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A semi-analytical method to predict net-tension failure of mechanically fastened joints in composite laminates

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Abstract

A Finite Fracture Mechanics model is proposed to predict the net-tension strength of composite mechanically fastened joints. A modification of coupled stress–energy criteria is suggested to take properly into account the effect of the R-curve in determining the crack length corresponding to the unstable crack propagation. The proposed model works for quasi-isotropic laminates and it requires as input the longitudinal strength and the fracture toughness of the laminate. The predictions obtained show good agreement when compared with the experimental results.

Key words:
A. Polymer-matrix composites (PMCs), B. Fracture, B. Strength

1 Introduction

Mechanically fastened joints are widely used in the aeronautical industry and, despite the fact that fasteners cause an increase of weight of aircraft structures, such joints are often the only feasible solution. For example, bonded joints should not be used in thick composite parts or when the joint needs to be disassembled for the maintenance or inspection of the structure.

Several studies on mechanically fastened joints in polymer composites have been conducted over the past 40 years. An extensive review of all the relevant studies in the field is reported in [1–3].

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Figure 1 shows the typical geometry of the specimen and the relevant parameters: the width, $w$, the end distance, $e$, the diameter of the hole, $d$. The effects of these parameters on the strength of composite mechanically fastened joints were discussed by several authors [4–9].

Hart-Smith [4] showed that when the diameter is large compared with the width of the specimen net-tension failure mode occurs. The critical value of the $w/d$ ratio that defines the change in the failure modes from net-tension to bearing depends on the material and on the lay-up used. For quasi isotropic CFRP laminates the $w/d$ ratio corresponding to the change in failure mode is about 3–4. Modifications of the geometry and of the layup of the laminate change the failure mode of the joint and consequently the strength of the joint.

The strength prediction methods developed in previous studies for the fast design and optimization of composite bolted joints use empirical parameters: the tensile strength of the laminate, $X_L$, and one or more characteristic dimensions. The strength prediction methods used are based on modifications of the point– [10] and average–stress [11] models proposed by Whitney and Nuismer [12].

A combination of a characteristic distance [12] and a failure theory were used by several authors [13,15–19]. Chang et al. [13] used the Yamada-Sun [14] failure criterion together with a proposed characteristic curve in a bi-dimensional Finite Element (FE) model. The Yamada-Sun [14] failure criterion is expressed as:

$$\left(\frac{\sigma_{11}}{X_L}\right)^2 + \left(\frac{\sigma_{12}}{S_c}\right)^2 = e_f^2$$

(1)

where $\sigma_{11}$ and $\sigma_{12}$ are respectively the longitudinal and in-plane shear stress in a ply, $X_L$ is the ply longitudinal strength and $S_c$ the ply shear strength measured from a cross-ply laminate. Failure occurs when $e_f \geq 1$. The characteristic curve was defined as:

$$r_c(\theta) = \frac{d}{2} + d_0t + (d_{0c} + d_{0t}) \cos \theta$$

(2)

where $d_{0t}$ and $d_{0c}$ are the characteristic dimensions for tension and compression, respectively. Both parameters must be obtained experimentally. The nonlinear shear stress-shear strain was also taken into account in [19].

Camanho and Lambert [20] using the concept of characteristic curve together with the LaRC04 failure criteria [21] developed a methodology able to predict
the elastic limit of the joint and the ultimate failure load of the joint.

All the previously mentioned methods are able to predict the strength of composite mechanically fastened joints. The drawback of those methods is that the empirical characteristic dimensions have to be experimentally determined for each lay-up and the accuracy of the solution depends on the particular geometry used to identify the model. When the material is changed, also the characteristic dimensions change and a further recalibration of the model is required. Moreover the physical meaning of the characteristic dimension is not clear.

Therefore the objective of this paper is to present a new model that can predict the net-tension strength of a mechanically fastened joint without the use of a characteristic dimension. The model proposed is developed in the framework of the Finite Fracture Mechanics (FFM) [22–30].

