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Large damage capability of non-crimp fabric thin-ply laminates

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Abstract

The residual strength of thin-ply non-crimp fabric laminates with large through-the-thickness cracks is investigated in this paper. Analysis methods such as the linear elastic fracture mechanics, inherent flaw, point-stress, average-stress and Finite Fracture Mechanics are used to predict large damage capability. Experiments show that thin-ply non-crimp fabric laminates exhibit high resistance to propagation of large cracks and a remarked crack-bridging effect. Due to the unsuitability of existing models to capture such bridging processes, large damage capability is systematically underestimated by the traditional analysis methods. By taking into account the $\mathcal{R}$ -curve of the material in the Finite Fracture Mechanics analysis good predictions of the notched response of laminated plates with large through-penetration damage are obtained using only independently measured material properties; in addition, the Finite Fracture Mechanics model does not require fitting parameters or complex finite element analyses.

Keywords: A. Laminates, B. Stress concentrations, C. Analytical modelling, D. Mechanical testing, E. Tow.

1. Introduction

The use of the recently developed non-crimp fabric (NCF) thin-ply laminates may result in considerable improvements of the structural integrity of composite structures such as those used in airframe construction. Recently, an extensive experimental test program [1] and a study of the applicability of traditional analysis methods to predict the open-hole tensile and compressive response of CFRP laminates [2] were conducted to assess the notched response, size effects and brittleness of NCF carbon/epoxy C-PLYTM T700/AR-2527 thin-ply laminates. A size effect on the strength was observed

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for both the open-hole tension and compression tests, and the associated notch sensitivity of thin-ply NCFs were similar to those observed in typical aerospace grade unidirectional pre-impregnated CFRPs.

However, the previous studies were restricted to the notched response of small scale samples. As it is well known, the tensile residual strength of composite laminates in the presence of through-the-thickness notches is significantly affected by size [3]. In general, it is possible to distinguish between small-notch (i.e., “strength-dominated”) and large-notch (i.e., “toughness-dominated”) strengths. The analysis methods should be able to capture and predict these distinct material behaviors, providing failure predictions beyond the notch sizes and structural geometries tested during material characterization. To ensure this extrapolation capability, suitable models must be based on the actual physics of the problem. Additionally, it is desirable to minimize the number of material properties required for the analysis models to reduce the material testing requirements [3]. The existing analytical models, such as the inherent flaw model (IFM) [4] and the point-stress (PS) and average-stress (AS) models [5], use empirical characteristic distances that, in practice, are determined from notched strength data. They are often thought as correction factors that account for apparent changes in the stress intensity with increasing crack size, with limited physical meaning.

An alternative approach is the Mar-Lin model [6]. This model captures the reduced sensitivity of composite materials to large changes in notch size by allowing the singularity, n , to be other than the square-root (as in LEFM). This model was successfully used to predict large damage capability (LDC) from smaller notch data, avoiding excessive conservatism. However, the exponent n is an additional empirical parameter that also needs to be experimentally determined (typically by curve-fitting). In addition, a precise, verified method for determining the appropriate singularity value in the absence of large notch data has not been developed. In fact, caution should be exercised in selecting exponents for extrapolation, since it is possible to select values that over-predict LDC (unconservative results). In general, verification tests should be conducted [3].

The objectives of this paper are to study the residual strength and the LDC of NCF thin-ply laminates, and to assess the validity of traditional analysis methods in predicting the notched response of CFRP laminates with large through-thickness cracks. The Finite Fracture Mechanics (FFMs) model [7] is extended to address the notched response of center-notched plates in tension, and it is used, together with the LEFM model, IFM [4] and PS and AS models [5], in the analysis of

the LDC of NCF thin-ply laminates. The Mar-Lin model [6] is not addressed in this work because it requires extensive experimental testing in center-cracked plates to obtain all necessary parameters. The LDC analysis results are compared against detailed experimental testing of an NCF thin-ply center-notched plate.

2. Experimental study

The thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_s$ 16-ply thick laminate, manufactured at VX Aerospace using vacuum bagging, is tested in a large damage capability (LDC) experimental study. This laminate is made of the carbon NCF C-PLYTM T700/AR-2527 epoxy system prepreg material from Aldila (material reference P/N C-PLY BX 0/-45 150 T3,4 12K HS-AR2527), which includes the tow-spread C-PLYTM T700 $[0/-45]$ NCF from Chomarat, with an areal weight of 150 g/m² per bi-angle layer (or 75 g/m² per ply). It has a fiber volume fraction around 50% and a nominal ply thickness of 0.08 mm. The 0° fiber orientation is coincident with the loading direction and the ply properties were reported in Ref. [1].

