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Interaction between parallel Gaussian electromagnetic beams in the ionosphere

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Abstract

In this paper, the propagation of two initially \((z = 0)\) parallel Gaussian electromagnetic beams, propagating in the \(z\)-direction in ionosphere, has been investigated. The nonlinearity in the dielectric function, responsible for the interaction between the beams arises from the redistribution of the electron density, caused by the nonuniform distribution of electron temperature determined by the Ohmic heating of the electrons and the energy loss of electrons on account of collisions. The wave frequencies have been assumed to be much larger than the electron collision frequency and gyrofrequency. A self-consistent solution of the electromagnetic wave equation and energy balance equation (considering the solar radiation) has been obtained in the paraxial approximation. Second-order coupled ordinary differential equations have been obtained for the distance between the centers of the beams and the beam widths in the \(x\) and \(y\) directions as a function of the distance of propagation along the \(z\)-axis. Using the available database for the mid-latitude daytime ionosphere, the equations have been solved numerically for a range of parameters and a discussion of the results has been presented. The physical basis for the fact that the beams move towards each other, when the resulting irradiance distribution of the two beams has a maximum in the space between the two beams, has been highlighted.

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Keywords: Ionosphere; Parallel Gaussian beams; Self-focusing; Nonlinearity

1. Introduction

Recent interest in self-focusing of electromagnetic beams in plasmas has spilled over to the field of mutual interaction between electromagnetic beams in a plasma. The cross focusing of coaxial electromagnetic beams in a plasma, on account of the dielectric function of either beam depending on the irradiance of both has been investigated for the predominance of collisional (Gurevich, 1978; Sodha and Sharma, 2006), ponderomotive (Sodha et al., 1979) and relativistic (Esarey et al., 1988; Gibbon, 1990; Kumar et al., 1998; Shvets and Pukhov, 1999; Gupta et al., 2005) nonlinearity. The parametric instabilities and ponderomotive nonlinearities have been authoritatively discussed by Shukla et al. (1975) and Yu et al. (1974).

A relatively recent development in this field is the investigation of the interaction between two laser beams, which are not coaxial (Ren et al., 2000, 2002;
Dong et al., 2002; Sodha et al., 2006a). The motivation behind these studies, is to use the results as a first step towards the understanding of the interaction between individual filaments of a beam or individual speckles in a random-phase plate beam. Ren et al. (2000) have demonstrated the mutual attraction between two skew electromagnetic beams, which may cause the beams to spiral around each other using the variational method as well as a three-dimensional particle in cell (PIC) model of laser light in a plasma and considering the relativistic dependence of the electronic mass on the quiver speed of the electrons. The coherent laser beams, linearly polarized along the same direction are subject to attraction or repulsion depending on their being in phase or antiphase. However, this nonlinearity has been approximated by a quadratic expression. Ren et al. (2002) took into account the relativistic nonlinearity and the nonlinearity on account of density modulations from a plasma wake in an exhaustive study of the interaction between two laser beams. It is seen that incoherent beams and coherent beams, which are linearly polarized in perpendicular directions or are circularly polarized get attracted to each other (when their axes are close enough). For coherent beams, linearly polarized but not in mutually perpendicular directions, the beams attract or repel depending on whether the beams are in phase or antiphase. In this analysis also the relativistic nonlinearity was taken as quadratic in nature. Dong et al. (2002) took into account the relativistic [explained by Hora (1975) and Hauser et al. (1988)] as well as the ponderomotive nonlinearity for the study of the time evolution of two interacting beams in a plasma. The detailed results of this transient analysis based on the quadratic nonlinearity and simulation can hardly be compared with the steady-state analysis incorporating saturating nonlinearity (Sodha et al., 2006a, 2007a); however, both the theories lead to situations for attraction and repulsion of the beams. Shukla et al. (2006) have recently investigated the nonlinear interaction between two weakly relativistic crossed laser beams in plasmas. A set of nonlinear mode coupled equations and nonlinear dispersion relations have been derived and analyzed for Raman (backward and forward) scattering instabilities as well as Brillouin and modulation self-focusing instabilities. Sodha et al. (2006a) have investigated this phenomenon analytically in the paraxial approximation. The dielectric function was expressed as a function of the irradiances of the two beams for three types of nonlinearities (viz. collisional, ponderomotive and relativistic); this expression for the dielectric function was substituted in the wave equation and a solution of the resulting nonlinear equation was obtained in the paraxial approximation.

The analyses (Ren et al., 2000, 2002; Dong et al., 2002; Shukla et al., 2006) based on numerical simulation do not bring out the physics of the mutual interaction between the beams. Also the complexity and the corresponding necessary efforts in the numerical computation account for their not being quite appropriate for parametric studies. Following Sodha et al. (2006a) it is interesting to consider the basic physics of interaction for the case of incoherent and circularly polarized or cross-polarized coherent beams; the argument can be extended to the general case.