A simple analytical model to predict the strength of a mechanically fastened joint using only the strength $X_T$ and the fracture toughness $K_C$ of the laminate as input parameters is proposed. The model is valid for the entire family of quasi-isotropic laminates; therefore, it can be used as a quick-design tool in an industrial environment for preliminary design and optimization.

2 Finite Fracture Mechanics Model for mechanically fastened joints

The following assumptions are made:

• only quasi-isotropic laminates will be addressed. This is not a severe limitation because even if in theory the layup of the composite joint can be a different one the common practice in the aeronautical industry is to use quasi-isotropic laminates to enhance the joint performance under different loading cases and to minimize the stress concentration factor. However the method is general and it can be extended to different layups and stacking sequences with some modifications.

• only the net-tension failure mechanism is investigated. This failure mechanism occurs in highly optimized composite joints and in relevant structural details such as composite lugs.

It should be noted that for the case of net-tension failure there is a clear macro-crack and therefore Finite Fracture Mechanics is applicable. This is not the case of bearing failure mode where permanent deformation of the hole caused by ply-level failure mechanisms and delamination occur [20,31].
The problem to solve in the Finite Fracture Mechanics framework is shown in Figure 1 where a mechanically fastened joint under tensile loading and the associated coordinate system are shown. The contact pressure around the pin is assumed to be cosinusoidal, as it has been suggested by Waszczak and Cruse [32] for both isotropic and anisotropic materials. Moreover, it is assumed that the $x$ and $y$ axes correspond to the preferred axes of the material and that the end distance, $e$, is sufficiently large to prevent shear-out failure. Finally, all the variables used are defined per unit thickness of the laminate. The contact pressure is given as:

$$ p = p_{\text{max}} \cos \vartheta $$

(3)

where $p_{\text{max}}$ is the maximum value of the contact pressure and $\vartheta$ is the angle shown in Figure 1. The load applied to the joint along the $y$-direction is simply obtained as:

$$ F_y = \frac{1}{2} dp_{\text{max}} \int_{-\pi/2}^{\pi/2} \cos^2 \vartheta \, d\vartheta = \frac{\pi}{4} dp_{\text{max}} $$

(4)

The remote stress, $\sigma_{\infty}$, and the bearing stress, $\sigma_b$, are given as:

$$ \sigma_{\infty} = \frac{F_y}{w} = \frac{\pi d p_{\text{max}}}{4 w} $$

(5)

$$ \sigma_b = \frac{F_y}{d} = \frac{\pi}{4} p_{\text{max}} $$

(6)

It is noted that the bearing strength depends only on the maximum contact pressure applied and not on the dimension of the joint.

A modification of the coupled stress-energy criterion proposed in [26] and applied in [30] for the prediction of open-hole strength of composite laminates is used to predict the strength of the joint. However, the knowledge of both the longitudinal stress distribution along the net-tension plane (Figure 2) and the stress intensity factor for the case of two cracks emanating from a hole for a composite joint loaded in tension (see Figure 3) is required.

[Fig. 2 about here.]

[Fig. 3 about here.]

It is convenient to define a new coordinate system $(\xi, y)$, with the origin at the edge of the hole where $\xi$ is defined as:
\[
\xi = \frac{2}{w - d} \left( x - \frac{d}{2} \right)
\]  
(7)

It should be noted that \( \xi \) is dimensionless and takes the value of \( \xi = 0 \) at the hole edge and \( \xi = 1 \) at the outer edge of the specimen. In this coordinate system the coupled stress-energy criterion [26] reads:

\[
\begin{align*}
\frac{1}{l} \int_0^l \sigma_y (\xi) \, d\xi &= X_L^T \\
\frac{1}{l} \int_0^l K_I^2 (\xi) \, d\xi &= K_{IC}^2
\end{align*}
\]  
(8)

where \( l \) is the crack extension just before its unstable propagation. It should be noted that the fracture toughness \( K_{IC} \) in the right-hand side of the second equation in (8) is considered constant in [26] and it can be considered equal to the toughness propagation value when the length of process zone, \( l_{pz} \), is small with respect to the characteristic dimensions of the specimen. In that case the actual shape of the R-curve can be neglected and a constant value of the fracture toughness \( K_{IC} \) is considered.

