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Self-focusing instability in ionospheric plasma with thermal conduction

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In this communication, an expression for the growth rate of self-focusing instability in the ionospheric plasma has been derived after taking finite thermal conduction into account. The instability arises on account of the depletion of electrons from regions where the irradiance of the perturbation is large. In contrast to earlier work, an appropriate energy balance equation for electrons and ions and the proper dependence of thermal conductivity on electron temperature have been used. The dependence of the growth rate of the filamentation instability on the background irradiation, thermal conductivity, and the wave number of transverse perturbation has been investigated. The mid-latitude daytime ionospheric model of Gurevich has been used for numerical computations, corresponding to a height of 200 km. The gradient of irradiance perturbations is assumed to be along the magnetic field of the Earth. The numerical results have been illustrated graphically and discussed. © 2007 American Institute of Physics. [DOI: 10.1063/1.2727449]

I. INTRODUCTION

There has been considerable interest in the growth of an instability (associated with the propagation (along the z axis) of a high power electromagnetic beam in a nonlinear medium), which is characterized by beam irradiance and electron density fluctuations in a direction (x axis) transverse to that of the beam propagation. The fact that higher beam irradiance in a region causes lower electron density and consequently higher refractive index leads to the increase in irradiance in this region; this is synonymous with self-focusing. Hence, such instabilities are known as self-focusing instabilities. The saturated state of such an instability results in light filamentation. Several publications^{1–12} in this field have appeared in the scientific literature since 1962. Apart from the scientific point of view, the results in this field have a bearing on ionospheric modification experiments,^{13–19} beams from proposed satellite power stations passing through the ionosphere,²⁰ and laser plasma interaction phenomena.

It is of interest to consider the mechanism of self-focusing instability in a plasma. A small perturbation $E_1(\mathbf{r})$, superposed on a uniform beam, with electric vector E_0 , causes the electrons to concentrate towards the region, where E_1 is maximum ($E_{1\max}$). This redistribution of electrons causes a refractive index gradient, which if large enough to overcome diffraction, leads to a continuous increase in $E_{1\max}$ as the beam propagates; this phenomenon is referred to as self-focusing instability.

This communication presents an investigation of the self-focusing instability in the ionosphere. The energy balance of the electrons includes contributions from the solar radiation, Ohmic heating, energy loss in collisions with neu-

tral atoms/molecules/ions, and energy loss by thermal conduction. The energy balance for the ions is obtained by equating the energy gained from the electrons to the energy lost to neutral species. Since the heat capacity (proportional to the number density) of the neutral species is several (10^2 to 10^3) times the heat capacity of the electrons and ions, the neutral species act as a constant temperature sink. The mid-latitude ionospheric model of Gurevich¹⁹ has been used to provide the basic data for the computations.

Perkins and Valeo²¹ were the first to highlight the role of electronic thermal conduction in filamentation; however their analysis ignores the change in the temperature of the electrons on account of the main beam. As may be seen from the present analysis this is a serious omission. Tewari *et al.*²² have studied filamentation, in the case, when the electron energy loss by collisions has been neglected; further, the temperature of the electrons in the region where the irradiance of the filament is highest has been implicitly assumed as the temperature in the absence of the beam. Using the available theory of filamentation on account of ponderomotive nonlinearity and taking thermal conduction into account Schmidt²³ has analytically and numerically studied the effect of induced spatial incoherence and random phase screen on controlling filamentation. Cornolti and Lucchesi²⁴ have considered the energy loss by collisions as well as by electronic thermal conduction in the analysis of filamentation; however, like Perkins and Valeo,²¹ they have also ignored the change in the electron temperature on account of the main beam. Epperlein²⁵ has, in his elegant analysis of filamentation, assumed the electron and ion temperature to be the same, which is not a good approximation for the ionospheric plasma subjected to the field of a high power electromagnetic wave.

