e2e,k}^{k,k} = \frac{\gamma_{e2e,k}^{k,d}}{\gamma_{e2e,k}^{k,d} + \gamma_{e2e,k}^{k,d}} \leq \min \left( \frac{\sigma_{e2e,k}^{k,k}}{\gamma_{e2e,k}^{k,k}}, \gamma_{e2e,k}^{k,d} \right),
\]

where \( \sigma_{e2e,k}^{k,k} \) denotes the SNR from the SS to the \( k \)th SR as \( \gamma_{e2e,k}^{k,k} = \gamma_{F,k} W_k \), and the INR at the \( k \)th SR as \( \gamma_{e2e,k}^{k,d} = \gamma_{F,k} R_k \). Note that \( X_k \sim \alpha_{x,k} \|G_{N,k}^{k,k}\|^2, W_k \sim \alpha_2 \|G_{N,k}^{k,k}\|^2, R_k \sim |h_k|^2, \gamma_{e2e,k}^{k,k} \sim \frac{P_R}{\sigma_n^2}, \) and \( \gamma_{e2e,k}^{k,d} \sim \frac{P_R}{\sigma_n^2} \).

### B. CogHRR

In the half-duplex mode, the SS and the SRs transmit signals in different channels and time slots. The maximum interference imposed on the PUs by the SS or the SR is equal to the peak interference power constraint \( I_{th} = 2Q \) at the PUs. As such, the transmit power at the SS and the \( k \)th SR in CogHRR is given by

\[
P_S^H = \min \left( \frac{Q}{Y_1}, \frac{P_F}{Y_k} \right),
\]

\[
P_{R,k}^H = \min \left( \frac{Q}{Y_k}, \frac{P_F}{Y_k} \right),
\]

respectively. With AF relaying, the received signals at the \( k \)th SR and at the SD via the \( k \)th SR are given by

\[
y_{r,k} = \sqrt{P_S^H} \alpha_{x,k} G_{N,k}^{k,k} x_k + n_{r,k},
\]

\[
y_{r,d} = \sqrt{\alpha_{x,k}} G_{N,k}^{k,k} G_k y_k + n_{r,d},
\]

The delay is not taken into account in our model, and thus our results give the achievable minimum outage probability. Note that the delay can be mitigated in practical scenario by using the self interference cancellation technique proposed in [37].

III. DISTRIBUTIONS OF SNR AND SINR

In this section, we first derive the CDFs and PDFs of the \( Y_1 \) and \( Y_k \) based on the Definition 1 and Definition 2 in the following. We then utilize these CDFs and PDFs to facilitate the derivations of CDFs of \( \gamma_{e2e,k}^{k,k} \) and \( \gamma_{e2e,k}^{k,d} \).

**Definition 1:** The PDF and the CDF of a RV \( X \) distributed as a gamma distribution with shape \( N \) and scale \( \alpha \) are given, respectively, as

\[
f_X(x) = \frac{1}{\Gamma(N)\alpha^N} x^{N-1} e^{-x/\alpha} U(x),
\]

\[
F_X(x) = \left( 1 - e^{-x/\alpha} \right) \sum_{i=0}^{N-1} \frac{1}{\alpha^i} U(x),
\]

where \( U(\cdot) \) denotes the discrete unit step function. In the sequel, a RV \( X \) distributed according to a gamma distribution with shape \( N \) and scale \( \alpha \) is denoted by \( X \sim \text{Ga}(N, \alpha) \). Here, shape \( N \) is a positive integer.

**Definition 2:** Let \( X_i \sim \text{Ga}(N_i, 1) \), then the PDF and the CDF of a RV \( X_{\max} = \max \{X_1, X_2, \ldots, X_{N_L} \} \) are given, respectively, as

\[
f_{X_{\max}}(x) = 1 + \sum_{L_{ij}, \{N_i\}, \{a_i\}} \left[ x^j e^{-bx} U(x) \right],
\]

\[
F_{X_{\max}}(x) = \sum_{L_{ij}, \{N_i\}, \{a_i\}} e^{-bx} U(x - bx) U(x),
\]

where

\[
\sum_{L_{ij}, \{N_i\}, \{a_i\}} \left[ \frac{l!}{j!} \right] = \sum_{l=1}^{L} \frac{(-1)^j}{j!} \sum_{n_1=1}^{N_1-1} \cdots \sum_{n_L=1}^{N_L-1} \sum_{j_1=0}^{N_1-1} \cdots \sum_{j_L=0}^{N_L-1} \frac{1}{j_1! \cdots j_L!} \cdots [n_1 \cup n_2 \cup \ldots \cup n_L],
\]

and \( j \) denotes the dimension of the union of \( L \) indices \( \{n_1, \ldots, n_L\} \).

Note that the magnitudes of the four channel vectors \( \|g_{N,k}^{k,k}\|^2, \|g_{N,k}^{k,d}\|^2, \|f_{N,k}^{j}\|^2, \) and \( \|f_{N,k}^{j}\|^2 \) are distributed as...
gamma distributions with shapes $N_{1,k}, N_{2,k}, N_{3,l},$ and $N_{4,k,l},$ respectively, and scale 1. Also, $|h_k|^2$ is distributed as a gamma distribution with shape 1 and scale 1. We have also defined the two RVs $X_k \sim \alpha_{1,k} \|\mathbf{x}_k\|^2 \sim \text{Ga}(N_{1,k}, \alpha_{1,k})$ and $Y_1 \sim \max_i \{ \alpha_{3,l} \|\mathbf{x}_l^i\|^2 \}$. For notational purposes, in the sequel, we have defined the normalized powers $\gamma_1 \triangleq \gamma \gamma_T = \frac{\gamma}{\gamma_T},$ and $\gamma_T = \frac{P_T \gamma_T}{\gamma_T},$ with $\gamma_T \triangleq \frac{\gamma \gamma_T}{\gamma_T}$. According to the distribution of $\|\mathbf{x}_k\|^2$, the CDF and the PDF of $Y_1$ are given by

$$F_{Y_1}(x) = 1 + \sum_{L_{jk},\{N_{i,j}\},\{\alpha_{3,l}\}} x^\gamma \frac{\beta_1}{\gamma_T} F_{U}(x),$$

and $f_{Y_1}(x) = \sum_{L_{jk},\{N_{i,j}\},\{\alpha_{3,l}\}} e^{\gamma_1 x} \left[ x^\gamma \frac{\beta_1}{\gamma_T} F_{U}(x) - \beta_1 x \gamma \frac{\beta_1}{\gamma_T} F_{U}(x) \right].$

where $\gamma_1 \triangleq \sum_{l=1}^{L} \gamma_1$ and $\beta_1 \triangleq \sum_{l=1}^{L} \frac{1}{\alpha_{3,l}}$.