Figure 4(a) shows qualitatively the R-curve of the material and the energies associated with the crack propagation. Considering the fracture toughness constant and equal to \( K_{IC}^p \), the energy associated to the crack propagation will be proportional to the area of the rectangle \( (\Delta a_c \times (K_{IC}^p)^2) \). In the presence of a R-curve, the energy is proportional to the area under the R-curve (striped area in Figure 4(a)). When \( \Delta a_c \gg l_{pz} \) this difference can be neglected.

However, in the case of bolted joints that fail by net-tension, and in particular in the presence of small diameters, the length of process zone, \( l_{pz} \) and the critical crack length, \( \Delta a_c \), may be of the same order than the characteristic dimension of the specimen \( (l_{pz} \sim \Delta a_c \sim (w - d)/2) \) and in that case the actual shape of the R-curve must not be neglected (see Figure 4(b)).

Therefore, to take into account the R-curve of the material a modification of the coupled stress-energy criterion, equation (8), is proposed:

\[
\begin{align*}
\frac{1}{l} \int_0^l \sigma_y (\xi) \, d\xi &= X_L^T \\
\int_0^l K_I^2 (\xi) \, d\xi &= \int_0^{\Delta a} K_{IC}^2 (\Delta \xi) \, d\Delta \xi
\end{align*}
\]  
(9)

where \( K_{IC} (\Delta \xi) \) is the R-curve expressed in the \((\xi, y)\) coordinate system. Assuming that the R-curve, \( K_{IC} (\Delta a) \) where \( \Delta a \) the crack increment, is known,
the fracture toughness $K_{IC}(\Delta \xi)$ is obtained taking into account that $\Delta a = \Delta \xi (w - d)/2$.

For the elastic orthotropic case the stress $\sigma_y$ and the stress intensity factor $K_I$ of equation (9) depend, for a given bearing strength, $\sigma_b$, only on the dimensions and on the orthotropy of the material.

For quasi-isotropic laminates the dependence of the stress intensity factor on the material can be neglected. In fact the response of an orthotropic solid in plane stress only depends on two dimensionless parameters [33,34]:

$$\lambda = \frac{E_y}{E_x}$$

$$\rho = \frac{\sqrt{E_x E_y}}{2 G_{xy}} - \sqrt{\nu_{xy} \nu_{yx}}$$

where $E_x$ and $E_y$ are respectively the laminate longitudinal and transverse Young’s modulus, $G_{xy}$ is the laminate shear modulus, and $\nu_{xy}$ and $\nu_{yx}$ are the laminate Poisson’s ratios. For the quasi-isotropic laminate and for a crack running along a preferred axis of the material $\lambda$ and $\rho$ take the values $\lambda = \rho = 1$. Therefore the dependence on the material orthotropy can be eliminated from the calibration of the model.

Therefore the stress distribution and the stress intensity factor only depend on the two dimensionless variables $\xi$ and $\omega = w/d$ and on the diameter $d$. It should be noted that $\xi$ and $\omega$ define the relations between the diameter, the width and the material point on the net-tension plane where the stress and stress intensity factors are calculated.

To compute the stress distribution $\sigma_y(\xi, \omega)$ it is convenient to use the superposition of the stress field. It is possible to compute numerically, using the Finite Element Method, the stress distribution when a unitary remote load is applied at a joint with a unitary diameter of the hole:

$$\sigma_y(\xi, \omega)|_{\sigma_b=1} = \phi(\xi, \omega) \quad \sigma_b|_{\sigma_b=1} = \phi(\xi, \omega)$$

The polynomial fitting function $\phi$ is calculated using the commercial software Abaqus 6.8-3 [35]. The two variables to investigate are $\omega (1 < \omega < \infty)$ and $\xi (0 \leq \xi < 1)$. However, for practical application $1.5 < \omega \leq 4$. In fact for $\omega > 4$ bearing failure occurs [4] instead of net-tension failure.
A parametric Finite Element model was created using Python [36] together with Abaqus 6.8-3 CAE [35]. The 8-node biquadratic element, CP S8, was used to mesh the models and a unitary bearing stress $\sigma_b = 1$ was applied.