Ghanshyam and Tripathi²⁶ have analyzed the phenom-

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enon of filamentation in a collisional plasma, taking into account thermal conduction as well as collisions with heavier particles as the mechanisms of power loss by the electrons, heated by the beam; however, their energy balance equation [Eq. (42) of their paper] and the expression for the electron temperature in the absence of perturbation [their Eq. (45)] are not correct. This is on account of the use of the modified parameter δ' , which has justification when the space dependence of the amplitude of the field is the same as that of the perturbation E_1 (this is not true for E_0 , which is uniform) and also when the dependence of thermal conductivity on the electron temperature is ignored. Further, this analysis does not take into account the change in the ion temperature on account of Ohmic heating of electrons by the electric field of the beam and subsequent collisions of these hot electrons with ions. These omissions necessitate an improved version of the analysis. This has been presented in this communication with reference to the ionosphere. Expressions for the growth rate of the instability and the conditions for the instability to occur, maximum value of the growth rate and the corresponding value of $q_\perp$ have been derived and a discussion of numerical results (corresponding to an ionospheric height of 200 km and the magnetic field of the earth along the x axis) has been presented.

II. THE ANALYSIS

Let the electric field of a beam of uniform illumination and that of a small perturbation (filament) be represented by $\hat{j}E_0$ and $\hat{j}E_1$, respectively, so that the resultant field E propagating in the z direction through a plasma can be expressed as

$$E = \hat{j}(E_0 + E_1) = \hat{j}(A_0 + A_1)\exp i(\omega t - kz), \quad (1)$$

where A_0 , without loss of generality, is a real constant and $A_1(|A_1| \ll A_0)$ is complex, $\hat{j}$ is a unit vector and k is the wave number defined later. Neglecting the small contribution $A_1A_1^*$, one can write

$$E \cdot E^* = A_0^2 + A_0(A_1 + A_1^*). \quad (2)$$

The dielectric function of the plasma depends on $E \cdot E^*$ and hence can be expressed as

$$\epsilon(z, E \cdot E^*) = \epsilon_0(z) + \epsilon_2(z)A_0(A_1 + A_1^*), \quad (3)$$

where

$$\epsilon_2 = \left[\frac{\partial \epsilon}{\partial (E \cdot E^*)} \right]_{E \cdot E^* = A_0^2}.$$

The wave equation for the total field can be separated for A_0 and A_1 . On choosing

$$k = \frac{\omega}{c} \sqrt{\epsilon_0},$$

the wave equation for A_0 yields a solution

$$A_0 = \text{constant};$$

without loss of generality, A_0 can be assumed to be real.

The wave equation for A_1 , on neglecting $\partial^2 A_1 / \partial z^2$ (assuming A_1 to be slowly varying) and $A_1 A_1^*$, reduces to

$$-2ik \frac{\partial A_1}{\partial z} + \nabla_\perp^2 A_1 + \frac{\omega^2}{c^2} \epsilon_2 A_0^2 (A_1 + A_1^*) = 0, \quad (4)$$

where $\nabla_\perp^2 = \nabla^2 - \partial^2 / \partial z^2$.

To solve Eq. (4), one can express the complex amplitude A_1 of the perturbation as

$$A_1 = A_{1r} + iA_{1i},$$

where A_{1r} and A_{1i} are real. Assuming A_{1r} and A_{1i} to be independent of y and proportional to $\exp i(q_\parallel z + q_\perp x)$; i.e.,

$$A_{1r} \propto \exp i(q_\parallel z + q_\perp x), \quad A_{1i} \propto \exp i(q_\parallel z + q_\perp x), \quad (5)$$

and following Ghanshyam and Tripathi,²⁶ one obtains two homogeneous equations in A_{1r} and A_{1i} , which on elimination of these amplitudes lead to the dispersion relation

$$q_\parallel = \frac{-iq_\perp}{2k} \left(2 \frac{\omega^2}{c^2} \epsilon_2 A_0^2 - q_\perp^2 \right)^{1/2}. \quad (6)$$

For the sake of convenience, the magnetic field of the Earth is assumed to be along the x axis and the electron cyclotron frequency to be much less than ω . As discussed later, in the ionosphere thermal conduction is important only when the magnetic field lies nearly along the x axis (otherwise it is negligible).

The parameter $q_\parallel$ will be imaginary, when

$$q_\perp^2 < 2(\omega^2/c^2) \epsilon_2 A_0^2. \quad (7)$$

In this case the field of the filament, $A_1 \exp iq_\parallel z$ will grow exponentially with z at a space rate

$$\Gamma = iq_\parallel = \frac{q_\perp}{2k} \left(2 \frac{\omega^2}{c^2} \epsilon_2 A_0^2 - q_\perp^2 \right)^{1/2}. \quad (8)$$

As pointed out earlier, Ghanshyam and Tripathi's²⁶ analysis of the dielectric function of the plasma determining ϵ_2 and ϵ_0 is not correct because the axial electron temperature T_{e0} [their Eq. (45)] in the absence of the perturbing field E_1 involves the coefficient of thermal conductivity and the wave number q of the perturbing field through the modified electron energy transfer parameter δ' [their Eq. (43)]; this is obviously not correct because A_0 and hence T_{e0} in the field of the main beam only, do not depend on (x, y, z) ; hence, thermal conduction should not enter into the expression for T_{e0} . Further, the use of δ' makes sense only when the strong dependence of the thermal conductivity on the electron temperature is neglected. This analysis also does not take into account the heating of ions by electron-ion collisions.