A center-notched plate, 270 mm wide (W) and 400 mm long (L), was tested in tension. The nominal length of the notch ($2a$) was 36 mm. The central notch was obtained using a milling machine, ensuring a distance of 1 mm between the notch faces. The notch tip was not sharpened, since, based on the previous work of Camanho and Catalanotti [8], no relevant difference between the fracture toughness of specimens with and without sharpened notch tips is expected.

The plate was instrumented with seven strain gages, placed in the -45° surface-ply along the longitudinal (y -)direction, as shown in figure 1. In addition, the digital image correlation (DIC) technique was used to evaluate the displacement and strain fields of the 0° surface-ply, and assess damage formation and development near the notch tips. The ARAMIS DIC-2D v6.0.2 developed by GOM [9] was used. The adopted configuration, which leads to a displacement resolution in the order of 10^{-2} and a strain resolution of about 0.01–0.05% [10], is summarized in table 1. In order to ensure the perpendicularity between the image grabbing setup and the sample's surface, a laser pointer fixed to the camera support and a mirror placed on the sample's surface and parallel to it were used to achieve a correct alignment.

[Table 1 about here.]

[Figure 1 about here.]

The LDC test was performed under displacement control, at a controlled speed of 1.0 mm/min, in a servohydraulic MTS 810 testing machine, using a 250 kN load cell and an acquisition frequency set to 5 Hz. The load introduction device is made of an upper and a bottom tool, each of which has two rows of holes used to fasten M5 bolts through the CFRP plate, applying pressure in a region corresponding to the gripping area — see figure 2. The resulting grip-to-grip length is of approximately 250 mm, and the width-to-gage length ratio is 1.1. According to a numerical study conducted by Catalanotti and co-authors in Ref. [11] addressing the effect of finite gage length on the finite width and orthotropy correction factors obtained from the orthotropy rescaling technique [12, 13], neglecting the effect of finite gage length for a width-to-gage length ratio of 1.1 incurs an error of approximately 5%. Nevertheless, taking into account that the analysis becomes much simpler, the authors believe this is an acceptable error for the objectives of the present paper. No bearing damage was observed in the holes of the CFRP plate after testing.

[Figure 2 about here.]

Figure 3 shows the longitudinal displacement (u_y)-remote longitudinal stress (σ_y^∞) and the far-field ($SG0$) longitudinal strain (ε_y)-remote longitudinal stress (σ_y^∞) relations for the tested center-notched plate. Observing figure 3(a), it can be seen that the longitudinal displacement-remote longitudinal stress relation is linear up to approximately 75% of the peak load. On the other hand, observing figure 3(b), it can be seen that the onset of nonlinear behavior in figure 3(a) coincides with the onset of unstable behavior of the far-field longitudinal strain-remote longitudinal stress relation. This behavior is related with the onset of a fracture process zone in the notched region, whose propagation can be analysed observing figure 4¹. Here, the surface longitudinal strain (ε_y) fields measured using the DIC technique for four different stages of the development of the fracture process zone are shown, together with the time-stress curve showing the points corresponding to these stages. The surface longitudinal strain (ε_y) measured along a cutting line centered along the notch (red line in the left-hand charts) are shown in the right-hand plots. At the onset of the fracture process

¹For a video showing the evolution of the surface longitudinal strain (ε_y) while the crack propagates, the reader is referred to the electronic version of this article.

(figure 4(b)), corresponding to the first load drop, the strain concentration moves from the notch tip, perpendicularly to the loading direction. A visible, finite region of high strained material ahead of the notch tip can be observed. In the subsequent stages, the location of the strain concentration can be used to identify the tip of the fracture process zone. Observing figure 4(a), it can be seen that the point in the time-stress relation for the onset of the fracture process (point (b)) coincides with the onset of nonlinear behavior (compare with figure 3). Thereafter, the strain concentration does not increase further, and, as shown in figure 4, the front of the fracture process zone propagates perpendicularly to the loading direction (i.e., in the x -direction).

[Figure 3 about here.]

[Figure 4 about here.]

Figure 5 shows the longitudinal strain (ε_y)-remote longitudinal stress (σ_y^∞) relations near the notch tips, obtained using *physical* (figures 5(a) and (b)) and *virtual* (figures 5(c) and (d)) strain gages. The far-field longitudinal strain is also shown. The measurements for the virtual strain gages were obtained using the DIC technique, averaging the strain values in the area equivalent to the gage area of the physical strain gages. It should be noted that, in the unloading regime, the longitudinal strain (ε_y) in the notch plane cannot be obtained because, after unstable fracture of the plate, the signal from both physical and virtual strain gages is lost due to the presence of a transverse intralaminar crack in that plane.