It can be readily seen that the irradiance distribution resulting from two parallel incoherent beams at a distance $2x_0$, having the ratio of axial irradiances $k$ and the widths of the beams as $r_0$ and $s_0$ has (in between the beams) a maximum when $2x_0 < wr_0$, and a minimum when $2x_0 > wr_0$; the parameter $w$ as a function of $k$ and $s$ can be determined (Sodha et al., 2007a) from the condition $d/dx = 0$ and $d^2I/dx^2 = 0$ [where $R(x, wr_0)$ is the sum of the irradiances of the two beams]. Hence in a nonlinear plasma (in which effective dielectric function is an increasing function of the irradiance) the dielectric function has a maximum or minimum in the space between the two beams, depending on whether $2x_0 < wr_0$ or $2x_0 > wr_0$. Since light bends towards regions of higher refractive index (i.e. higher dielectric function) the beams move towards the maximum irradiance region i.e. towards each other when $2x_0 < wr_0$. Similarly, when $2x_0 > wr_0$ the beams should repel each other; however, the paraxial theory is not relevant to validating this result as it may not be applicable in the regime $2x_0 > wr_0$. Thus, it is not the nature of nonlinearity, but the distance between the axes of the two beams, which determines whether the beams will attract or repel each other.

On account of the abundance and importance of nonlinear processes (Gurevich, 1978) in the field of interaction of high power electromagnetic beams with ionospheric plasma, it is natural to investigate the interaction of close parallel electromagnetic beams in the ionosphere also. Such an investigation is not a straightforward extension of the theory applicable to plasmas on account of considerable...
complexity in the kinetics, collisions and energy balance in the ionospheric plasma. The additional factors, which have to be considered, are

(i) the role of solar radiation in the energy balance,
(ii) the dependence of the frequency of electron–neutral species, electron–ion and ion–neutral species collisions on electron/ion temperatures,
(iii) the dependence of the fraction of energy transferred per collision on electron/ion temperature and
(iv) the number of neutral and ionic species.

In this communication the following simplifying assumptions have been made:

(i) the interaction lasts for periods less than the electron density relaxation time \( \tau_{eN} \) and hence the electron density is constant; the variation of \( \tau_{eN} \) with ionospheric height has been given in Table 13 of Gurevich’s book (\( \tau_{eN} \) lies between 52 s and \( 3.7 \times 10^4 \) s for heights 100–500 km);
(ii) the ion–neutral species collision frequency \( v_{em} \gg \delta_{q1} v_{q1} \), where \( v_{q1} \) is the electron–ion collision frequency and \( \delta_{q1} \) is the mean fraction of excess energy transferred by the electron, during a collision with an ion (this leads to the ion temperature and temperature of the neutral species to be the same); this is specifically true for an ionospheric height of 150 km for which computations have been made and
(iii) the heat capacity (\( \propto \) number density) of the neutral species is much larger than that of ions and electrons (this ensures a constant temperature neutral species heat sink); this is true for the ionosphere, at least up to 1000 km [Tables 1 and 2 Gurevich (1978)].

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
</tr>
</thead>
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<td>141,666.66</td>
<td>977,954.54</td>
<td>-13,257.57</td>
</tr>
<tr>
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<td>110</td>
<td>3465.00</td>
<td>9762.04</td>
<td>-144.31</td>
</tr>
<tr>
<td>120</td>
<td>1949.00</td>
<td>4782.40</td>
<td>-72.50</td>
</tr>
<tr>
<td>130</td>
<td>1070.83</td>
<td>1951.70</td>
<td>-29.35</td>
</tr>
<tr>
<td>150</td>
<td>761.16</td>
<td>606.23</td>
<td>-6.40</td>
</tr>
<tr>
<td>200</td>
<td>516.73</td>
<td>-16.68</td>
<td>6.31</td>
</tr>
</tbody>
</table>

A paraxial theory of interaction between two initially parallel electromagnetic Gaussian beams in the ionosphere has been presented here. The redistribution of the electron density, caused by the nonuniform distribution of electron temperature, determined by the Ohmic heating of the electrons and the energy loss of electrons on account of collisions results in the nonlinear dielectric function, which is responsible for the interaction between the beams. The electron energy loss by thermal conduction has been neglected on account of the electron cyclotron frequency being much greater than the electron collision frequency \( v_{ec} \). The Ohmic heating of the electrons by the nonuniform field of the beams and the loss of electron energy by the collisions gives rise to a nonuniform distribution of the electron temperature, which creates pressure gradients of the electron and ion gases. The balancing of these pressure gradients by the space charge field in the steady-state results in a nonuniform field-dependent distribution of electron density and hence the dielectric function. The present analysis to determine the space-dependent distribution of the electron temperature and thereby the dielectric function is similar to a recent study of Sodha et al. (2006b), which investigates self-focusing of a single beam in a collisional plasma with significant thermal conduction.