A. CogFRN

From the definition of the SNR from the SS to the kth SR $\gamma_{F,k} \triangleq \min(\gamma_1, \gamma_T) X_k \gamma_T$, we have the following CDF of $\gamma_{F,k}$ as

$$F_{\gamma_{F,k}}(\gamma) = 1 - e^{-\gamma_{F,k}/\gamma_T} \sum_{i=0}^{N_{1,k}-1} \frac{1}{i!} \left( \frac{\gamma}{\gamma_T} \right)^i \frac{1}{\alpha_{1,k}} \frac{\beta_1}{\gamma_T} F_{U}(x),$$

where $\gamma_T \triangleq \gamma_T$ and $\Gamma(\cdot, \cdot)$ denotes the incomplete gamma function.

Proof: See Appendix A.

B. CogHRN

In cooperative CP-SC CogHRN, we have $\gamma_{H,k} \triangleq \min(2\gamma_1, \gamma_T) X_k \gamma_T$. We derive the CDF of $\gamma_{H,k}$ as

$$F_{\gamma_{H,k}}(\gamma) = 1 - e^{-\gamma_{H,k}/\gamma_T} \sum_{i=0}^{N_{1,k}-1} \frac{1}{i!} \left( \frac{\gamma}{\gamma_T} \right)^i \frac{1}{\alpha_{1,k}} \frac{\beta_1}{\gamma_T} F_{U}(x).$$

Next, $\gamma_{H,k}$ is written as $\gamma_{H,k} \triangleq \min(2\gamma_1, \gamma_T) X_k \gamma_T$. We derive the CDF of $\gamma_{H,k}$ as

$$F_{\gamma_{H,k}}(\gamma) = 1 - e^{-\gamma_{H,k}/\gamma_T} \sum_{i=0}^{N_{1,k}-1} \frac{1}{i!} \left( \frac{\gamma}{\gamma_T} \right)^i \frac{1}{\alpha_{1,k}} \frac{\beta_1}{\gamma_T} F_{U}(x).$$

IV. ASYMPTOTIC DESCRIPTION

In this section, we assume $N_1 = N_{1,k}, N_2 = N_{2,k}, N_3 = N_{3,k}, N_4 = N_{4,k,l}$ and $\alpha_1 = \alpha_{1,k}, \alpha_2 = \alpha_{2,k}, \alpha_3 = \alpha_{3,k}, \alpha_4 = \alpha_{4,k,l}$. To examine the effect of power scaling on the outage probability, we have also defined $\gamma_T \rightarrow \infty$. When $\gamma_T \rightarrow \infty$, we can easily observe $\gamma_T \rightarrow \infty$ and $\gamma_Q \rightarrow \infty$. This will benefit the secondary network without violating the transmission of the primary network [8].

A. CogFRN

To derive the asymptotic results, (8) is simplified to one term for high SNRs. Since the second order term is dominating compared with the linear terms (i.e., $E[\gamma_{F,k}] E[\gamma_{F,k}] \gg E[\gamma_{F,k}] + E[\gamma_{F,k}] + E[\gamma_{F,k}]$), at high SNRs, we can obtain an approximate e2e-SNR expression as

$$\gamma_{F,k}^{\text{e2e}} \approx \frac{\gamma_{Q,k}^{\text{e2e}} \gamma_{F,k}^{\text{e2e}}}{\gamma_{F,k}^{\text{e2e}}} = \frac{\gamma_{F,k}^{\text{e2e}}}{\gamma_{F,k}^{\text{e2e}}}.\gamma_{F,k}$$

We see that the high e2e-SNR is only determined by the first hop and residual loop interference, and is independent of the second hop. By eliminating $\gamma_T$ in (23), we derive the new expressions $\gamma_{F,k}^{\text{e2e}} = \min(\gamma_T, 1) X_k$, and $\gamma_{F,k}^{\text{e2e}} = \min(\gamma_T, 1) \rho R_k$. To derive the closed-form expression for $\gamma_{F,k}^{\text{e2e}}$, we first derive the closed-form expressions for $\gamma_{F,k}^{\text{e2e}}$ and $\gamma_{F,k}^{\text{e2e}}$.

1) Asymptotic SNR From the SS to the kth SR: From the definition of $\gamma_{F,k}^{\text{e2e}} = \min(\gamma_T, 1) X_k$, we have the following asymptotic CDF of $\gamma_{F,k}^{\text{e2e}}$ as

$$F^{\infty}_{\gamma_{F,k}^{\text{e2e}}}(\gamma) = 1 - e^{-\gamma_{F,k}^{\text{e2e}}/\gamma_T} \sum_{i=0}^{N_{1,k}-1} \frac{1}{i!} \left( \frac{\gamma}{\gamma_T} \right)^i \frac{1}{\alpha_{1,k}} \frac{\beta_1}{\gamma_T} F_{U}(x).$$

2) Asymptotic INR at the kth SR: From the definition of $\gamma_{F,k}^{\text{e2e}} = \min(\gamma_T, 1) \rho R_k$, we have the following
asymptotic CDF of $\gamma_p^{k,l}$ as
\[
\mathbb{P}^{\infty}_{\gamma_p^{k,l}}(\gamma) = 1 - e^{-\frac{\gamma}{\gamma_p^{k,l}}} - \frac{\gamma}{\mu_T} \sum_{L, d, [N_1], [d_3]} \Gamma(\tilde{d} + 1, \frac{\mu_T}{\mu_T + \tilde{\beta}_2}) \frac{\mu_T}{\mu_T + \tilde{\beta}_2}^{\tilde{d} + 1}.
\]
\[
(25)
\]

The derivation of (24) and (25) are similar to those provided in Appendix A.

B. CogHRN

Different from the approach used in deriving the asymptotic e2e-SINR of CogFRN, in CogHRN, we use the first order expansion for the CDFs of $\gamma_H^{k,\bar{N}}$ and $\gamma_H^{k,d}$ to derive the asymptotic e2e-SINR of CogHRN.