III. EVALUATION OF ϵ_0 AND ϵ_2

A. Energy balance

Following Gurevich,¹⁹ the energy balance for the electrons in the ionosphere in the presence of the electric field E of a high irradiance electromagnetic wave may be expressed as

$$\begin{aligned}
Q + \overline{\mathbf{J} \cdot \mathbf{E}} &= Q + \frac{e^2 N_e \nu_e}{2m\omega^2} EE^* \\
&= N_e \sum \nu_{em} \delta_{em} \left(\frac{3}{2} k_B T_e - \frac{3}{2} k_B T \right) \\
&\quad + N_e \nu_{ei} \bar{\delta}_{ei} \left(\frac{3}{2} k_B T_e - \frac{3}{2} k_B T_i \right) - \nabla \cdot (K_e \nabla T_e), \quad (9)
\end{aligned}$$

where Q is the net power per unit volume gained by the electrons from solar radiation, on account of excess energy of electrons, produced by photo-ionization, above the thermal energy $3N_e k_B T_{e00}/2$.

- The subscript “00” corresponds to the regions away from the beam,
- $\mathbf{J} \cdot \mathbf{E}$ is the time averaged Ohmic loss per unit volume,
- ν_{em} is the collision frequency of electrons with the m th neutral species,
- δ_{em} is the fraction of excess energy of an electron (above that of neutral species) lost in a collision with the m th neutral species,
- ν_{ei} is the electron collision frequency with ions,
- $\bar{\delta}_{ei}$ is the mean fraction of excess energy of an electron (above that of an ion) lost in a collision with an ion,
- the collision and cyclotron frequency of the electrons is much less than the wave frequency ω ,
- K_e is the electronic thermal conductivity,
- N_e is the number density of electrons,
- T_e is the electron temperature,
- T_i is the ion temperature, and
- T is the temperature of neutral species.

It may be remembered (Ref. 19, pp. 212–213) that in the ionosphere, the main contribution of the energy flux is made by thermal conduction $K_e \nabla(T_e)$ and the contribution of thermal force is negligible. In particular, for heat propagation along the magnetic field of the Earth (x axis), the thermal conduction is the dominant process for heights up to 500 km; the expression for the relevant coefficient of heat conduction is also the same as that for an isotropic plasma.

The energy balance equation for the ions is

$$N_e \nu_{ei} \bar{\delta}_{ei} \left(\frac{3}{2} k_B T_e - \frac{3}{2} k_B T_i \right) = N_e \bar{\nu}_{im} \delta_{im} \left(\frac{3}{2} k_B T_i - \frac{3}{2} k_B T \right).$$

Hence,

$$T_i = \frac{\nu_{ei} \bar{\delta}_{ei} T_e + \bar{\nu}_{im} \delta_{im} T}{\nu_{ei} \bar{\delta}_{ei} + \bar{\nu}_{im} \delta_{im}}. \quad (10)$$

ν_{im} is the collision frequency of an ion with a neutral molecules/atoms and δ_{im} (≈ 1) is the fraction of excess energy transferred per ion-neutral species collision. Substituting for T_i from Eq. (10) in Eq. (9), one obtains

$$\begin{aligned}
Q + \overline{\mathbf{J} \cdot \mathbf{E}} &= Q + \frac{e^2 N_e \nu_e}{2m\omega^2} EE^* \\
&= N_e \left\{ \sum \nu_{em} \delta_{em} + \frac{\nu_{ei} \bar{\delta}_{ei} \cdot \bar{\nu}_{im}}{\nu_{ei} \bar{\delta}_{ei} + \bar{\nu}_{im}} \right\} \left(\frac{3}{2} k_B T_e - \frac{3}{2} k_B T \right) \\
&\quad - \nabla \cdot (K_e \nabla T_e), \quad (11a)
\end{aligned}$$