The introduction may be concluded by a passing reference to the important fact that the stochastic pulsations are suppressed in laser plasma interaction by laser beam smoothing, using a random-phase plate (Hora, 2006), in which the thermal collisions wash out the density ripples (Hora and Aydin, 1992; Glowacz et al., 2006). The randomization of phase of neighboring beams also leads to smoothing of the beam as shown experimentally for laser plasma interaction by Labaune et al. (1992) and Hora (2006). This topic is also important for
suppression of self-focusing in generation of plasma blocks for laser-driven fusion (Badziak et al., 2004, 2005; Hora et al., 2007).

In general the magnetic field plays an important part in the magnetized plasmas (Shukla and Stenflo, 1984). However, this communication considers the case of the wave frequency much higher than the electron cyclotron frequency and in this approximation the beams can to a satisfactory degree be assumed to be linearly polarized. Since the electron cyclotron frequency is much higher than the electron collision frequency, thermal conduction by electrons can be neglected in the energy balance equation. Gurevich has shown that in the ionosphere the ponderomotive nonlinearity is $10^{-3}$ times the collisional nonlinearity (Gurevich, 1978, Table 3).

2. Analysis

2.1. Radial distribution of electron density

On account of the nonuniform radial distribution of the beam irradiance, there is a corresponding distribution of electron and ion temperatures, which creates pressure gradients in the electron and ion gas; in the steady state these gradients are balanced by the space charge field. This phenomenon, along with the assumption of charge neutrality and efficient energy exchange between the ions and neutral species leads to an expression for the electron density in terms of the electron temperature $T_e$; thus, following earlier workers (Sodha et al., 2007b) one obtains

$$\frac{N_e}{N_{e0}} = \frac{T_{e0} + T}{T_e + T},$$

(1)

where the suffix e refers to electrons and the suffix double zero refers to the regions, which are not affected by the beams. $T$ is the temperature of the neutral species and $N$ refers to the electron density. Eq. (1) is based on assumptions (ii) and (iii), as given in Section 1; these assumptions are indeed valid at the ionospheric height of 150 km (for which computations have been made), as indicated in Section 1.

2.2. Propagation of two initially parallel Gaussian electromagnetic beams in the ionosphere

The dielectric function, corresponding to a linearly polarized electromagnetic wave of frequency much higher than the collision and cyclotron frequencies of electrons and propagating along the direction of geomagnetic field in the ionosphere is given by

$$\varepsilon_j = \varepsilon_{ij} - \varepsilon_{ej},$$

(2a)

where suffixes r and i indicate real and imaginary parts and $j$ refers to the $j$th beam

$$\varepsilon_{ij} = 1 - \left( \frac{\omega_p^2}{\omega_j^2} \right) \left( \frac{N_e}{N_{e0}} \right),$$

(2b)

$$\varepsilon_{ej} \approx \left( \frac{\omega_p^2}{\omega_j^2} \right) \left( \frac{v_{e0}}{\omega_j} \right),$$

(2c)

where $e$ is electronic charge, $m$ is the electronic mass, $\omega_p = \sqrt{(4\pi e^2 N_{e0}/m)}$ and $v_{e0}$ are respectively the plasma frequency and the electron collision frequency in the absence of the field of the waves.

When $\varepsilon_{ej} \ll 1$, its dependence on the temperature of the electrons and the irradiance can be neglected as a good approximation.

The irradiance of the two Gaussian beams propagating along the $z$-axis can at $z = 0$, be represented by

$$E_1 E_1^* = E_{10}^2 \exp \left[ -\frac{(x + x_0)^2}{r_{10}^2} - \frac{y^2}{r_{10}^2} \right]$$

(3a)

and

$$E_2 E_2^* = E_{20}^2 \exp \left[ -\frac{(x - x_0)^2}{r_{20}^2} - \frac{y^2}{r_{20}^2} \right],$$

(3b)

where $E_1$ and $E_2$ are the amplitudes of the electric vectors of the two beams, $2x_0$ is the distance between the axes of the beams at $z = 0$ and $r_{10}$ and $r_{20}$ are the widths of the two beams at $z = 0$.

The electron temperature $T_e$ is a function of $\sum z_j \varepsilon_j E_j E_j^*$ [$z_j$ being a parameter (defined later) and $E_j$ being the amplitude of the electric vector of the $j$th beam of frequency $\omega_j$]. Since $E_j E_j^*$ is a function of $x$ and $y$, one can expand $T_e$ and $N_e$ and hence $\varepsilon_{ej}$ [referring to Eqs. (1) and (2b)] in powers of $x$ and $y$ and in the paraxial approximation retain terms only up to those having $x^2$ and $y^2$. Thus in the paraxial approximation $T_e$, $N_e$ and $\varepsilon_{ej}$ can be expressed as

$$T_e = T_{e0} - \frac{x}{r_{10}} T_{e1x} - \frac{x^2}{r_{10}^2} T_{e2x} - \frac{y^2}{r_{10}^2} T_{e2y},$$

(4)

$$N_e = N_{e0} - x N_{e1x} - x^2 N_{e2x} - y^2 N_{e2y}$$
and
\[ e_{ij} = e_{0j} - \frac{x}{r_{10}} e_{1xj} - \frac{x^2}{r_{10}^2} e_{2xj} - \frac{y^2}{r_{10}^2} e_{2yj}, \quad (5) \]
where \( T_{e0}, T_{e1x}, T_{e2x}, T_{e1y}, N_{e0}, N_{e1x}, N_{e2x}, e_{0j}, e_{1xj}, e_{2xj} \) and \( e_{2yj} \) are functions of \( z \), \( E_1 \) and \( E_2 \).