1) Asymptotic SNR From the SS to the kth SR: When $\bar{Y}_T \to \infty$ and $\bar{Y}_Q \to \infty$, an asymptotic expression of $F_{X_i}(\gamma / \bar{Y}_T)$ is derived by applying [38, eq. (1.211.1)] and [38, eq. (3.354.1)]
\[
F_{X_i}(\gamma / \bar{Y}_T) \approx \frac{1}{\Gamma(N_1 + 1)} \left( \frac{\gamma}{\alpha_1 \bar{Y}_T} \right)^{N_1}.
\]
\[
(26)
\]
The asymptotic CDF of $\gamma_H^{S,k}$ is derived as
\[
F_{\gamma_H^{S,k}}(\gamma) = \frac{1}{\Gamma(N_1 + 1)} \left( \frac{\gamma}{\alpha_1 \bar{Y}_T} \right)^{N_1} \left( 1 - e^{-\frac{2\mu_T}{\alpha_3}} \sum_{j=0}^{N_1-1} \left( \frac{2\mu_T}{\alpha_3} \right)^j \right)^L
+ \frac{1}{\Gamma(N_1 + 1)} \left( \frac{\gamma}{2\alpha_1 \bar{Y}_Q} \right)^{N_1} \left( \frac{\beta_1}{\alpha_3} \right)^{(N_1 + \tilde{j})} \sum_{L, d, [N_1], [d_3], [d_3]} \left[ \tilde{j} \Gamma(N_1 + \tilde{j}, 2\mu_T \beta_1) - \Gamma(N_1 + \tilde{j} + 1, 2\mu_T \beta_1) \right].
\]
\[
(27)
\]
2) Asymptotic SNR From the kth SR to the SD: When $\bar{Y}_R \to \infty$ and $\bar{Y}_Q \to \infty$, the asymptotic CDF of $\gamma_H^{k,d}$ is derived as
\[
F_{\gamma_H^{k,d}}(\gamma) = \frac{1}{\Gamma(N_1 + 1)} \left( \frac{\gamma}{\alpha_2 \bar{Y}_R} \right)^{N_2} \left( 1 - e^{-\frac{2\mu_R}{\alpha_4}} \sum_{j=0}^{N_2-1} \left( \frac{2\mu_R}{\alpha_4} \right)^j \right)^L
+ \frac{1}{\Gamma(N_1 + 1)} \left( \frac{\gamma}{2\alpha_2 \bar{Y}_Q} \right)^{N_2} \left( \tilde{\beta}_2 \right)^{(N_2 + \tilde{d})} \sum_{L, d, [N_1], [d_3]} \left[ \tilde{d} \Gamma(N_2 + \tilde{d}, 2\mu_R \beta_2) - \Gamma(N_2 + \tilde{d} + 1, 2\mu_R \beta_2) \right].
\]
\[
(28)
\]
Having (27) and (28) for the CDFs of $\gamma_H^{S,k}$ and $\gamma_H^{k,d}$ in closed-form, respectively, we derive the lower bound on the outage probability of CogHRN in Section VI.

V. OUTAGE PROBABILITY OF CogFRN

In this section, we derive the expression for the lower bound on the outage probabilities of CogFRN with various relay selection policies based on the max-min criterion, partial relay selection criterion, and maximum interference criterion. We then derive the corresponding asymptotic outage probabilities to observe the diversity gains of the three selection policies.

A. CogFRN With MM

Compared with the conventional MM policy in CogHRN, the MM policy in CogFRN takes into account the loop interference. Let $k_{MM}$ be the selected relay based on the max-min criterion. The employed relay selection is mathematically given by
\[
k_{MM} = \arg\max_{k=1,\ldots,K} \min \left( \frac{\gamma_F^{S,k}}{\gamma_F^{T,k} + 1}, \frac{\gamma_F^{k,d}}{\gamma_F^{k,d} + 1} \right).
\]
\[
(29)
\]
1) Outage Probability: The lower bound on the outage probability of CogFRN at a given threshold $\eta_F$ is given by
\[
\mathbb{P}_{\text{out}}^{\text{MM}}(\eta_F) = \prod_{k=1}^{K} \int_{0}^{\infty} \left( 1 - \left( 1 - F_{\gamma_F^{T,k}}(\eta_F) \right) \left( 1 - F_{\gamma_F^{k,d}}(\eta_F) \right) \right) dy.
\]
\[
(30)
\]
\[\text{Theorem 1: The lower bound on the outage probability of CogFRN with MM policy is derived as}
\]
\[
\mathbb{P}_{\text{out}}^{\text{MM}}(\eta_F) = \int_{\mu_R}^{\infty} \left\{ 1 - \left[ \frac{\eta_F}{\gamma_R} \sum_{i=0}^{N_1+k-1} \Pi_1(i, t) \Gamma(t+1) \right] \times \left( \frac{\eta_F}{\alpha_1 \bar{Y}_T} + \frac{\gamma}{\bar{Y}_Q} \right)^{-i-1} + \frac{\gamma}{\bar{Y}_Q} \sum_{m=0}^{N_2+k-1} \sum_{n=0}^{N_1+k-1} \Pi_2(n, m, h) \Pi_3(h, \frac{\eta_F}{\alpha_1 \bar{Y}_T} + \frac{\gamma}{\bar{Y}_Q}) \times \frac{\Gamma(N_2+k, \frac{\eta_F}{\alpha_2 \bar{Y}_Q} \gamma) \bar{Y}_Q}{\Gamma(N_2, k)} \right] \sum_{L, d, [N_1], [d_3]} \left[ \tilde{d} \Gamma(\tilde{d}, 2\mu_R \beta_2) - \Gamma(\tilde{d} + 1, 2\mu_R \beta_2) \right]} \right. dy
+ \left. \left[ 1 - \left[ \frac{1}{\gamma_R} \sum_{i=0}^{N_1+k-1} \sum_{t=0}^{N_1+k-1} \Pi_1(i, t) \frac{\eta_F}{\alpha_1 \bar{Y}_T} + \frac{1}{\gamma_R} \right] \times \Gamma(t+1) + \frac{1}{\gamma_R} \sum_{m=0}^{N_2+k-1} \sum_{n=0}^{N_1+k-1} \Pi_2(n, m, h) \Pi_3(h, \frac{\eta_F}{\alpha_1 \bar{Y}_T} + \frac{1}{\gamma_R}) \times \frac{\Gamma(N_2+k, \frac{\eta_F}{\alpha_2 \bar{Y}_Q} \gamma) \bar{Y}_Q}{\Gamma(N_2, k)} \right] \sum_{L, d, [N_1], [d_3]} \left[ \tilde{d} \Gamma(\tilde{d}, 2\mu_R \beta_2) - \Gamma(\tilde{d} + 1, 2\mu_R \beta_2) \right] \right],
\]
\[
(31)
\]
where
\[
\Pi_1(i, t) = \frac{1}{i!} \left( \frac{\eta_F}{\alpha_1 \bar{Y}_T} \right)^i \frac{1}{t} e^{-\frac{\eta_F}{\alpha_1 \bar{Y}_T}},
\]
\[
(32)
\]
\[ \Pi_2(m, n, h) = \frac{(n_F/\gamma_Q)^{N_{1,k}}}{(a_{1,k})^{N_{1,k}}} \left( \frac{N_{1,k} + \tilde{j} - 1}{1} \right)! \left( \frac{1}{m!} \right)^{m^2} \left\{ \begin{array}{l} \sum_{n=0}^{m} n! \beta_1^n \left( \frac{n_F}{a_{1,k} \gamma_Q} \right)^n \times h^{\tilde{j} + N_{1,k} - \tilde{j} - 1} \Gamma(h + 1) \times \psi \left( h + 1, h + 2 - N_{1,k} \right) \times \left( \frac{\eta_F}{a_{1,k} \gamma_Q} + \tilde{\beta}_1 \right) \left( \frac{1}{\eta_F} \right) \end{array} \right. \]

