Integrated Optimal Power Flow for Distribution Networks in Local and Urban Scales

Xiandong Xu, Kang Li
School of Electronics, Electrical Engineering and Computer Science
Queen’s University Belfast
Belfast BT9 5AH, UK
Email: {x.xu, k.li}@qub.ac.uk

Yang Liu
Department of Control Science and Engineering
Harbin Institute of Technology
Harbin 150001, China
Email: hitlg@hit.edu.cn

Hongjie Jia
Key Laboratory of Smart Grid of Ministry of Education
Tianjin University
Tianjin, China, 300072
Email: hjjia@tju.edu.cn

Abstract—This paper develops an integrated optimal power flow (OPF) tool for distribution networks in two spatial scales. In the local scale, the distribution network, the natural gas network, and the heat system are coordinated as a microgrid. In the urban scale, the impact of natural gas network is considered as constraints for the distribution network operation. The proposed approach incorporates unbalance three-phase electrical systems, natural gas systems, and combined cooling, heating, and power systems. The interactions among the above three energy systems are described by energy hub model combined with components capacity constraints. In order to efficiently accommodate the nonlinear constraint optimization problem, particle swarm optimization algorithm is employed to set the control variables in the OPF problem. Numerical studies indicate that by using the OPF method, the distribution network can be economically operated. Also, the tie-line power can be effectively managed.

Keywords—distribution network, spatial scales, natural gas network, integrated optimal power flow, energy hub

I. INTRODUCTION

The integration of different energy sources and loads has been intensively researched and developed in the last decade within the context of sustainable energy systems [1]. As a major energy supply network in urban areas, the distribution network (DN) plays a significant role in the urban energy system, especially in the background of electrification development [2]. It has been shown that the coordination among various energy systems can affect the DN operation [3] [4]. However, distributed energy resources (DERs) in local areas may be owned by customers or some third parties in addition to utilities, which indicates different interest concerns. Therefore, efficient management is required in the distribution network (DN) operation in order to satisfy the demands of different owners.

Optimal power flow (OPF) has been widely used to optimize the electric power system operation. In order for the DN to work efficiently, a number of OPF methods have been proposed to manage various resource in the DN including DERs, energy storage devices, loads, and energy networks. In the urban scale, the DN is usually operated from the view point of the utility [5]. For example, the OPF is employed to evaluate the amount of the wind power curtailment which is required to ensure the DN operation security [6]. In [7], DERs and energy storage are managed by the OPF incorporating both active and reactive power. However, all the above methods are not designed to analyze the impact of the natural gas (NG) network and to support the operation of DN with combined heat and power (CHP) units. Further, due to issues like untransposed lines, unbalanced loads and single-phase distributed generation plants, etc., the three-phase unbalanced characteristics needs to be incorporated in the DN management [8] [9].

In the local scale, low voltage distribution networks and DERs are integrated for supplying electrical and heat loads to customers in district areas, also known as microgrids [10]. It features locally produced, stored, and consumed cleaner energy, higher renewable penetration and less reliance on the energy from the utility grid. By providing peak shaving and other ancillary services, microgrids presents as a mechanism to fully utilize the benefits of the DERs and to promote a highly efficient energy delivery and supply system [11]. As one of the most successful commercial applications for microgrids [12], the optimization of CHP systems has drawn a lot of interests in recent years. An economical and coordinated dispatch method for CHP based microgrid was presented in [13] [14], taking the fluctuation and non-determinacy of renewables and loads into account. Based on the energy hub (EHub) [15] concept that was first proposed for interrelated energy system description, an integrated optimal energy flow model was developed for multi-carrier energy network optimization in an island [3]. Using the EHub model, a hierarchical framework [4] was designed for the management of microgrid with CHPs. However, most of the published work focused on the analysis of the electric power system or the energy balance of various energy systems. Few studies have been reported so far to examine the three-phase electric power system management and the integrated coordination of NG and electric power systems in microgrids.