In the steady state, the amplitude of the electric vector \( E_j \) satisfies the scalar wave equation
\[ \nabla^2 E_j + \frac{\omega_0^2}{c^2} e_j E_j = 0. \quad (6) \]

Following earlier workers (Sodha et al., 2006a), the solution of the wave equation in the paraxial approximation, consistent with the dependence of the dielectric function on transverse coordinates, expressed by Eq. (5) is
\[ E_j E_j^* = \frac{E_{j0}^2}{f_{jx}(\xi)f_{jy}(\xi)} \sqrt{\frac{\omega_0(0)}{\omega_0(\xi)}} \exp \left[ -\frac{(x - (-1)^{\frac{j}{2}} \chi_0 F_j(\xi))^2}{r_{j0}^2 f_{jx}(\xi)} - \frac{y^2}{r_{j0}^2 f_{jy}(\xi)} \right], \quad (7a) \]
\[ E_j E_j^* = \frac{E_{j0}^2}{f_{jx}(\xi)f_{jy}(\xi)} \exp \left[ -\frac{(x - (-1)^{\frac{j}{2}} \chi_0 F_j(\xi))^2}{r_{j0}^2 f_{jx}(\xi)} - \frac{y^2}{r_{j0}^2 f_{jy}(\xi)} \right], \quad (7b) \]
\[ E_j E_j^* = A_{j0}^2(\xi) \exp \left[ -\frac{(x - (-1)^{\frac{j}{2}} \chi_0 F_j(\xi))^2}{r_{j0}^2 f_{jx}(\xi)} - \frac{y^2}{r_{j0}^2 f_{jy}(\xi)} \right], \quad (7c) \]

Further the beam width parameters \( f_{jx}, f_{jy} \) corresponding to \( x \) and \( y \) directions, respectively and parameter \( F_j \) representing the axial separation of \( j \)th beam from \( z \)-axis are given by
\[ \frac{d^2 f_{jx}}{d\xi^2} = -\rho^2 \frac{e_{2xj} f_{jx}^2}{e_{0j} f_{jx}} \left( \frac{p_{jx}^2}{\rho^2 f_{jx}} \right) - \frac{1}{2e_{0j}} \frac{d\omega_j}{d\xi} \frac{df_{jx}}{d\xi}, \quad (8a) \]
\[ \frac{d^2 f_{jy}}{d\xi^2} = -\rho^2 \frac{e_{2yx} f_{jy}^2}{e_{0j} f_{jy}} \left( \frac{p_{jy}^2}{\rho^2 f_{jy}} \right) - \frac{1}{2e_{0j}} \frac{d\omega_j}{d\xi} \frac{df_{jy}}{d\xi}, \quad (8b) \]

and
\[ \frac{d^2 F_j}{d\xi^2} = -\frac{\rho^2}{e_{0j}} \left( e_{2xj} F_j + (-1)^{\frac{j}{2}} e_{1xj} \right) - \frac{1}{2e_{0j}} \frac{d\omega_j}{d\xi} \frac{dF_j}{d\xi}, \quad (8c) \]

where \( \xi = (c/\omega_0 r_{j0}^2) z \) is the dimensionless distance and \( \rho = (r_{10}^2 \rho_0 / c) \) is the initial dimensionless width of the first beam,
\[ p_j = \omega_0^2 / \omega_0^2, \quad s_j = r_{j0}^2 / r_{j0}^2, \quad \mu = x_0^2 / r_{j0}^2 \]
and at \( z = 0, F_j = f_{jx} = f_{jy} = 1. \)

Eq. (7) indicates that the curve \( x = (-1)^{\frac{j}{2}} \chi_0 F_j(\xi), y = 0 \) represents the axis of the \( j \)th beam.