\[ \Pi_3(h, \xi) = \left( \frac{\eta_F}{a_{1,k} \gamma_Q} + \tilde{\beta}_1 \right) \left( \frac{\alpha_{1,k} \gamma_Q}{\eta_F} \right)^{h+1} \Gamma \left( h + 1 \right) \times \psi \left( h + 1, h + 2 - N_{1,k} \right) \times \left( \frac{\eta_F}{\alpha_{1,k} \gamma_Q} + \tilde{\beta}_1 \right) \left( \frac{1}{\eta_F} \right). \]

**Proof:** See Appendix B.

Note that our derived outage probability with the MM policy is valid for different types of SRs and PUs having arbitrary channel lengths and path loss components.

2) **Asymptotic Outage Probability:** Based on (23), the asymptotic outage probability can be written as

\[ \mathbb{P}_{\text{MM}}^\infty,\text{out}(\eta_F) = \left( \mathbb{F}_{\gamma_{\text{MM}}}^\infty (\eta_F) \right)^K. \]  

Having (24) and (25), we derive the asymptotic CDF of \( \gamma_{\text{MM}}^K \) as

\[ \mathbb{F}_{\gamma_{\text{MM}}}^\infty (\gamma) = \int_0^\infty \mathbb{F}_{\gamma_{\text{MM}}}^\infty (\gamma | x) \mathbb{F}_{\gamma_{\text{MM}}}^\infty (x) dx \]

\[ = 1 - e^{-n_{\text{PS}}^{-1} y_{\text{PS}}} \sum_{i=0}^{n_{\text{PS}}-1} \left( \frac{\gamma y_{\text{PS}}}{\alpha_{1}} \right)^i \mathbb{F}_{\gamma_{\text{MM}}}^\infty (y_{\text{PS}}) \left( \frac{\gamma y_{\text{PS}}}{\alpha_{1}} \right)^i \mathbb{F}_{\gamma_{\text{MM}}}^\infty (x) dx \]

\[ \times \sum_{L_{d,c} \in \{ | \alpha_{1} | \}} \left[ \frac{\gamma y_{\text{PS}}}{\alpha_{1}} \left( \frac{\gamma y_{\text{PS}}}{\alpha_{1}} + \tilde{\beta}_1 \right) \right]^{N_{1,k} + \tilde{j}} \times \mathbb{F}_{\gamma_{\text{MM}}}^\infty (x) dx = 1 - \mathbb{P}_1 - \mathbb{P}_2, \]

where the two terms \( \mathbb{P}_1 \) and \( \mathbb{P}_2 \) are derived in Appendix C. Substituting the derived closed-form expression of \( \mathbb{F}_{\gamma_{\text{MM}}}^\infty (\gamma) \) in (36) at a given \( \eta_F \) into (35), we obtain the asymptotic outage probability with MM policy. Since \( \mathbb{P}_{\text{MM}}^\infty (\eta_F) \) is independent of \( \gamma_T, \gamma_R, \) and \( \gamma_Q \) (as shown in (24) and (25)), which are independent of \( \gamma_T, \gamma_R, \) and \( \gamma_Q \) (as shown in (24) and (25)), the diversity gain drops to zero regardless of the spatial diversity and multipath diversity in the high SNR regime.

**B. CogFRN With PS**

In this policy, partial CSI is required, the SR which has the maximum SNR from the SS to the 4th SR is selected. Thus, the index of the selected relay is denoted as

\[ k_{\text{PS}} = \arg_{k=1, \ldots, K} \max \left( \gamma_{\text{PS}}^{k}, \frac{\eta_F}{\gamma_T} \right). \]

To see the diversity gain of the outage probability, in the rest of this section we have assumed that \( N_1 = N_{1,k}, N_2 = N_{2,k}, N_3 = N_{3,k}, N_4 = N_{4,k} \) and \( \alpha_1 = \alpha_{1,k}, \alpha_2 = \alpha_{2,k}, \alpha_3 = \alpha_{3,k}, \alpha_4 = \alpha_{4,k} \). As such, we have the same distribution for each SR to the SD link, that is, \( \mathbb{P}_{\gamma_{\text{PS}}}^\infty (\eta_F) = \mathbb{P}_{\gamma_{\text{PS}}}^\infty (\eta_F) \) at a given \( \eta_F \).

1) **Outage Probability:** The lower bound on the outage probability is evaluated as

\[ \mathbb{P}_{\text{PS}}^\infty (\eta_F) = \int_0^\infty \mathbb{P}_{\text{PS}}(\eta_F) \left( 1 - F_{\gamma_{\text{PS}}} (\eta_F) \right) \left( 1 - F_{\gamma_{\text{PS}}} (\eta_F) \right) dy, \]

where \( \sigma_{\text{PS}}^2 = \max_{x=1, \ldots, K} \left( \gamma_{\text{PS}}^{k} \right) \).

**Theorem 2:** The lower bound on the outage probability of CogFRN with PS policy is derived as

\[ \mathbb{P}_{\text{PS}}^\infty (\eta_F) = \int_0^\infty \left( 1 - \int_0^\infty \frac{y}{\gamma_Q} e^{-\frac{\eta_F}{\gamma_Q}} \left[ 1 - e^{-\frac{\eta_F}{\gamma_Q}} \right] \right) dy \]

\[ \times \sum_{L_{d,c} \in \{ | \alpha_{1} | \}} \left( \frac{\gamma y_{\text{PS}}}{\alpha_{1}} \right)^{N_{1,k} + \tilde{j}} \times \mathbb{F}_{\gamma_{\text{MM}}}^\infty (x) dx \]

\[ \Gamma \left( \frac{N_{2,k}}{n} \right) \Gamma \left( \frac{N_{2,k}}{\gamma_{\text{PS}}} \right) \times \sum_{L_{d,c} \in \{ | \alpha_{1} | \}} \left( \frac{\gamma y_{\text{PS}}}{\alpha_{1}} \right)^{N_{1,k} + \tilde{j}} \times \mathbb{F}_{\gamma_{\text{MM}}}^\infty (x) dx \]

\[ \left( \frac{\gamma y_{\text{PS}}}{\alpha_{1}} \right)^{N_{1,k} + \tilde{j}} \times \mathbb{F}_{\gamma_{\text{MM}}}^\infty (x) dx \]

where \( \Pi_1(i, t), \Pi_2(m, n, h), \) and \( \Pi_3(h, \xi) \) are given in (32), (33), and (34), respectively.