This paper aims to develop OPF models and associated algorithms to coordinate power, gas, and heat systems in both district and urban areas. The main contributions of this paper are as follows: 1) The DN is formulated as a microgrid or a DN with multiple energy systems according to its spatial scale; 2) An integrated DN model is developed, incorporating three-phase electric power system, gas system, and heat system which are coupled by the EHub; 3) OPF models are proposed, taking into account various requirements of the DN in local and urban scales. Experimental results demonstrate that the developed OPF tools are effective in the DN management. For successful planning and operation of the DN, the OPF
will help engineers making proper decisions assisted with operators experiences. The study also shows that it is necessary to consider NG network constraints in the DN management when gas-powered units are used.

II. SYSTEM DESCRIPTION AND MODELING

In this paper, the DNs are presented as a combination of energy networks and EHubs.

A. Energy Network

Two types of fundamental infrastructure, namely the electric power network and the NG network, are considered for urban energy distribution. With the development of DERs in urban areas, electric power systems will be composed of the DN and microgrids [16]. To analyze the interrelated energy network, a NG network model consisting of compressors and pipelines is employed [17].

B. EHub

In the DN, power, gas, and heat systems are coupled by CHP units with different structures [18]. The CHP units can be represented by the EHub, as shown in Fig. 1.

![Fig. 1. The basic structure of the EHub.](image)

It includes power/NG input \( (P_e, P_g) \) and power/heat outputs \( (L_e, L_h) \). Also, other types of input and output \( (P_0, L_0) \), energy storage \( (S_1, S_2) \), and energy conversion units \( (C_1, C_2) \) can be included. The energy conversion process in the EHub can be formulated by

\[
\begin{bmatrix}
L_e \\
L_h \\
L_0
\end{bmatrix} =
\begin{bmatrix}
C_{ee} & C_{ge} & C_{0e} \\
C_{eh} & C_{gh} & C_{0h} \\
C_{e0} & C_{g0} & C_{00}
\end{bmatrix}
\begin{bmatrix}
P_e \\
P_g \\
P_0
\end{bmatrix}
\]  

(1)

where \( P \) and \( L \) represents inputs and outputs power of the EHub; \( C \) represents the energy conversion matrix.

Based on the widely adopted CHP layout, this paper will focus on two types of EHubs, as shown in Fig. 2. It should be noted that other devices such as heat storage devices and batteries are not considered here. In addition, the heat generated by the air-conditioner (AC), gas-boiler (GB) and microturbine (MT) will first be transmitted to the heat exchanger. The heat load studied in this paper will be the input to the heat changer.

As shown in Fig. 2(a), the EHub I is composed of the MT and the AC. The energy conversion process is presented by

\[
\begin{bmatrix}
L_e \\
L_h
\end{bmatrix} =
\begin{bmatrix}
(1 - \nu_{AC})\eta^T_{AC} \\
\nu_{AC}\eta^T_{AC} \eta^T_{MT}
\end{bmatrix}
\begin{bmatrix}
P_e \\
P_g
\end{bmatrix}
\]  

(2)

where \( L_e \) and \( L_h \) represent the electric and heat loads supplied by the EHub; \( P_e \) and \( P_g \) represent exchange values of electricity and NG between the EHub and energy networks; \( \nu_{AC} \) represents dispatch factor of the power to the AC, \( 0 \leq \nu_{AC} \leq 1 \); \( \nu_{AC}P_e \) represents the electricity consumed by the AC; \( (1 - \nu_{AC})P_e \) represents the electricity consumed by the MT; \( \eta^T_{AC} \) represents the transformer efficiency; \( \eta^T_{AC} \) represents the AC efficiency.