In the absence of the wave (i.e., in undisturbed ionosphere) $\overline{\mathbf{J} \cdot \mathbf{E}} = 0$; hence, Eq. (11a) reduces to

$$Q = N_{e00} \left(\frac{3}{2} k_B T_{e00} - \frac{3}{2} k_B T \right) \left\{ \sum \nu_{em0} \delta_{em0} + \frac{\nu_{ei0} \bar{\delta}_{ei0} \bar{\nu}_{im0}}{\nu_{ei0} \bar{\delta}_{ei0} + \bar{\nu}_{im0}} \right\}. \quad (11b)$$

From Eqs. (11a) and (11b), one obtains

$$\begin{aligned}
\alpha EE^* &= 2000 \nu_e^{-1} \left\{ \left(\frac{T_e}{T} - 1 \right) \left(\sum \nu_{em} \delta_{em} + \frac{\nu_{ei} \bar{\delta}_{ei} \cdot \bar{\nu}_{im}}{\nu_{ei} \bar{\delta}_{ei} + \bar{\nu}_{im}} \right) \right. \\
&\quad \left. - \left\{ \left(\frac{T_{e00}}{T} - 1 \right) \left(\frac{N_{e00}}{N_e} \right) \left(\sum \nu_{em0} \delta_{em0} \right. \right. \right. \\
&\quad \left. \left. + \frac{\nu_{ei0} \bar{\delta}_{ei0} \cdot \bar{\nu}_{im0}}{\nu_{ei0} \bar{\delta}_{ei0} + \bar{\nu}_{im0}} \right) \right\} - \frac{2 \nabla (K_e \nabla T_e)}{3 k_B N_e T} \right\}, \quad (12)
\end{aligned}$$

where $\alpha = 2000 e^2 / 3 m \omega^2 k_B T$ and the factor 2000 ($\approx M_H / m$) has been introduced for numerical convenience. M_H is the mass of a hydrogen atom.

B. Distribution of electron density

On account of the nonuniform spatial distribution of the irradiance of the instability, there is a corresponding nonuniform distribution of electron and ion temperatures, which creates pressure gradients of electron and ion gas. Initially these gradients push the electrons and ions from regions of higher irradiance and thus temperature (and hence pressure) to regions of lower irradiance; this transport of electrons and ions continues until a steady state is reached; i.e., when these gradients are balanced by the space charge field. With this reasoning in mind and following earlier workers,²⁷ one can write

$$\frac{N_e}{N_{e00}} = \frac{T_{e00} + T_{i00}}{T_e + T_i}. \quad (13)$$

This equation is valid (Ref. 19, p. 234) even when the electron-ion force is taken into account. However, the above equation is not quantitatively consistent with the condition of hydrodynamic equilibrium or kinetic theory (for the simple dependence of ν_e on electron density) based on Boltzmann's transfer equation.

C. Electron temperature

The electron-ion collision frequency and electronic thermal conductivity²⁸ (for heat propagation along the direction of the magnetic field) may be expressed as

$$\nu_{ei} = \nu_{ei0} (N_e / N_{e00}) [T_e / T_{e00}]^{-3/2}, \quad (14)$$

$$K_e = \frac{5N_e k_B^2 T_e}{2m\nu_e}, \quad (15)$$

and

$$\nu_e = \nu_{ei} + \sum \nu_{em}. \quad (16)$$

When the magnetic field B makes an angle θ with the direction of heat propagation (say, the x axis), $1/\nu_e$ in Eq. (15) is replaced by $\nu_e/(\nu_e^2 + \omega_B^2 \sin^2 \theta)$, where $\omega_B = eB/mc$. Since $\nu_e \ll \omega_B$ for ionosphere, thermal conduction is important only when B lies almost along the x axis.

The temperature dependence of ν_{em} , δ_{em} , and ν_{im} has been tabulated by Gurevich.¹⁹

In the presence of the electric field with the perturbation given by Eqs. (1) and (5), the electron temperature can be expressed as

$$T_e = T_{e0} + T_{e1} \exp(iq_{\parallel}z + iq_{\perp}x), \quad (17)$$

where $T_{e1} \ll T_{e0}$.