In view of Eq. (8c) the parameter \( \Sigma F_j \) representing the axial separation between the beams is given by
\[ \frac{d^2}{d\xi^2} \sum F_j = -\rho^2 \sum \frac{1}{2e_{0j}} \left( e_{2xj} F_j + (-1)^{\frac{j}{2}} e_{1xj} \right) - \frac{1}{2e_{0j}} \frac{d\omega_j}{d\xi} \frac{dF_j}{d\xi}. \quad (8d) \]

It is evident from Eq. (8d) that for the beams to attract each other
\[ \sum \frac{1}{2e_{0j}} \left( e_{2xj} F_j + (-1)^{\frac{j}{2}} e_{1xj} \right) > 0. \quad (9) \]

2.3. Evaluation of \( e_{0j}, e_{1xj}, e_{2xj} and e_{2yj} \)

From Eq. (7) the irradiance of \( j \)th beam at a point \( (x, y) \), for any value of \( z \) can in the paraxial approximation be expressed as
\[ E_j E_j^* \approx \frac{E_{j0}^2}{f_{jx}f_{jy}} \exp \left( -\frac{\chi_0 F_j^2}{r_{j0}^2 f_{jx}^2} \right) \left[ 1 + x(-1)^{\frac{j}{2}} \left( \frac{2\chi_0 F_j}{r_{j0}^2 f_{jx}} \right) - \frac{\chi_0^2}{r_{j0}^2 f_{jx}^2} \left( 1 - \frac{2\chi_0^2 F_j^2}{r_{j0}^2 f_{jx}^2} \right) - \frac{y^2}{r_{j0}^2 f_{jy}^2} \right]. \quad (7d) \]

Further, from Eqs. (1), (2), (4) and (5), one obtains
\[ e_{0j} = 1 - \Omega p_j \left( \frac{T_{e00} + T}{T_{e0} + T} \right), \quad (10a) \]
\[ e_{1xj} = \Omega p_j \left( \frac{T_{e1x} + T}{T_{e0} + T} \right) \left( \frac{T_{e00} + T}{T_{e0} + T} \right), \quad (10b) \]
\[ e_{2xj} = \Omega p_j \left( \frac{T_{e2x} + T}{T_{e0} + T} \right) \left( \frac{T_{e00} + T}{T_{e0} + T} \right)^2. \quad (10c) \]
and

\[ \psi_{2yj} = \Omega p_j \frac{T_{e2y} (T_{e00} + T)}{(T_{e0} + T)^2}, \]

(10d)

where \( \Omega = \omega_0^j/\omega_j^e \).

\( T_{e0}, T_{e1x}, T_{e2x} \) and \( T_{e2y} \) depend upon the axial irradiances and widths of the two beams. Thus, \( \psi_{0yj}, \psi_{1xj}, \psi_{2xj} \) and \( \psi_{2yj} \) can be obtained as functions of \( z, f_{jx}, f_{jy} \) and \( F_j \) for given axial irradiances \( E_{p00} \) (at \( z = 0 \)) at a given height in the mid-latitude daytime ionosphere, from a knowledge of the dependence of \( T_{e00}, T_{e1x}, T_{e2x} \) and \( T_{e2y} \) on the irradiance \( \sum z_j E_j E_j^* \) and the beam widths.

2.4. Evaluation of electron temperature

The energy balance for the electrons in the ionosphere (Gurevich, 1978) can be expressed as

\[
Q + \mathbf{J} \cdot \mathbf{E} = Q + \frac{e^2 N_e v_e \sum E_j E_j^*}{2m} \frac{1}{\omega_j^2} = N_e \left\{ \sum v_{em} \delta_{em} + v_{ei} \bar{\delta}_{ei} \right\} \times \frac{3}{2} k_B (T_e - T),
\]

(11a)

where \( \mathbf{J} \cdot \mathbf{E} \) is the time-averaged Ohmic loss per unit volume, \( Q \) is the net power per unit volume gained by the electrons [above the thermal energy (3/2) \( N_e k_B T_{e00} \)] from solar radiation, on account of excess energy of electrons produced by photo ionization, \( v_{em} \) is the frequency of electron collision with the \( m \)th neutral species, \( \delta_{em} \) is the fraction of excess energy of an electron (above that of neutral species) lost in a collision with the \( m \)th neutral species, \( v_{ei} \) is the electron collision frequency with ions and \( \bar{\delta}_{ei} \) is the mean fraction of excess energy of an electron (above that of an ion) lost in a collision with an ion.

The temperature of ions and neutral species is taken to be the same and independent of the field of the beams in view of the assumptions (ii) and (iii) of Section 1. Further the electron cyclotron frequency \( \omega_j \leq \omega_j^e \) but it is much greater than the electron collision frequency \( v_e \) and hence the electron energy loss by thermal conduction has been neglected. In the absence of the waves (i.e. in undisturbed ionosphere) \( \mathbf{J} \cdot \mathbf{E} = 0 \) and Eq. (11a) reduces to

\[
Q = \frac{3}{2} k_B (T_{e00} - T) N_e \left\{ \sum v_{em0} \delta_{em0} + v_{ei0} \bar{\delta}_{ei0} \right\},
\]

(11b)

where suffix zero indicates the values in the absence of the beams, which have been tabulated by Gurevich (1978). From Eqs. (11a) and (11b) one obtains

\[
\sum \frac{z_j E_j E_j^*}{M_H} = \frac{1}{v_e} \left\{ \left( \sum v_{em} \delta_{em} + v_{ei} \bar{\delta}_{ei} \right) \left( \frac{T_e}{T} - 1 \right) \right\} - \frac{N_e}{N_e} \left( \sum v_{em0} \delta_{em0} + v_{ei0} \bar{\delta}_{ei0} \right) \times \left( \frac{T_{e00}}{T} - 1 \right),
\]

(12)

where \( z_j = (2000 e^2/3 m_o \omega_0^j k_B T) \).