As shown in Fig. 2(b), the EHub II is composed of the MT and the GB. The energy conversion process can be formulated as

\[
\begin{bmatrix}
L_e \\
L_h
\end{bmatrix} =
\begin{bmatrix}
\eta^T_{MT} \\
0
\end{bmatrix}
\begin{bmatrix}
u_{MT}\eta^T_{MT} + (1 - \nu_{MT})\eta_{GB}
\end{bmatrix}
\begin{bmatrix}
P_e \\
P_g
\end{bmatrix}
\]  

(3)

where \( \nu_{MT} \) represents the dispatch factor of the gas to the GB, \( 0 \leq \nu_{MT} \leq 1 \); \( \nu_{MT}P_g \) represents the NG consumed by the MT; \( (1 - \nu_{MT})P_g \) represents the NG consumed by the GB; \( \eta_{GB} \) represents the GB efficiency; other variables are the same as (2).

III. PROBLEM FORMULATION

Here, we present a mathematical model and formulate the optimization problem for the DN in local and urban scales.

A. Objective Function

In the local scale, the DN and other energy systems could be owned by various business entities, which indicates different interests. Two objective functions are adopted as examples to optimize the local DN with DERs (also known as microgrid as mentioned before). In the urban scale, the network losses is considered as an example from the view point of utilities. The impact of NG network is handled as constraints in the DN management.

1) Operating cost minimization:

\[
\min [\mu_e P_{e,t} + \mu_g P_{g,t}]
\]  

(4)

where \( P_{e,t} \) and \( P_{g,t} \) are the electricity and gas exchange between the microgrid and the electric/gas network.
2) Tie-line power exchange minimization:
\[
\min \left[ \omega_c (P_{e,t}^{\text{set}} - P_{e,t})^2 + \omega_g (P_{g,t}^{\text{set}} - P_{g,t})^2 \right]
\]
(5)
where \( P_{e,t}^{\text{set}} \) and \( P_{g,t}^{\text{set}} \) are tie line power set-points; \( \omega_c \) and \( \omega_g \) are weighting factors for the microgrid to track the electric/gas tie line power, and when \( \omega_g \) is zero, the OPF is used to follow the electric tie-line power set-points.

3) Network loss minimization:
\[
\min \left[ \sum_{i=1}^{n_{\text{GEN}}} P_{\text{GEN},i} - \sum_{j=1}^{n_{\text{LD}}} P_{\text{LD},j} \right]
\]
(6)
where \( n_{\text{GEN}} \) and \( n_{\text{LD}} \) represent the number of generators and loads. \( P_{\text{GEN},i} \) and \( P_{\text{LD},j} \) stand for the power of \( i \)th generator and \( j \)th load.

B. Constraints

The equality constraints include three-phase electric power and NG flow equations as follows.
\[
\begin{align*}
&h_c(P,Q,V,\theta) = 0 \\
&h_g(M,p,k_{\text{cp}}) = 0
\end{align*}
\]
(7)
According to the EHub topology, the upper and lower bounds of the CHP system power consumption can be obtained. The bounds can be expressed as
\[
\begin{align*}
P_{\text{min},C\text{HP}} & \leq P_{e,C\text{HP}} \leq P_{\text{max},C\text{HP}} \\
P_{\text{min},G\text{HP}} & \leq P_{g,C\text{HP}} \leq P_{\text{max},G\text{HP}}
\end{align*}
\]
(8)
The other inequality constraints include bounds for three-phase voltage, NG pressure, and compressor ratio as follows.
\[
\begin{align*}
V_{\text{min}} & \leq V_i^a \leq V_{\text{max}} \\
V_{\text{min}} & \leq V_i^b \leq V_{\text{max}} \\
V_{\text{min}} & \leq V_i^c \leq V_{\text{max}} \\
p_{j}^{\text{min}} & \leq p_j \leq p_{j}^{\text{max}} \\
k_{\text{cp}}^{\text{min}} & \leq k_{\text{cp}} = \frac{p_m}{p_k} \leq k_{\text{cp}}^{\text{max}}
\end{align*}
\]
(9)
(10)
(11)
where \( V_i^a, V_i^b, V_i^c \) are the three-phase voltages at bus \( i \); \( V_{\text{min}} \) and \( V_{\text{max}} \) are the minimum and maximum voltages; \( p_{j}^{\text{min}} \) and \( p_{j}^{\text{max}} \) are the lower and upper bounds of the NG network pressure at node \( j \); \( k_{\text{cp}}^{\text{min}} \) and \( k_{\text{cp}}^{\text{max}} \) are the lower and upper bounds of the compressor ratio.