To proceed further, it is useful to express (empirically) ν_e [$\sum \nu_{em} \delta_{em} + \nu_{ei} \overline{\delta_{ei}} \cdot \overline{\nu_{im}} / (\nu_{ei} \overline{\delta_{ei}} + \overline{\nu_{im}})$] and $T_e + T_i$ as a polynomial in T_e/T_{e00} , using Eqs. (10) and (12)–(17) and the database of Gurevich¹⁹ (Tables 1, 2, 6, 8, and 11) for an ionospheric height of 200 km. Thus,

$$\left(\sum \nu_{em} \delta_{em} + \frac{\nu_{ei} \overline{\delta_{ei}} \cdot \overline{\nu_{im}}}{\nu_{ei} \overline{\delta_{ei}} + \overline{\nu_{im}}} \right) = a + \frac{T_{e1}}{T_{e00}} A \exp(iq_{\parallel}z + iq_{\perp}x), \quad (18a)$$

$$\nu_e = b + \frac{T_{e1}}{T_{e00}} B \exp(iq_{\parallel}z + iq_{\perp}x), \quad (18b)$$

$$\begin{aligned} T_e + T_i &= \left[\frac{T_e}{T_{e00}} (2\nu_{ei} \overline{\delta_{ei}} + \overline{\nu_{im}}) + \frac{T}{T_{e00}} \overline{\nu_{im}} \right] \frac{T_{e00}}{(\nu_{ei} \overline{\delta_{ei}} + \overline{\nu_{im}})} \\ &= d + \frac{T_{e1}}{T_{e00}} D \exp(iq_{\parallel}z + iq_{\perp}x), \end{aligned} \quad (18c)$$

where

$$a = a_0 + a_1 \left(\frac{T_{e0}}{T_{e00}} \right) + a_2 \left(\frac{T_{e0}}{T_{e00}} \right)^2 + a_3 \left(\frac{T_{e0}}{T_{e00}} \right)^3,$$

$$A = a_1 + 2a_2 \left(\frac{T_{e0}}{T_{e00}} \right) + 3a_3 \left(\frac{T_{e0}}{T_{e00}} \right)^2,$$

$$b = b_0 + b_1 \left(\frac{T_{e0}}{T_{e00}} \right) + b_2 \left(\frac{T_{e0}}{T_{e00}} \right)^2,$$

$$B = b_1 + 2b_2 \left(\frac{T_{e0}}{T_{e00}} \right),$$

$$d = d_0 + d_1 \left(\frac{T_{e0}}{T_{e00}} \right),$$

and

$$D = d_1,$$

where the parameters $a_0, a_1, a_2, a_3, b_0, b_1, b_2, d_0$, and d_1 can be evaluated, as indicated above. Substituting for N_e/N_{e00} , K_e , T_e , [$\sum \nu_{em} \delta_{em} + \nu_{ei} \overline{\delta_{ei}} \cdot \overline{\nu_{im}} / (\nu_{ei} \overline{\delta_{ei}} + \overline{\nu_{im}})$], ν_e , and $T_e + T_i$ from Eqs. (13), (15), (17), and (18a)–(18c) in Eq. (12) and equating the terms with and without $\exp(iq_{\parallel}z + iq_{\perp}x)$ on both sides of the resulting equation, one obtains

$$\frac{T_{e0}}{T} = \left[\left\{ \frac{\alpha A_0^2}{2000} + \frac{\Lambda d \left(\frac{T_{e00}}{T} - 1 \right)}{b \left(\frac{T_{i00}}{T_{e00}} + 1 \right) T_{e00}} \right\} \frac{b}{a} + 1 \right], \quad (19)$$

and

$$\frac{T_{e1}}{T} = \frac{\alpha A_0 (A_1 + A_1^*)}{F(q_{\perp})}, \quad (20)$$

where

$$\begin{aligned} F(q_{\perp}) &= 2000 \left[\frac{a}{b} \left(\frac{T}{T_{e00}} \right) \left\{ \frac{T_{e00}}{T} + \left(\frac{A}{a} - \frac{B}{b} \right) \left(\frac{T_{e0}}{T} - 1 \right) \right\} \right. \\ &\quad \left. - \left\{ \frac{\Lambda}{b} \frac{T}{T_{e00}} \frac{\left(D - \frac{Bd}{b} \right) \left(\frac{T_{e00}}{T} - 1 \right)}{\left(\frac{T_{i00}}{T_{e00}} + 1 \right) T_{e00}} \right\} + \frac{5k_B T}{3mb^2} q_{\perp}^2 \frac{T_{e0}}{T} \right], \end{aligned} \quad (21a)$$

$$\Lambda = \left(\sum \nu_{em0} \delta_{em0} + \frac{\nu_{ei0} \overline{\delta_{ei0}} \cdot \overline{\nu_{im0}}}{\nu_{ei0} \overline{\delta_{ei0}} + \overline{\nu_{im0}}} \right) \quad (21b)$$

and $q_{\perp} \gg q_{\parallel}$.