**Proof:** See Appendix D.

2) **Asymptotic Outage Probability:** The asymptotic outage probability with PS policy is given as

\[ \mathbb{P}_{\text{PS}}^\infty (\eta_F) = \int_0^\infty \mathbb{F}_{\gamma_{\text{PS}}}^\infty (\gamma | x) \mathbb{E}_{\gamma_{\text{PS}}}^\infty (x) dx. \]

Having (24) and (25), we derive the asymptotic outage probability. The asymptotic diversity gain with PS policy is zero.
The e2e-SINR expression of CogFRN with the MI policy is derived as (42) at the top of the page. In this policy, the relay with the maximum e2e-SNR is selected in order to achieve the minimum loop interference, thus the index of the selected relay is given as

\[ k_{\text{MI}} = \arg_{k=1, \ldots, K} \max(Y_k). \] (41)

1) Outage Probability:

**Theorem 3:** The lower bound on the outage probability of CogFRN with MI policy is derived as (42) at the top of the page.

In (42), \( \Pi_1(i, t) \), \( \Pi_2(m, n, h) \), and \( \Pi_3(h, \xi) \) are given in (32), (33), and (34), respectively.

**Proof:** See Appendix E.

2) Asymptotic Outage Probability: In the high SNR regime, the e2e-SINR expression of CogFRN with the MI policy becomes

\[ \gamma_{\text{Fr2p}}^{\text{MI}} \approx \gamma_{\text{Fr2p}}^{\text{MI,T}}. \] (43)

Where \( \gamma_{\text{Fr2p}}^{\text{MI}} = \min(\frac{\eta_F}{\alpha_{1,k}^2}, 1)X_k \), \( \gamma_{\text{Fr2p}}^{\text{MI,T}} = \min(\frac{\mu_T}{\max_{k \in \mathcal{K}T}, \rho} R_k). \)

With the derived CDF of \( \gamma_{\text{Fr2p}}^{\text{MI}} \) in (24) and the PDF of \( \gamma_{\text{Fr2p}}^{\text{MI,T}} \) as

\[
\begin{align*}
& f_{\gamma_{\text{Fr2p}}^{\text{MI,T}}}(x) = \frac{x}{\mu_T^2} \int_{\frac{x}{\mu_T}}^{\infty} y (1 + \sum_i \sum_j e^{-\beta_{2j}y})^K e^{-\frac{\gamma}{\alpha_{1,k}^2}y} dy \\
& - \frac{1}{\mu_T^2} \int_{\frac{x}{\mu_T}}^{\infty} y (1 + \sum_j \sum_i e^{-\beta_{2j}y})^K e^{-\frac{\gamma}{\alpha_{1,k}^2}y} dy.
\end{align*}
\] (44)

and we substitute them into

\[ \Pr_{\text{MI}}^{\infty, \text{out}}(\eta_F) = \int_0^{\infty} \Pr_{\gamma_{\text{Fr2p}}}^{\infty, \text{out}}(\eta_F) \Pr_{\gamma_{\text{Fr2p}}}^{\infty, \text{out}}(x) dx, \] (45)

we derive the asymptotic outage probability with MI policy. In CogFRN, the diversity gain of the MI policy is identical to those of the MM and PS policies.

### VI. OUTAGE PROBABILITY OF COGHRN

In this section, we present the lower bound on the exact and asymptotic outage probabilities of COGHRN with the MM policy and the PS policy.

**A. COGHRN With MM**

In this policy, a relay with the maximum e2e-SNR is selected based on the CSI from the SS to the 4th SR link and from the kth SR to the SD link. Thus, the index of the selected relay is denoted as

\[ k_{\text{MM}} = \arg_{k=1, \ldots, K} \max\left(\min(\gamma_{\text{Fr}}^{k,x}, \gamma_{\text{Fr}}^{k,d})\right). \] (46)

Based on (46), the lower bound on the outage probability at a given \( \eta_H \) is written as

\[ P_{\text{MM}}(\eta_H) = \prod_{k=1}^{K} \left(1 - (1 - F_{\gamma_{\text{Fr}}^{k,x}(\eta_H)})(1 - F_{\gamma_{\text{Fr}}^{k,d}(\eta_H)})\right). \] (47)

Substituting (21) and (22) into (47), we can easily derive the lower bound on the outage probability of COGHRN with the MM policy, which is applicable to different types of SRs and PUs having arbitrary channel lengths and pass loss components.

**Lemma 1:** For the proportional interference case, the asymptotic diversity gain of COGHRN with the MM policy is \( K \min(N_1, N_2) \).
Proof: As $\bar{\gamma}_Q \to \infty$, it can be seen that

$$P_{\text{MM}}^\infty(\eta_H) \approx \left( F_{\gamma_H}^\infty(\eta_H) + F_{\gamma_H}^\infty(\eta_H) \right)^K$$

where

$$d_k = \begin{cases} \frac{N_1}{a_1}, & \text{if } N_1 < N_2, \\ \frac{N_2}{a_2}, & \text{if } N_2 < N_1, \\ \left( d_3 + d_6 \right) \frac{\mu}{\alpha_3}, & \text{if } N = N_1 = N_2. \end{cases}$$

In (48), $d_3 = \frac{N_1}{a_1} + \frac{d_2}{a_1}$ and $d_6 = \frac{N_2}{a_2} + \frac{d_5}{a_2}$, where

$$d_1 \triangleq \frac{1}{\Gamma(N_1 + 1)} \left[ 1 - e^{-\frac{2\mu_T}{a_3}} \sum_{j=0}^{N_1-1} \frac{1}{j!} \left( \frac{2\mu_T}{a_3} \right)^j \right]^L,$$

$$d_2 \triangleq \frac{1}{\Gamma(N_1 + 1) / \beta_1^{N_1+1} / 2N_1} \sum_{L, \{N_1\}, \{a_3\}} \left[ \Gamma(N_1 + j, 2\mu_T), 1 - \Gamma(N_1 + j + 1, 2\mu_T) \right],$$

$$d_4 \triangleq \frac{1}{\Gamma(N_2 + 1)} \left[ 1 - e^{-\frac{2\mu_R}{a_4}} \sum_{j=0}^{N_2-1} \frac{1}{j!} \left( \frac{2\mu_R}{a_4} \right)^j \right]^L,$$

$$d_5 \triangleq \frac{1}{\Gamma(N_2 + 1) / \beta_2^{N_2+1} / 2N_2} \sum_{L, \{N_2\}, \{a_4\}} \left[ \Gamma(N_2 + j + d, 2\mu_R), 1 - \Gamma(N_2 + j + d + 1, 2\mu_R) \right].$$

Therefore, this policy provides $K \min(N_1, N_2)$ diversity gain.