C. Optimization Algorithm

Considering the power flow model, the NG flow model, and the EHub model, it is clear that there exists complex nonlinear relationships among variables of different energy systems in the DN. Thus, the particle swarm optimization (PSO) method which has high efficiency and is easy to be implemented, is used to solve the optimization problem [19].

The proposed procedures are given as (see Fig. 3):

Step 1: Initialize the EHub based on the system structure and operating modes;

Step 2: Specify lower and upper bound information for EHub control variables associated with the DN and the NG network. Initialize the PSO population including particle dimension, positions, and velocities. All initial individuals should satisfy the operating constraints;

Step 3: Call the EHub, OpenDSS, and NG flow modules to solve the energy flow;

Step 4: Calculate the fitness of each particle in the population.

Step 5: Update individual best position \( p_{\text{best},id} \) and global best position \( g_{\text{best},id} \).

Step 6: Modify the velocity \( v_{id} \) and position \( x_{id} \) to ensure that all individuals satisfy the constraints.

Step 7: If the iterations satisfy the stop criteria or the number of iterations reaches the maximum, then go to Step 8. Otherwise, go to Step 5.

Step 8: Obtain the optimal solution for EHub control variables based on the individual that generates the latest \( g_{\text{best},id} \).

![Fig. 3. Flowchart of the integrated optimal flow analysis algorithm.](image-url)
In this paper, the DN management is investigated using two cases to illustrate the effectiveness of the proposed method in different scenarios. In case A, the NG network and the small scale DN are coordinated as a microgrid owned by customers. In case B, the DN is managed from the view of the utility. The NG composed of 90% methane and 10% ethane with lower heating value (LHV) is used in the fuel consumption calculation. The power is converted to the volume flow of the NG by dividing the LHV. The electricity from the grid and the NG from the gas utility are supposed to be available under all circumstances. The energy price used in this paper is taken from Pacific Gas and Electric Company, as shown in Fig. 5(a). The NG price is 0.0425$/kWh [21]. The loads accessed to the CHP are changing while other loads are assumed to be constant. The electric/thermal loads in a whole day is shown in Fig. 6.

In this case, the dispatch objectives are set to be the operating cost minimization (objective I), and electric tie-line power minimization (objective II, set $\omega_g = 0$ in (5)). The obtained results of objective I and II are shown in Fig. 8. The relative electric/gas tie-line power in the microgrid is shown in Fig. 9 under objective I and objective II. The simulation results suggest that the electric tie-line power is reduced significantly under objective II, while the microgrid absorbs more electric power from the electric network under objective I. This is because in order to minimize the operating cost, the microgrid tends to purchase more electricity and less gas from the NG network in the current energy price conditions.

**B. Case B—Distribution Network**

In this case, the proposed OPF method is applied to a DN containing CHP units. The objective is set to minimize
electric power loss (as shown in (6)). By removing the heating, ventilating, and air conditioning units, the studied case in [17] are utilized to illustrate the proposed method in the DN as shown in Fig. 10.