D. Dielectric function

The dielectric function ϵ can now be obtained from the equation

$$\epsilon = 1 - 4\pi N_e e^2 / m\omega^2$$

by expressing N_e in terms of the electron temperature T_e [Eq. (13)] and using Eqs. (17) and (20); Thus

$$\epsilon = 1 - \frac{\omega_p^2 (T_{e00} + T_{i00})}{\omega^2 d} + \frac{\omega_p^2 (T_{e00} + T_{i00}) D}{\omega^2 d^2} \frac{\alpha A_0 (A_1 + A_1^*)}{F(q_{\perp})}. \quad (22)$$

Comparing Eqs. (3) and (22), one obtains

$$\epsilon_0 = 1 - \frac{\omega_p^2 (T_{e00} + T_{i00})}{\omega^2 d} \quad (23a)$$

and

$$\epsilon_2 = \frac{\omega_p^2 (T_{e00} + T_{i00}) D}{\omega^2 d^2} \frac{\alpha}{F(q_{\perp})}, \quad (23b)$$

where ω_p is the plasma frequency in absence of the electromagnetic field.

IV. GROWTH OF INSTABILITY

The condition for the growth is given by Eq. (7), with ϵ_2 expressed by Eq. (23b).

Substituting for ϵ_2 from Eq. (23b) in Eq. (8), one obtains an expression for the growth rate Γ of the perturbation in terms of q . Thus,

$$\frac{c}{\omega}\Gamma = \frac{q}{2\sqrt{\epsilon_0}} \left(2 \frac{\omega_p^2}{\omega^2} (T_{e00} + T_{i00}) \frac{D\alpha A_0^2}{d^2 F(q)} - q^2 \right)^{1/2}, \quad (24)$$

where $q = (c/\omega)q_\perp$,

$$F(q) = 2000 \left[\frac{a}{b} \left(\frac{T}{T_{e00}} \right) \left\{ \frac{T_{e00}}{T} + \left(\frac{A}{a} - \frac{B}{b} \right) \left(\frac{T_{e00}}{T} - 1 \right) \right\} - \left\{ \frac{\Lambda}{b} \frac{T}{T_{e00}} \frac{\left(D - \frac{Bd}{b} \right) \left(\frac{T_{e00}}{T} - 1 \right)}{\left(\frac{T_{i00}}{T_{e00}} + 1 \right) T_{e00}} \right\} + \beta q^2 \frac{T_{e00}}{T} \right],$$

and $\beta = (10k_B T \omega^2 / 3mc^2)(1/b^2)$.

For the instability to occur, $\Gamma > 0$; thus, the critical value of $q = (c/\omega)q_\perp$, below which the instability occurs (viz., q_{critical}) is given by putting the right-hand side of Eq. (24) equal to zero.

For Γ to be maximum, the optimum value of q ; i.e., q_{opt} is given by

$$\frac{d\Gamma}{dq} = 0,$$

which yields

$$q_{\text{opt}}^2 [F(q_{\text{opt}})]^2 = \frac{\omega_p^2}{\omega^2} (T_{e00} + T_{i00}) \frac{D\alpha A_0^2}{d^2} F_1, \quad (25)$$

where

TABLE II. Dependence of b_0 , b_1 , and b_2 on height [refer to Eq. (18b)]; mid-latitude daytime ionosphere.

Height (km)	b_0	b_1	b_2
80	141 666.66	977 954.54	-13 257.57
90	25 983.33	144 990.15	-2 200.75
100	9 116.66	41 827.42	-611.36
110	3 465.00	9 762.04	-144.31
120	1 949	4 782.40	-72.5
130	1 070.83	1 951.70	-29.35
150	761.16	606.23	-6.40
200	516.73	-16.68	6.31

$$F_1 = 2000 \left[\frac{a}{b} \left(\frac{T}{T_{e00}} \right) \left\{ \frac{T_{e00}}{T} + \left(\frac{A}{a} - \frac{B}{b} \right) \left(\frac{T_{e00}}{T} - 1 \right) \right\} - \left\{ \frac{\Lambda}{b} \frac{T}{T_{e00}} \frac{\left(D - \frac{Bd}{b} \right) \left(\frac{T_{e00}}{T} - 1 \right)}{\left(\frac{T_{i00}}{T_{e00}} + 1 \right) T_{e00}} \right\} \right].$$