B. CogHRN With PS

In this policy, the relay with the maximum SNR from the SS to the $k$th SR is selected. The corresponding relay index is given by

$$k_{\text{PS}} = \arg_{k=1, \ldots, K} \max \left( \gamma_H^{s,k} \right).$$

Here, we have assumed $N_1 = N_1, N_2 = N_2, N_3 = N_3, N_4 = N_4, a_1 = a_1, a_2 = a_2, a_3 = a_3, a_4 = a_4$. The lower bound on the outage probability is evaluated as

$$P_{\text{PS}}(\eta_H) = 1 - \left( 1 - F_{\gamma_H}^{s,k}(\eta_H) \right)^K \left( 1 - F_{\gamma_H^{\text{PS},d}}(\eta_H) \right).$$

Substituting (21) and (22) into (51), we can easily derive the lower bound on the outage probability of CogHRN with the PS policy.

**Lemma 2:** The diversity gain with the PS policy is $\min(N_1, N_2)$ as $\bar{\gamma}_Q \to \infty$.

Proof: Based on (27) and (28), we can easily see that

$$P_{\text{PS}}^\infty(\eta_H) \approx F_{\gamma_H}^\infty(\eta_H) + F_{\gamma_H^{\text{PS},d}}(\eta_H)$$

Thus, the diversity gain is $\min(N_1, N_2)$.

We can readily see that the number of PUUs has no effect on the diversity gain with the MM and the PS policies.

Table I highlights the required CSI for the three relay selection strategies of CogFDR and CogHDR.

**VII. SIMULATION RESULTS**

In this section, we present numerical results to verify our new analytical results for three different relay selection policies in cooperative CP-SC spectrum sharing systems with the link level simulation. We assume the symbol block size as $N_s = 512$ and CP length as $N_{CP} = 16$. For the purpose of comparison, we set the target data rate as $R_T = 1$ bit/s/Hz, thus the fixed SNR threshold for CogFRN is denoted as $\eta_T = 2R_T - 1$. However, in CogHRN, two different channels are needed for CP-SC transmission. We assume that both the SS and the SRs use half of the resource, therefore a fixed SNR threshold for CogHRN is denoted as $\eta_H = 2R_T - 1$. In order to examine the effects of power scaling on the outage probability, in the simulations we set $\bar{\gamma}_R = \mu R_T, \gamma_Q = \mu T R_T$, and $\bar{\gamma}_Q = \mu R T \bar{\gamma}_R$. The figures highlight the accuracy of our derived closed-form expressions for the relay selection policies. In all the figures, we assume $\{N_1, N_3\} = \{2, 0.5\}$ and $\{N_2, N_4\} = \{3, 0.3\}$. Fig. 2 shows the outage probability of CogFRN for various numbers of relays and different relay selection policies. The exact plots with MM, PS, and MI relay selection policies are numerically evaluated using (31), (39), and (42). The asymptotic outage probabilities are plotted from (35), (40), and (45). First, we observe error floors in the high SNR with zero outage diversity gain, which is due to the dominant effects of the residual loop interference. Second, for the same number of relays, for example $K = 6$, relay selection policy MM outperforms PS, and PS outperforms MI over all SNR values. The outage probabilities with MM policy and PS policy improve.
with increasing the number of SRs, while the outage probability with MI policy is not significantly improved by deploying more SRs. Interestingly, the performance gaps between each selection policy increase as the number of SRs increases.

In Fig. 3, we examine the outage probability of CogFRN for various numbers of PUs and different relay selection policies. It is easy to note that increasing the number of PUs deteriorates the outage performance of CogFRN since the secondary network has less chance to share the spectrum of the primary network when the number of PUs is large.

In Fig. 4, we compare the outage probability of CogFRN and CogHRN at the same target data rate under different relay selection policies. Interestingly, we notice that: 1) Compared with CogHRN, CogFRN sacrifice the outage probability to achieve the potential higher spectral efficiency; and 2) CogHRN overcomes the outage floors of CogFRN in the high SNRs. This is due to the fact that the dominating effect of residual loop interference is removed in CogHRN.

In Fig. 5, we examine the impact of the ratio between the peak interference power constraint at the PU and the maximum transmit power constraint at the SS \((Q/P_T)\) on the outage performance of CogFRN with the MM relay selection policy. We see that the outage probability for the same relay selection policy improves with a more relaxed peak interference power constraint at the PU. The higher ratio between the peak interference power constraint at the PU and the maximum transmit power constraint at the SS, the lower error floors and the bigger gaps among these three policies can be achieved. It is readily observed that the diversity gain is zero regardless of \(\mu_T\) in the high SNR regime.

In Fig. 6, we show the outage probability with FDR and HDR as a function of \(\rho\), which is the ratio between \(\bar{\gamma}_R\) and \(\bar{\gamma}_T\). For the same relay transmission mode and the same relay selection policy, the parallel slopes illustrate that the diversity gain is unrelated to \(\rho\). Interestingly, we observe that as \(\rho\) increases, a better outage performance is achieved in CogHRN, while a
worse outage performance in CogFRN, and the crossover point between full-duplex and half-duplex moves to the left. This is due to the fact that with $\rho$ increases, $\gamma_R$ increases, which results in the enhancement of the second hop transmission in CogHRN. However, due to increased residual loop interference with increasing $\rho$, the adverse effect of the residual loop interference grows with increasing the transmit power of SR.

In Fig. 7, we examine the outage probability with FDR with various relay selection policies and $\rho$. Similar phenomenon in CogFRN is observed as Fig. 6. As $\rho$ decreases, the outage probability with the PS policy and the MI policy degrade. This is because the residual loop interference is a detrimental characteristic of FDR, which is shown in (29), (37), and (41). We define $\bar{\gamma}_T < 12$ dB as the SNR dominant region, and $\bar{\gamma}_T > 25$ dB as the residual loop interference dominant region. In the diversity achievable SNR dominant region, we observe that the outage probability decreases as increasing $\bar{\gamma}_T$. In the residual loop interference dominant region, we observe the zero diversity gain, which restricted the decreasing trend of outage probability.