Finally, the parameters that best fit the data are obtained via a least-squares curve fitting method in Matlab 7.6.0 [37]. Our calculation shows that $\phi(\xi, \omega)$ can be approximated with high accuracy with a polynomial fitting function:

$$\phi(\xi, \omega) = \sum_{i=1}^{M} \sum_{j=1}^{N} \Phi_{ij} \omega^{j-1} \xi^{i-1}$$  \hspace{1cm} (13)

where $\Phi_{ij}$ is the element of the matrix $\Phi$ at the row $i$ and at the column $j$, and $M$ and $N$ are the number of rows and columns of $\Phi$ respectively. The matrix $\Phi$ is reported in Appendix A. Figure 5 shows the numerical results obtained and the corresponding fitting surface, $\phi(\xi, \omega)$. It should be noted that the only variable that varies in the model is $\omega$; therefore, in Figure 5 the numerical points corresponding to the same $\omega$ are calculated in the same analysis.

For a general bearing stress $\sigma_b$, and for any diameter, $d$, the stress distribution reads:

$$\sigma_y(\xi, \omega) = \sigma_b \phi(\xi, \omega)$$  \hspace{1cm} (14)

In an analogous way, for a crack of length $\xi$ the stress intensity factor when a unitary bearing stress is applied reads:

$$K_1(\xi, \omega)|_{\sigma_b=1} = \sigma_b|_{\sigma_b=1} \sqrt{d}|_{\xi=1} \psi(\xi, \omega) = \frac{\sqrt{\xi}}{1-\xi} \sum_{i=1}^{M} \sum_{j=1}^{N} \Psi_{ij} \omega^{j-1} \xi^{i-1}$$  \hspace{1cm} (15)

To compute $\Psi$, a parametric Finite Element model was created using Python [36] together with Abaqus 6.8-3 CAE [35]. The 4-node quadratic, reduced integration element, CPS4R, was used to mesh the models and a unitary bearing stress $\sigma_b = 1$ has been applied. In this model the variables that vary are the diameter over width ratio, $\omega$ ($1.5 < \omega \leq 4$), and the crack length $l$ ($0 < l < 1$). Figure 6 shows the contour plot of $\sigma_y$ obtained in the model with $\omega = 2.75$ and a crack length $\xi = 0.45$.
It should be noticed that the expression of $\psi(\xi, \omega)$ respects the boundary conditions, i.e. $\psi = 0$ for $\xi \to 0$ and $\psi = \infty$ for $\xi \to 1$. Figure 7 shows the numerical results obtained and the corresponding fitting surface, $\psi(\xi, \omega)$ (each dot represents one numerical simulation). The matrix $\Psi$ of equation (15) is reported in Appendix A.

Finally, for a general bearing stress, $\sigma_b$, and for any diameter, $d$, the stress intensity factor reads:

$$K_1(\xi, \omega) = \sqrt{d} \sigma_b \psi(\xi, \omega)$$  \hspace{1cm} (16)

Inserting equations (14) and (16) in (9):

$$\begin{cases}
\sigma_b \int_0^l \phi \, d\xi = X^T_L l \\
d \sigma_b^2 \int_0^l \psi^2 \, d\xi = \int_0^l K_{IC}^2 \, d\Delta\xi
\end{cases}$$  \hspace{1cm} (17)

where the bearing stress at failure, $\sigma_b$, and the crack length at failure, $l$, are the two unknowns. Dividing the first of equation (17) for the squared root of the second gives:

$$\frac{\int_0^l \phi \, d\xi}{X^T_L} = l \left( \frac{d \int_0^l \psi^2 \, d\xi}{\int_0^l K_{IC}^2 \, d\Delta\xi} \right)^{1/2}$$  \hspace{1cm} (18)

Equation (18) can be solved for $l$ ($0 < l < 1$). Substituting the value of $l$ in one of (17) enables the calculation of the bearing stress at failure, $\sigma_b$.