For q_{opt} , given by Eq. (25), the maximum growth rate, namely, Γ_{max} is given by

$$\frac{c}{\omega}\Gamma_{\text{max}} = \left(\frac{q_{\text{opt}}}{2\sqrt{\epsilon_0}} \right) \left\{ 1 + \frac{2\beta q_{\text{opt}}^2}{F_1} \right\}^{1/2}. \quad (26)$$

V. NUMERICAL RESULTS AND DISCUSSIONS

All the numerical results obtained by us correspond to a height of 200 km for the mid-latitude daytime ionospheric model of Gurevich;¹⁹ the collision frequency and the cyclotron frequency of the electrons are also assumed to be much less than the wave frequency. Computations for other cases can be likewise made by using Tables I–III for the characterization of the electron temperature dependence on the parameters a , b , and d occurring in Eqs. (18a)–(18c) at different heights.

Figure 1 illustrates the dependence of the critical value of the wave number of the disturbance q_{critical} on the dimensionless irradiance αA_0^2 of the main beam. This may be

TABLE I. Dependence of a_0 , a_1 , a_2 , and a_3 on height [refer to Eq. (18a)]; mid-latitude daytime ionosphere.

Height (km)	a_0	a_1	a_2	a_3
80	5.8×10^3	-1156.9	218.9	-7.6
90	8.3×10^2	-226.4	47.8	-2.1
100	2.15×10^2	-60.6	13.7	-0.6
110	1.1×10^2	-40.0	7.87	-0.35
120	30.8	-10.1	2.634	-0.1
130	9.9	-3.2	1	0
150	-2.9	4.46	-1.16	0.1
200	0.05	0.2	-0.05	0.067

TABLE III. Dependence of d_0 , and d_1 on height [refer to Eq. (18c)]; mid-latitude daytime ionosphere.

Height (km)	d_0	d_1
80	180	180
90	190	200
100	210	240
110	270	320
120	360	400
130	460	500
150	670	800
200	1100	1300

FIG. 1. Dependence of critical wave number q_{critical} of a perturbation on dimensionless background irradiance αA_0^2 in daytime mid-latitude ionosphere at a height 200 km.

readily understood as follows. As q increases, the width of the disturbance ($\propto$ the wavelength) decreases, diffraction becomes more important, requiring larger magnitudes of non-linearity, corresponding to larger values of the irradiance αA_0^2 of the main beam.

Figure 2 illustrates the dependence of the growth rate Γ of the self-focusing instability with the wave number $q = (c/\omega)q_{\perp}$ in the transverse direction for different values of the dimensionless background irradiance $\alpha A_0^2 = 1, 5, 10, 20, 30, 40, 50$, and 100 for an ionospheric height of 200 km in the daytime.

It is seen that corresponding to a certain value of the beam irradiance (αA_0^2), there is an optimum value of q for which the growth rate is maximum; this is evident from a cursory look at Eq. (24).

Figure 3 expresses the dependence of the maximum growth rate Γ_{max} on the background irradiance αA_0^2 for the daytime mid-latitude ionospheric height of 200 km. The monotonic dependence is also evident from Eq. (26).

It may be reiterated that in the upper ionosphere (height, greater than 150 km), the electron collision frequency is much less than the electron cyclotron frequency due to the

FIG. 2. Dependence of the dimensionless growth rate $(c/\omega)\Gamma$ of self-focusing instability on q , the dimensionless wave number of the perturbation in the transverse direction corresponding to the dimensionless background irradiance $\alpha A_0^2 = 1, 5, 10, 20, 30, 40, 50$, and 100 in daytime mid-latitude ionosphere at a height 200 km.

FIG. 3. Dependence of the dimensionless maximum growth rate $(c/\omega)\Gamma_{\text{max}}$ on dimensionless background irradiance αA_0^2 in daytime mid-latitude ionosphere at a height 200 km.

Earth's magnetic field. Hence, if the magnetic field is not along the x axis, the relevant thermal conductivity, for conduction along the x axis will be much lower than that in the case considered here and the role of thermal conduction will be at best marginal.

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