[Figure 5 about here.]

As observed in previous works [14, 15], and in spite of having been obtained in plies with different orientations, no substantial difference can be found between the results obtained with the physical or virtual (from DIC) strain gages, except for the notch tips; however, it should be noted that, as the distance to the notch tips decreases, the strain gradient considerably increases, and a small difference between the positions of the physical strain gage and the averaged region used in the DIC technique may lead to large differences in the strain measurements. Still, the correlation between the measured strains at the notch tips is satisfactory. Indeed, both techniques show that the local longitudinal strains (ε_y) considerably increase with its proximity to the notch tips. In addition, observing the notch tip longitudinal strains (particularly for the virtual strain gages results, which

do not suffer from premature failure), it can be seen that the remote stress at onset of the fracture process (corresponding to a sudden increase in strain) relates very well with the onset of nonlinear behavior observed in figures 3 and 4.

Before final fracture, and contrary to what was observed for small notch specimens [1], the crack that propagates from the notch tips extends significantly along the notch plane, although not very regularly, as shown in figures 4 and 6². From a macromechanical point of view, the development of the fracture process zone occurs in discrete, finite steps, which result from a bridging process at the crack tip. This bridging process prevents unstable fracture along the ligament width, increasing the fracture toughness of the laminate as the crack length increases, until a maximum is reached, after which final fracture occurs. Therefore, the material tested shows a crack resistance-curve ($\mathcal{R}$ -curve). Accordingly, some small load drops can be observed, resulting from the development of the fracture process zone, which have some effect on the stiffness of the center-notched plate, causing the nonlinear behavior mentioned above. This bridging process and the $\mathcal{R}$ -curve effect are fundamental in the analysis of LDC, as it will be observed in the next sections.

[Figure 6 about here.]

Figure 7 shows the fractured center-notched plate. Near the notch tips, some fiber splitting can be observed in both 0° and -45° surface-ply, as well as some separated plies due to delamination. However, these damage mechanisms only occurred after unstable fracture along the ligament width (note that such damage mechanisms are not observed in figures 4 and 6), and their extent is reduced. In addition, similarly to what was observed in the fractured small notch specimens [1], a brittle type of net-section failure mode can be identified. Therefore, observing figure 7, it is possible to conclude that the development of the fracture process zone observed in figures 4 and 6 results from stable (although not regular) intralaminar damage, and that subcritical damage mechanisms such as delamination, transverse cracking or fiber splitting do not play an important role in the LDC of thin-ply laminates, even when stacking sequences more susceptible for damage growth are used, as it is the case. [In the future, related studies should include non-destructive inspection data of the coupons during testing \[16\] to verify the previous conclusions.](#)

²For a video showing the crack propagating, the reader is referred to the electronic version of this article.

[Figure 7 about here.]

3. Analysis methods

3.1. Linear elastic fracture mechanics model

The linear elastic fracture mechanics (LEFM) model is the classical method of predicting the residual strength of homogeneous materials, and therefore it can be applied to the analysis of LDC. Assume that a through-thickness crack of length $2a$ exists. For classical continuum theories, the stress distribution at the crack tip is singular. In LEFM, a square-root singularity is assumed, and the residual strength $\bar{\sigma}^\infty$ is predicted as:

$$\bar{\sigma}^\infty = \frac{\mathcal{K}_{Ic}}{\sqrt{\pi a}} \quad (1)$$

where $\mathcal{K}_{Ic}$ is the critical stress intensity factor, or fracture toughness, calculated from a specimen with a central crack. However, this approach suffers from the physically unacceptable situation of infinite stresses at the crack tip, and the residual strength increases rapidly with decreasing a . At the limit, as $a \rightarrow 0$, $\bar{\sigma}^\infty \rightarrow \infty$.

In order to account for finite-width and material orthotropy effects, correction factors have to be included in equation (1). According to the orthotropy rescaling technique [12, 13], the stress intensity factor (SIF) $\mathcal{K}_I(a)$ for a center-cracked plate can be written as:

$$\mathcal{K}_I(a) = \chi(\rho) Y(2a/W) \sigma^\infty \sqrt{\pi a} \quad (2)$$

where $Y(2a/W)$ is the finite-width correction (FWC) factor and $\chi(\rho)$ is the correction factor for the orthotropy of the material. Following the orthotropy rescaling technique [12, 13], the geometry dependence, $Y(2a/W)$, is the same of that of an isotropic material. The simple FWC factor proposed by Chen et al. [17] is used in this work. This FWC factor was derived imposing equilibrium between the remotely applied stress and the stress in the elastic material, and it is given as [17]:

$$Y(2a/W) = \frac{1}{\sqrt{1 - \left(\frac{2a}{W}\right)^2}} \quad (3)$$

where W is the plate width.