3 Model Validation

The material chosen for the experimental investigation is the Hexcel IM7-8552 carbon epoxy unidirectional pre-preg. The $[90/0/\pm 45]_{3s}$ lay-up, corresponding to laminates with a nominal thickness of 3mm, was selected.

The material was cured according to the specifications of the manufacturer. The curing cycle consisted in keeping the laminate at 110°C for 1 hour, increasing the temperature up to 180°C, keeping this temperature for 2 hours, and cooling at 3°C/min. The pressure of 7 bar was used during all the curing cycle.
After curing, the laminate was cut using a diamond saw and the specimens were drilled clamping sacrificial plates at the insertion and exit points of the drill to avoid damage at the hole edge.

The bolted joints were tested according to ASTM D-5961 standard [38] with a 2.2Nm torque applied to the bolt. The relevant dimension of the specimens are shown in Table 1.

The Finite Fracture Mechanic model described in the previous section was implemented in Maple 13.0 [39]. Knowing $\omega$ the functions $\phi$ and $\psi$ defined in equations (13) and (15) respectively can be calculated. Figures 8 and 9 show $\phi$ and $\psi$ as a function of $\xi$ for $\omega = 2$.

The strength of the laminate, $X_L = 845.1\text{MPa}$, and the fracture toughness of the laminate $K_{IC}$ are reported respectively in [40] and [41]. Without loss of generality it is proposed to use a Gompertz function [42] to obtain an analytical expression of the R-curve, $K_{IC}(\Delta a)$, to be inserted in equation (18). Therefore the R-curve can be expressed as:

$$K_{IC} = K_{IC}^p e^{\log(K_{IC}^i/K_{IC}^p) e^{b \Delta a}}$$

where $K_{IC}^i$ and $K_{IC}^p$ are respectively the initiation and the propagation values of the fracture toughness, $e$ is the Euler’s number, and $b$ is calculated fitting the experiments. Figure 10 shows the fracture toughness values obtained in [41] and the fitting function for the laminate under investigation. The parameters used in equation (19) are $K_{IC}^i = 36.7\text{MPa}\sqrt{m}$, $K_{IC}^p = 42.8\text{MPa}\sqrt{m}$, and $b = -1.27\text{mm}^{-1}$.

For a given $\omega$, the length $l$ can be obtained from equation (18). Consequently, substituting $l$, for example in equation (14), the bearing stress at failure can be obtained. Table 2 shows the comparison between the experimental value of the joint strength and the corresponding prediction. It is concluded that the proposed method, that is based on only two material properties (the strength and the fracture toughness), yields predictions that are in good agreement with the experimental data.
Figure 11 shows the specimen after testing. A straight crack in the net tension plane is observed.

[Fig. 11 about here.]

Figure 12 shows the predicted failure envelope, the experimental data and the bearing cut-off strength obtained in a previous investigation [20].

[Fig. 12 about here.]

From an inspection of equation (17) it is concluded that the bearing stress at failure will depend on the diameter of the joint. Using the proposed model it is possible to identify this dependency as shown in Figure 13 where the bearing stress at failure of a joint with $\omega = 2$ is plotted as a function of the diameter $d$.

[Fig. 13 about here.]

4 Conclusions

A Finite Fracture Mechanics model was proposed to predict the strength of a mechanically fastened joint when net-tension failure occurs. The model is applicable for quasi-isotropic laminates and it requires as input the longitudinal strength and the fracture toughness of the laminate. Using the Finite Element Method a parametric study was conducted to determine the stress distribution of a mechanically fastened joint and the stress intensity factor for the joint with two cracks emanating from the hole edge.

A modification of the coupled stress-energy criterion proposed in [26] was suggested to properly take into account the actual shape of the R-curve of the material. The predictions obtained are in good agreement with the experimental results.