The factor 2000 (\( \approx M_H/m, M_H \) being mass of a hydrogen atom) has been introduced for numerical convenience. Since all the terms on the right-hand side of Eq. (12) are known functions (Gurevich, 1978) of electron temperature \( T_e \) (at a given height), this equation expresses a useful relationship between the electron temperature \( T_e \) and the irradiance \( z_j E_j E_j^* \).

At 150 km, the significant ionic species in the ionosphere are \( \text{NO}^+, \text{O}_2^+ \) and \( \text{O}^+ \), with concentrations, tabulated by Gurevich (1978) (Table 2); further \( \bar{\delta}_{ei} \) is given by

\[
\bar{\delta}_{ei} = \frac{2}{M_H} \left\{ \left( \frac{n_{\text{O}^+}}{16} \right) + \left( \frac{n_{\text{O}_2^+}}{32} \right) + \left( \frac{n_{\text{NO}^+}}{30} \right) \right\},
\]

(13)

where 16, 30 and 32 being the molecular/atomic masses of the ionic species \( \text{O}^+ \), \( \text{NO}^+ \) and \( \text{O}_2^+ \), respectively in atomic mass units, \( n_{\text{O}^+}, n_{\text{O}_2^+} \) and \( n_{\text{NO}^+} \) being the fractional concentrations of ions, tabulated by Gurevich (1978) as functions of height.

The significant neutral species at 150 km height are \( \text{N}_2, \text{O}_2 \) and \( \text{O} \). Gurevich (1978) has described the temperature dependence of \( v_{em} \) and \( \delta_{em} \) corresponding to these neutral species. The electron–ion collision frequency \( v_{ei} \) is however given by

\[
v_{ei} = v_{e0} \frac{N_e}{N_{e00}} \left( \frac{T_e}{T_{e00}} \right)^{-3/2},
\]

(14)

and electron collision frequency \( v_e \) is given by

\[
v_e = v_e + \sum v_{em}.
\]

(15)

The mid-latitude daytime model of the ionosphere and the data base on the collision processes [collated by Gurevich (1978)] can be used to obtain a relationship between \( v_e \left( \sum v_{em} \delta_{em} + v_{ei} \delta_{ei} \right) \) and the electron temperature \( T_e \). The relationships (best fit) can be represented empirically for a given
height as
\[
\sum v_{em}\delta_{em} + v_{ei}\delta_{ei} = a_0 + a_1 \left( \frac{T_e}{T_{e00}} \right) + a_2 \left( \frac{T_e}{T_{e00}} \right)^2 + a_3 \left( \frac{T_e}{T_{e00}} \right)^3, \quad (16a)
\]
\[
\sum v_{em}\delta_{em} + v_{ei}\delta_{ei} = A_1 - \frac{x}{r_{10}} A_2 - \frac{x^2}{r_{10}^2} A_3 + \frac{y^2}{r_{10}^2} A_4, \quad (16b)
\]
and
\[
v_e = b_0 + b_1 \left( \frac{T_e}{T_{e00}} \right) + b_2 \left( \frac{T_e}{T_{e00}} \right)^2, \quad (17a)
\]
\[
v_e = B_1 - \frac{x}{r_{10}} B_2 - \frac{x^2}{r_{10}^2} B_3 - \frac{y^2}{r_{10}^2} B_4, \quad (17b)
\]
where
\[
A_1 = a_0 + a_1 \left( \frac{T_{e0}}{T_{e00}} \right) + a_2 \left( \frac{T_{e0}}{T_{e00}} \right)^2 + a_3 \left( \frac{T_{e0}}{T_{e00}} \right)^3, \quad (18a)
\]
\[
A_2 = \frac{T_{e1x}}{T_{e00}} \left( a_1 + a_2 \frac{2T_{e0}}{T_{e00}} + a_3 \frac{3T_{e0}^2}{T_{e00}^2} \right), \quad (18b)
\]
\[
A_3 = a_1 \frac{T_{e2x}}{T_{e00}} + a_2 \frac{2T_{e0}T_{e2x} - T_{e1x}^2}{T_{e00}^2} + a_3 \frac{3T_{e0}(T_{e0}T_{e2x} - T_{e1x}^2)}{T_{e00}^3}, \quad (18c)
\]
\[
A_4 = \frac{T_{e2y}}{T_{e00}} \left( a_1 + a_2 \frac{2T_{e0}}{T_{e00}} + a_3 \frac{3T_{e0}^2}{T_{e00}^2} \right), \quad (18d)
\]
\[
B_1 = b_0 + b_1 \left( \frac{T_{e0}}{T_{e00}} \right) + b_2 \left( \frac{T_{e0}}{T_{e00}} \right)^2, \quad (19a)
\]
\[
B_2 = \frac{T_{e1x}}{T_{e00}} \left( b_1 + b_2 \frac{2T_{e0}}{T_{e00}} \right), \quad (19b)
\]
\[
B_3 = b_1 \frac{T_{e2x}}{T_{e00}} + b_2 \frac{2T_{e0}T_{e2x} - T_{e1x}^2}{T_{e00}}, \quad (19c)
\]
and
\[
B_4 = \frac{T_{e2y}}{T_{e00}} \left( b_1 + b_2 \frac{2T_{e0}}{T_{e00}} \right). \quad (19d)
\]