In the previous study, the NG pressure constraints at node 003 and 004 are set in a large range considering the function of compressors in MTs. Since it is necessary to consider the requirements of pipeline pressure from other loads in the NG network, the OPF model of the DN containing EHubs is investigated taking the NG pressure into account. The real time NG pipeline pressure and corresponding compression ratio of the compressor located in node 001 after optimization are studied under different NG pipeline constraints (Constraint I: \(0.2 \leq p \leq 1.3\); Constraint II: \(0.8 \leq p \leq 1.2\)) in the following cases. The real time NG pipeline pressure and corresponding compression ratio after optimization are shown in Fig. 13. The simulation results demonstrate that the NG pipeline pressure fluctuates more obviously under the larger range constraint I. The pipeline pressure is close to 0.2 in individual moments, which can affect the surrounding NG loads seriously.

The electric/thermal loads of CHPs described by EHubs in a whole day are shown in Fig. 6. The rated power of the three PV groups are 500kW, 100kW (Phase A) and 1000kW. In order to simplify the problem, the same light intensity data over 24 hours (see Fig. 11) is utilized in the three groups. Simulation results for the OPF are shown in Fig. 12 compared with energy flow results of the DN in a normal state without optimization. The results demonstrate that the DN loss is reduced significantly using the proposed OPF. The reason is that EHubs generate more power to balance load in local areas, and thereby change the power flow in the electric network.

In the previous study, the NG pressure constraints at node 003 and 004 are set in a large range considering the function of compressors in MTs. Since it is necessary to consider the requirements of pipeline pressure from other loads in the NG network, the OPF model of the DN containing EHubs is investigated taking the NG pressure into account. The real time NG pipeline pressure and corresponding compression ratio of the compressor located in node 001 after optimization are studied under different NG pipeline constraints (Constraint I: \(0.2 \leq p \leq 1.3\); Constraint II: \(0.8 \leq p \leq 1.2\)) in the following cases. The real time NG pipeline pressure and corresponding compression ratio after optimization are shown in Fig. 13. The simulation results demonstrate that the NG pipeline pressure fluctuates more obviously under the larger range constraint I. The pipeline pressure is close to 0.2 in individual moments, which can affect the surrounding NG loads seriously.

![Fig. 8. System losses and operating cost comparison in Case A.](image)

![Fig. 9. Electric and gas tie-line power under objective I and II.](image)

![Fig. 10. Scheme of the studied Case B.](image)

![Fig. 11. Solar Radiation.](image)

![Fig. 12. The power losses of the DN under two operation states.](image)

![Fig. 13. Comparison of NG network pressure.](image)
Although the regulation of compressors can guarantee enough gas supply for the MT and similar OPF results under the two constraints, a larger range of NG pressure constraint causes the compression ratio rising significantly, as shown in Fig. 14. A high compression ratio would cause increased power consumption of the compressor and operating cost of the NG system. However, in case of heavy loads, the compressor operating close to the upper bound would result in a deficiency of NG supply for the MT, which will further cause the failure to reach the optimized set point. Moreover, if the NG compressor is shut down under high compression ratio, the NG network is unable to maintain the required pressure. When there is a high ratio of gas-powered generation units in the NG network, the effect will be even more distinctive. Therefore, considering the pipeline pressure in the OPF model has a significant impact on the security and reliable operation of the DN.

This paper investigates OPF models for the DN in two spatial scales for energy system owned by customers or utilities. In the local scale, two optimization models are presented for multiple energy system management in a microgrid. In the urban scale, the impact of gas network on the DN is investigated incorporating multiple energy supplies and unbalanced three-phase electric power system characteristics from the view of the utility. The coupling units are formulated by the EHub model to capture the interactions among various energy systems. Based on the models, an integrated simulation and dispatch platform is developed for the DN. The results confirm that the proposed method is effective in both district and urban areas. It can be concluded that electric power systems, gas systems, and heat systems can be well coordinated by the proposed OPF tool with the integration of CHP units.

ACKNOWLEDGMENT

This work was supported by Engineering and Physical Sciences Research Council (EPSRC) under Grant No. EP/L001063/1 and the National Science Foundation of China (No. 51377117).

REFERENCES