It should be pointed out that the proposed methodology can be easily extended to the case of an orthotropic mechanically fastened joint. In that case, the influence of the orthotropy of the material should be taken into account (the parameters $\rho$ and $\lambda$ of equation (11) will differ from unity) and consequently $\phi$ and $\psi$ will also be function of $\rho$ and $\lambda$ (or only $\rho$ is the orthotropic rescaling technique is used [33]). Moreover it should be pointed out that the hypothesis that the stress distribution at the hole be cosinusoidal [32] would be unrealistic for highly orthotropic laminates so a different stress distribution should be used for those laminates.
Acknowledgments

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A Polynomial fitting functions calculation

The matrices $\Phi$ and $\Psi$ of equations (13) and (15) respectively are:

$$
\Phi = \begin{bmatrix}
88.47259368 & -170.4688173 & 142.1541230 & -63.66745751 & 16.02655846 & -2.141735655 & 0.1184734754 \\
-122.1958288 & 153.5379000 & -62.54369626 & 12.1492145 & -0.400873416 & -0.1271694776 & 0.007169337386 \\
363.2685871 & -209.2179324 & -139.5989080 & 138.6183273 & -45.43450856 & 6.35612158 & -0.2992400294 \\
-606.3183747 & 78.63062116 & 799.2229915 & -521.2745009 & 146.4340404 & -18.11785833 & 0.7364993402 \\
816.2118494 & 19.49098364 & -834.3668216 & 527.9131694 & -132.9112667 & 12.59320117 & -0.1400671546 \\
-746.1678535 & 513.6516665 & 33.30630791 & -64.69929108 & 0.5204533361 & 5.777467829 & -0.8363149246 \\
\end{bmatrix}
$$

(A.1)

$$
\Psi = \begin{bmatrix}
22.83435806 & -29.81479531 & 18.40143327 & -5.758140163 & 0.9066311202 & -0.0571860696 \\
-37.50380850 & 23.07360348 & -8.22158301 & 0.6293260657 & 0.0973244630 & -0.0114088928 \\
301.6521309 & -222.6945915 & 84.43828034 & -14.02728352 & 1.448093939 & -0.1084130419 \\
-1273.889397 & 884.2488556 & -273.6088052 & 28.35795338 & -0.7156219526 & 0.07507960542 \\
2843.096695 & -1803.563243 & 416.1988366 & 15.0310666 & -13.55753986 & 0.9925783371 \\
-3669.049920 & 2346.185565 & -529.1567783 & -27.31814770 & 20.82634120 & -1.697989115 \\
2518.166947 & -1714.235036 & 452.2469504 & -13.23735312 & -9.695981378 & 0.961030382 \\
-704.7412797 & 516.0148956 & -159.8012277 & 16.15772934 & 0.7170582248 & -0.1554093946
\end{bmatrix}
$$

(A.2)

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Fig. 3. Model for the stress intensity factor of two cracks emanating from a hole.
Fig. 4. R-curve and energies associated to the crack propagation

(a) $\Delta a_c >> l_{pz}$

(b) $\Delta a_c \sim l_{pz}$
Fig. 5. FE results and fitting surface $\phi(\xi, \omega)$
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Fig. 10. R-curve of the laminate: experiments and fitting
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<th>$d$ [mm]</th>
<th>$\omega$</th>
<th>$w$ [mm]</th>
<th>$e$ [mm]</th>
<th>length [mm]</th>
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<td>NT1</td>
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<tr>
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<td>12</td>
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</table>

*Experiments presented in [20]*

Table 1
Relevant dimension of the specimen
<table>
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<tr>
<th>Specimen</th>
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<th>Strength [MPa]</th>
<th>STDV [MPA]</th>
<th>Predicted strength [MPa]</th>
<th>$\epsilon$ [%]</th>
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<td>NT1</td>
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<td>322.3</td>
<td>0.26</td>
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</tr>
<tr>
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<td>20.7</td>
<td>549.5</td>
<td>+4.3</td>
</tr>
</tbody>
</table>

*Experiments presented in [20]*

Table 2

Comparison between experimental and predicted strength of the joint