Table 1 and 2 represent the values of the parameters \(a_0, a_1, a_2, a_3, b_0, b_1, b_2\), which have been evaluated for 80–200 km height using database of Gurevich (1978).

Substituting for \((N_e/N_{e00}), E_j E_k^*, v_e, \sum v_{em}\delta_{em} + v_{ei}\delta_{ei}\) and \(T_e\) from Eqs. (1), (7d), (17b), (16b) and (4), respectively in Eq. (12), expanding the terms in powers of \(x\) and \(y\), equating the coefficients of \(x, x^2, y^2\) and the term independent of \(x\) and \(y\) on both sides of the equation one obtains
\[
\eta_1 - B_1 \sum K_j = 0, \quad (20a)
\]
\[
\eta_2 B_1 - \eta_1 B_2 + B_1^2 \sum (-1)^j K_j \frac{2s_j F_j \sqrt{\mu}}{f_{jx}^2} = 0, \quad (20b)
\]
\[
B_1^2 \eta_3 + \eta_2 B_1 - \eta_1 (B_3 B_1 + B_2^2) \quad - B_1^2 \sum s_j K_j \frac{s_j F_j^2}{f_{jx}^2} \left( 1 - \frac{2s_j F_j^2}{f_{jx}^2} \right) = 0, \quad (20c)
\]
and
\[
\eta_4 B_1 - B_4 \eta_1 - B_1^2 \sum s_j K_j \frac{s_j F_j^2}{f_{jy}^2} = 0, \quad (20d)
\]
where
\[
A = (a_0 + a_1 + a_2 + a_3)(t - 1),
\]
\[
\eta_1 = A_1(t_1 - 1) - A_1(t_1 + 1),
\]
\[
\eta_2 = (A_1 - A)t_2 + A_2(t_1 - 1),
\]
\[
\eta_3 = (A_1 - A)t_3 + A_3(t_1 - 1) - A_2 t_2,
\]
\[
\eta_4 = (A_1 - A)t_4 + A_4(t_1 - 1),
\]
\[
K_j = \frac{s_j F_j^2}{f_{jx}^2} \exp \left( \frac{-\mu s_j F_j^2}{f_{jx}^2} \right); \quad j = 1, 2, \quad (21)
\]
\[
t_1 = (T_{e0}/T), \quad t_2 = (T_{e1x}/T), \quad t_3 = (T_{e2x}/T),
\]
\[
t_4 = (T_{e2y}/T) \quad \text{and} \quad t = (T_{e00}/T).
\]

Thus, for a given set of parameters, the dependence of the parameters \(T_{e0}, T_{e1x}, T_{e2x}\) and \(T_{e2y}\) on \(\frac{z_0^2}{F_{j00}}\) (at \(z = 0\)) may be evaluated from the above-coupled equations.

3. Computations for the interaction between identical and incoherent beams

The mid-latitude daytime model of the ionosphere detailed by Gurevich (1978) and the database on collision processes (Gurevich, 1978) has been used for numerical computations. The values of the parameters \(a_0, a_1, a_2, a_3, b_0, b_1, b_2\) consistent with the empirical relationships represented by Eqs. (16a) and (17a) have been evaluated and
tabulated (Tables 1 and 2) for different heights (80–200 km) using database of Gurevich (1978).

Using C++ programming the computations have been made to investigate the simultaneous propagation of two initially parallel identical and incoherent Gaussian electromagnetic beams with \( r_{10} = r_{20} = r_0 \), \( x_1 E_{100}^2 = x_2 E_{200}^2 = xE_{00}^2 \) and wave frequency \( \omega_1 = \omega_2 = \omega = 5 \times 10^7 \text{ rad/s} \). Eqs. (20) have been solved numerically to determine the dependence of the parameters \( T_{e0} \), \( T_{e1x} \), \( T_{e2x} \) and \( T_{e2y} \) [i.e. the electron temperature \( T_e \) as defined in Eq. (4)] on axial irradiance \( zE_0^2 \) for a given set of parameters; Eqs. (10) have been used to evaluate the dependence of the dielectric function defined by Eq. (5) on axial irradiance \( zE_0^2 \). From the knowledge of the dependence of the dielectric function on axial irradiance the variation of the beam width parameters \( f_{1x} = f_{2x} = f_x \) and \( f_{1y} = f_{2y} = f_y \), and the parameter \( F_1 = F_2 = F \) (representing the separation of either beam from the \( z \)-axis) with dimensionless distance of propagation \( \xi = (c/\omega r_0^2)z \) has been determined for the set of parameters described above.