VIII. CONCLUSION

We have examined the effects of residual loop interference in cooperative CP-SC spectrum sharing with FDR. The lower bound on the outage probabilities and asymptotic outage probabilities for the MM policy requiring global CSI, as well as the PS and the MI policies requiring partial CSI have been derived and quantitatively compared. Interestingly, we observe that the diversity gain results from spatial diversity and multipath diversity can be achieved in the SNR dominant region, whereas the diversity gain lost in the residual loop interference dominant region. For comparison purposes, the lower bound on the outage probabilities and the corresponding asymptotic outage probabilities of cooperative CP-SC spectrum sharing with HDR have been derived for each of the relay selection policies. Our results show that CogFDR is a good solution to achieve the spectral efficiency and bearable outage probability for the systems that operate at low to medium SNRs, while CogHDR is more favorable to those operate in the high SNRs.

APPENDIX A

DETAILED DERIVATION OF (20)

We start from the definition of the CDF of $\gamma_F^{s,k}$, which is given by

$$F_{\gamma_F^{s,k}}(\gamma) = \Pr(\min(Q/Y_1, P_T)X_k\gamma \leq \gamma) = \mathbb{E}_{X_k}(\gamma/\bar{\gamma}_T)\mathbb{E}_{Y_1}(\mu_T) + \int_{\mu_T}^\infty \mathbb{E}_{X_k}(\gamma/\bar{\gamma}_Q)\mathbb{E}_{Y_1}(\gamma/\bar{\gamma}_Q)\ dy. \quad (A.1)$$

We use the integration by parts to solve $I_1$ of (A.1), which is given by

$$I_1 = \mathbb{E}_{X_k}(\gamma/\bar{\gamma}_Q)\mathbb{E}_{Y_1}(\gamma)\bigg|_{\mu_T}^\infty - \int_{\mu_T}^\infty \mathbb{E}_{X_k}(\gamma/\bar{\gamma}_Q)\mathbb{E}_{Y_1}(\gamma)\ dy = 1 - \mathbb{E}_{Y_1}(\mu_T)\mathbb{E}_{X_k}(\gamma/\bar{\gamma}_T) - \left[1 - \mathbb{E}_{Y_1}(\gamma/\bar{\gamma}_T)\right]$$

$$- \sum_{L,i,\{N_k,j,\{a_k,j\}\}} \frac{\gamma}{\bar{\gamma}_Q} \int_{\mu_T}^\infty \mathbb{E}_{X_k}(\gamma/\bar{\gamma}_Q)\mathbb{E}_{Y_1}(\gamma/\bar{\gamma}_Q)\ dy. \quad (A.2)$$

Substituting (A.2) into (A.1), we first obtain

$$F_{\gamma_F^{s,k}}(\gamma) = \mathbb{E}_{X_k}(\gamma/\bar{\gamma}_T)$$

$$- \sum_{L,i,\{N_k,j,\{a_k,j\}\}} \frac{\gamma}{\bar{\gamma}_Q} \int_{\mu_T}^\infty \mathbb{E}_{X_k}(\gamma/\bar{\gamma}_Q)\mathbb{E}_{Y_1}(\gamma/\bar{\gamma}_Q)\ dy. \quad (A.3)$$

Then using [38, eq. 3.351.2] and the PDF of $X_k$, the closed-form expression for the CDF of $\gamma_F^{s,k}$ can be derived as (20).
APPENDIX B

Detailed Derivation of (31)

Based on (30), the outage probability with MM policy is given as

\[
\mathbb{P}_{\text{MM}}^\text{out}(\eta F) = \prod_{k=1}^{K} \int_{\mu_F}^{\infty} \left( 1 - \left( 1 - F_{\sigma_F^k}[y > \mu_F(\eta F)] \right) \right) f_{y}(y) dy + \int_{0}^{\mu_F} \left( 1 - \left( 1 - F_{\sigma_F^k}[y \leq \mu_F(\eta F)] \right) \right) f_{y}(y) dy ,
\]

where \( \sigma_F^k[y > \mu_F = \frac{y_k \gamma}{\sum_{i=1}^{K} y_i}, \quad \sigma_F^k[y \leq \mu_F = \frac{\gamma W_k}{\sum_{i=1}^{K} y_i},\quad \mu_F = \frac{\gamma W_k}{R_N y + \gamma R_F} \), and \( \gamma_F^d[y \leq \mu_F = W_k \gamma_R F \).

In (E.1), \( F_{\sigma_F^k}[y > \mu_F(\eta F)] \) and \( F_{\sigma_F^k}[y \leq \mu_F(\eta F)] \) are presented as

\[
F_{\sigma_F^k}[y > \mu_F(\eta F)] = \int_{0}^{\infty} F_{\gamma F^k}(y(x + 1)) f_{MM}[\gamma_F, y \leq \mu_F] (y) dy ,
\]

and

\[
F_{\sigma_F^k}[y \leq \mu_F(\eta F)] = \int_{0}^{\infty} F_{\gamma F^k}(y(x + 1)) f_{MM}[\gamma_F, y \leq \mu_F] (y) dy .
\]

respectively.

Based on the distribution of \( W_k, R_k, \gamma_F^d, \) and \( Y_k \), we derive \( \mathbb{P}_{\text{MM}}^\text{out}(\eta F) \).

APPENDIX C

Detailed Derivation of (36)

Similar as the analysis in Appendix B, the first term \( R_1 \) is evaluated as

\[
R_1 = \sum_{i=0}^{N_1-1} \frac{1}{i!} \left( \frac{\gamma}{\alpha_1} \right)^i \times \left[ \frac{1}{\rho} \left( \frac{1 + \frac{\gamma}{\alpha_1}}{\rho} \right)^{-i-1} \Gamma(i+1) - \sum_{L,d,[N_k],[a_k]} \mu_{T}^{\tilde{w}_{i}^n} \tilde{w}_{i}^{n-d} \right] \\
\times \left[ \sum_{r=0}^{d} \sum_{w=0}^{r} \gamma \left( \tilde{d}, \frac{\mu_T}{\rho}, \frac{1}{\mu_T} \right) \right. \\
\Gamma(wi + 1) \Psi \left( wi + 1, wi + 1 \right) \\
\left. + \sum_{r=0}^{d} \sum_{w=0}^{r} \gamma \left( \tilde{d}, 1, \frac{\mu_T}{\rho}, \frac{1}{\mu_T} \right) \right] \\
\Gamma(wi + 2) \Psi \left( wi + 2, wi + 1 \right) \\
\left. - \tilde{d}, \left( \frac{1 + \frac{\gamma}{\alpha_1}}{\rho} \right) \mu_T \tilde{g}_2 \right] .
\]

where \( w \equiv w + i, \quad \Psi(\sigma, \tau, \varepsilon) = \sigma e^{-\tilde{g}_2 t} \left( \frac{\epsilon}{\alpha_1} \right)^{\tilde{w}_i^n} \tilde{g}_2^{n-w} \).