4. Numerical results and discussion

It has been discussed in the introduction that the beams will move towards each other, when the resulting irradiance distribution of the two beams has a maximum in the space between the two beams. The numerical computations have been made to investigate the interaction of identical Gaussian beams. In case of identical beams Eqs. (20b) and (20c) give

\[ T_{e1x} = 0 \]  

(22a)

and

\[ \frac{T_{e2x}}{T_{e0}} = \frac{B_1}{B_2} \left\{ (A_1 - A) \frac{T_{e0}}{T} + \left( a_1 + 2\frac{A_1}{T_{e0}} + a_2 \frac{3T_{e0}^2}{T_{e0}^2} \right) (t_1 - 1) \right\} 
- \eta_1 \left( b_1 + b_2 \frac{2T_{e0}}{T_{e0}} \right) 
= \frac{2K}{f_x^2} \left( 1 - \frac{2\mu F^2}{f_x^2} \right), \]  

(22b)

where

\[ K = \frac{zE_0^2}{f_x f_y} \exp \left( -\frac{\mu F^2}{f_x^2} \right). \]

In view of Eqs. (22a) and (10b) the condition (9) of the attraction between the two beams reduces to

\[ \frac{2\Omega T_{e2x}(T_{e0} + T)F}{\varepsilon_0(T_{e0} + T)^2} > 0, \]  

(23a)

where

\[ \varepsilon_0 = 1 - \Omega \frac{(T_{e0} + T)}{T_{e0} + T}. \]

It is evident from Eq. (22b) and condition (23) that the beams bend towards each other when

\[ \mu < \frac{F^2}{2\rho^2} \]  

(23b)

at \( z = 0, \mu < 0.5 \).

The dependence of \( 2\varepsilon_{2z}/\varepsilon_0 \) on \( \mu = x_0^2/r_0^2 \) is illustrated in Fig. 1.

Evidently, under the condition (23) the resulting irradiance distribution of the two beams also has a maximum in the space between the two beams.

Fig. 2 illustrates the dependence of \( F, (\varepsilon_0 F) \) being the distance of the axis of the beam from \( x = 0 \) axis in the \( xz \)-plane) on \( \rho^2 \), the parameter which characterizes the initial width of the beam. It is seen that the bending of the beams towards each other increases with increasing \( \rho^2 \). This can be readily explained by the fact that increasing \( \rho^2 \) leads to higher electron temperature and associated nonlinearity.

Fig. 3 represents the dependence of \( F \) on the initial separation of the beams; it can be seen that with decrease of the parameter \( \mu = x_0^2/r_0^2 \) i.e. with beams getting initially closer, the interaction of the

Fig. 1. Critical condition for the bending of the identical beams towards each other at \( z = 0 \); variation of \( (2\varepsilon_{2z}/\varepsilon_0) \) with \( \mu = x_0^2/r_0^2 \) at \( z = 0 \). The dependence of \( (2\varepsilon_{2z}/\varepsilon_0) \) on \( \mu \) corresponding to various beam irradiances \( zE_0^2 = 10, 50, 100 \) are, respectively shown by dotted, dash-dotted and solid curves. For the attraction between the two beams \( (2\varepsilon_{2z}/\varepsilon_0) > 0 \) or \( \mu < 0.5 \).
beams gets stronger. This is qualitatively an obvious conclusion.

Fig. 3 illustrates the dependence of the parameter $F$ on the axial irradiance $zE_{00}^2$ of the two beams (taken as equal). It is seen that the bending of the beams towards each other increases with increasing irradiance of the beams. The greater irradiance of the beams leads to greater nonlinearity and hence increased bending of the beams.

Fig. 5 shows the axes of the two beams; for the specific case of equal irradiances $zE_{00}^2 = 100$, width $\rho^2 = 10$ and frequency $\omega = 5 \times 10^7$ rad/s of the two incoherent beams. It is seen that the axes are symmetrical about $z$-axis in the $xz$-plane.

Fig. 6 represents the variation of $f_x$ and $f_y$ with distance of propagation $\xi$. It is seen that as $\xi$ increases, $f_x$ and $f_y$ become significantly different. This indicates that a symmetrical Gaussian beam turns into an elliptical Gaussian beam on account of the interaction with the other beam.

The computations for other heights can be readily made by using Tables 1 and 2 for the relevant coefficients.
Fig. 6. Dependence of beam width parameters $f_x$ and $f_y$ corresponding to $x$- and $y$-axis, respectively with distance of propagation $\xi = (c/\omega_0^2)z$, corresponding to $\omega_0^2 = 100$, $\rho^2 = 10$, $\alpha = 5 \times 10^7 \text{rad/s}$ and $\mu = x_0^2/r_0^2 = 0.3$ for the daytime mid-latitude ionospheric height of 150 km.

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References