Applying [38, eq. 9.211.4] and [38, eq. 8.352.2], we derive \( R_2 \) as

\[
R_2 = \sum_{L,d,[N_k],[a_k]} \sum_{m=0}^{N_1-1} \sum_{n=0}^{m} \Phi(\mu_T) \\
\times \left[ \frac{1}{\rho} \tilde{g}_1^{-N_1-i} \lambda \left( N_1 + n + 1, n + 2 - j, \frac{\alpha_1 \mu_T \tilde{g}_2}{\gamma} \right) \right] \\
\times \sum_{L,d,[N_k],[a_k]} \left[ \sum_{r=0}^{d} \sum_{w=0}^{r} \gamma \left( \tilde{d}, \frac{\mu_T}{\rho}, \frac{1}{\mu_T} \right) \right] \\
\times \left[ \sum_{l=1}^{d+1} c_l \left( \frac{\alpha_1 \mu_T \tilde{g}_2}{\gamma} \right)^{-l} \right] \\
\times \left[ \sum_{l=1}^{d+1} d_l \left( \frac{\alpha_1 \mu_T \tilde{g}_2}{\gamma} \right)^{-l} \right] \\
\times \lambda \left( N_1 + i, N_1 + 1, N_1 + 1 - l_3, \frac{\alpha_1 \mu_T \tilde{g}_2}{\gamma} \right) \right] .
\]

where \( N_1, n \equiv N + n, \lambda(\vartheta, \tau, \varsigma) = \Omega(\vartheta, \tau, \varsigma) + \frac{\vartheta}{\rho} \).

\( \Phi(\delta) = \frac{\Omega(\delta) \left( N_1 + j - 1 \right) \mu_T}{(\alpha_1 \mu_T \tilde{g}_2)^n / (\alpha_1 \mu_T \tilde{g}_2)^n} \)

where

\[
c_l \triangleq \left( \begin{array}{c} -1 \delta - l + j \delta - l + j - 1 \delta - l + j - 2 \end{array} \right) / \left( \begin{array}{c} \delta - l - 1 \delta - l - 1 + j \delta - l - 1 + j - 1 \end{array} \right) \\
do_l \triangleq \left( \begin{array}{c} -1 \delta - l - 2 \delta - l - 3 \delta - l - 4 \end{array} \right) / \left( \begin{array}{c} \delta - l - 1 \delta - l - 2 \delta - l - 3 \end{array} \right) \\
dc_1 \triangleq \left( \begin{array}{c} \alpha_1 \mu_T \tilde{g}_2 - \alpha_1 \mu_T \tilde{g}_2 \end{array} \right) / \left( \begin{array}{c} \delta - l - 1 \delta - l - 1 + j \delta - l - 1 + j - 1 \end{array} \right) \\
dc_2 \triangleq \left( \begin{array}{c} -1 \delta - l - 2 \delta - l - 3 \delta - l - 4 \end{array} \right) / \left( \begin{array}{c} \delta - l - 1 \delta - l - 2 \delta - l - 3 \end{array} \right) \\
dc_3 \triangleq \left( \begin{array}{c} \alpha_1 \mu_T \tilde{g}_2 - \alpha_1 \mu_T \tilde{g}_2 \end{array} \right) / \left( \begin{array}{c} \delta - l - 1 \delta - l - 1 + j \delta - l - 1 + j - 1 \end{array} \right) \\
dc_4 \triangleq \left( \begin{array}{c} \alpha_1 \mu_T \tilde{g}_2 - \alpha_1 \mu_T \tilde{g}_2 \end{array} \right) / \left( \begin{array}{c} \delta - l - 1 \delta - l - 1 + j \delta - l - 1 + j - 1 \end{array} \right) .
\]

\[\]
APPENDIX D
Detailed Derivation of (39)

Based on (37), the outage probability with PS policy is given as

$$
\mathbb{P}_{\text{out}}^{\text{PS}}(\eta_F) = \int_{\mu_R}^{\infty} \left(1 - \left(1 - F_{\sigma_F}^{\text{PS}} | y > \mu_R (\eta_F)\right)\right) dy \times \left(1 - F_{\gamma_F}^{\text{PS}} | y > R_k \right) \left(\eta_F\right) dy + \int_{0}^{\mu_R} \left(1 - \left(1 - F_{\sigma_F}^{\text{PS}} | y \leq \mu_R (\eta_F)\right)\right) dy \times \left(1 - F_{\gamma_F}^{\text{PS}} | y \leq \mu_R \right) \left(\eta_F\right) dy,
$$

\[\text{(D.1)}\]

where $\sigma_F^{\text{PS}} | y > \mu_R = \max_{k=1, \ldots, K} \{\gamma_F^{k}\}$ and $\sigma_F^{\text{PS}} | y \leq \mu_R = \max_{k=1, \ldots, K} \{\gamma_F^{k}\}$.

Thus, $\mathbb{P}_{\text{out}}^{\text{PS}}(\eta_F)$ can be derived by using the distribution of $W_k$, $R_k$, $\gamma_F^{k}$, and $Y_k$.

APPENDIX E
Detailed Derivation of (42)

Based on (41), the outage probability with MI policy is given as

$$
\mathbb{P}_{\text{out}}^{\text{MI}}(\eta_F) = \int_{\mu_R}^{\infty} \left(1 - \left(1 - F_{\sigma_F}^{\text{MI}} | y > \mu_R (\eta_F)\right)\right) dy \times \left(1 - F_{\gamma_F}^{\text{MI}} | y > \mu_R \right) \left(\eta_F\right) dy + \int_{0}^{\mu_R} \left(1 - \left(1 - F_{\sigma_F}^{\text{MI}} | y \leq \mu_R (\eta_F)\right)\right) dy \times \left(1 - F_{\gamma_F}^{\text{MI}} | y \leq \mu_R \right) \left(\eta_F\right) dy,
$$

\[\text{(E.1)}\]

where $\sigma_F^{\text{MI}} | y > \mu_R = \gamma_F^{k}$, $\sigma_F^{\text{MI}} | y \leq \mu_R = \gamma_F^{k}$, and $Y_{\text{MI}} = \max_{k=1, \ldots, K} \{Y_k\}$.

Thus, $\mathbb{P}_{\text{out}}^{\text{MI}}(\eta_F)$ can be derived by using the distribution of $W_k$, $R_k$, $\gamma_F^{k}$, and

$$
\tilde{f}_{Y_{\text{MI}}} (y) = K' \left(1 + \sum_{\ell_1, \ell_2} y^{\ell_1} e^{-\beta_{21}}\right)^{-1} \times \sum_{L_d, \{N_{\text{CI}}, \{a\}\}} e^{-\beta_{21}} \left[\tilde{d} y^{\ell_1 - 1} - \beta_{21} \tilde{d} y^{\ell_2}\right].
$$

\[\text{(E.2)}\]

REFERENCES


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