Regarding the orthotropy dependence, $\chi(\rho)$, it can be expressed as a single function of a dimensionless elastic parameter ρ [13]:

$$\chi(\rho) = 1 + 0.1(\rho - 1) - 0.016(\rho - 1)^2 + 0.002(\rho - 1)^3 \quad (4)$$

where:

$$\rho = \frac{(E_x E_y)^{1/2}}{2G_{xy}} - (\nu_{xy} \nu_{yx})^{1/2} \quad (5)$$

and E_x and E_y are the Young's moduli, G_{xy} the shear modulus, and ν_{xy} and ν_{yx} the Poisson's ratios in the respective preferred axes, as shown in figure 8.

[Figure 8 about here.]

Note that, as reported in Ref. [13], equations (2) and (4) are only valid for materials with $0 \leq \rho \leq 4$. As discussed in Ref. [11], for highly orthotropic materials, with $\rho > 4$ (which is typically the case of cross-ply laminates, for example), a new correction factor for equation (2), κ , which takes into account both geometry and orthotropy dependence, should be determined. Since a quasi-isotropic laminate ($\rho = 1.0$ and $\chi = 1.0$) is used here, equations (2) and (4) are used, and such additional correction factor is not needed.

For the general case, taking both geometry (Y) and orthotropy (χ) correction factors, $\bar{\sigma}^\infty$ is now predicted by the LEFM model as:

$$\bar{\sigma}^\infty = \frac{\mathcal{K}_{Ic}}{\chi Y \sqrt{\pi a}} \quad (6)$$

3.2. Inherent flaw model

In order to address the physically unacceptable situation of infinite stresses at the crack tip predicted by LEFM models, several theories, which generally rely on a “*characteristic distance*”, were proposed. One of these theories is the *inherent flaw model* (IFM). It was originally developed by Waddoups et al. [4], in the early 1970's, to predict the residual strength of notched or center-cracked laminates with infinite widths under tensile loads, and extended to finite widths by introducing a finite-width correction factor [18]. The IFM is based on LEFM; it assumes that a composite laminate containing a notch is analogous to a crack in an isotropic material. According to this model, the

noncritical damage mechanisms can be lumped into a constant “intense energy region”, or “inherent flaw”, of length a_0 [4]. Approximating the central notch of length $2a$ by a through-thickness crack, the SIF is given by equation (2). For a quasi-isotropic laminate ($\rho = 1.0$ and $\chi = 1.0$), assuming that an inherent flaw exists at each notch tip, at failure, the critical SIF, or fracture toughness, is:

$$\mathcal{K}_{Ic} = \frac{\bar{\sigma}^\infty \sqrt{\pi(a + a_0)}}{\sqrt{1 - \left[\frac{2(a + a_0)}{W} \right]^2}} \quad (7)$$

The critical SIF of an unnotched specimen can be predicted by letting the notch length tend to zero, i.e.:

$$\mathcal{K}_{Ic} = \frac{X_T^L \sqrt{\pi a_0}}{\sqrt{1 - \left(\frac{2a_0}{W} \right)^2}} \quad (8)$$

where X_T^L is the tensile laminate unnotched strength. Notice that, unlike Aronsson and Bäcklund [18], a FWC is used to calculate the critical SIF when the notch length tends to zero. Combining equations (7) and (8) results in:

$$\bar{\sigma}^\infty = X_T^L \sqrt{\frac{a_0}{a + a_0}} \sqrt{\frac{1 - \left[\frac{2(a + a_0)}{W} \right]^2}{1 - \left(\frac{2a_0}{W} \right)^2}} \quad (9)$$

Accordingly, the strength of a notched laminate is predicted using two parameters, the length of the inherent flaw, a_0 , that needs to be calculated from a baseline specimen, and the unnotched tensile strength of the laminate, X_T^L . Although this is a simple approach, since it does not require complex Finite Element (FE) implementations, the IFM only provides reasonable predictions for specimens with geometries close to that used to calibrate the model [19].

3.3. Point-stress and average-stress models

The residual strength of homogeneous materials can also be determined using stress-based criteria, such as the point-stress (PS) and average-stress (AS) models of Whitney and Nuismer [5]. These models predict failure when the stress at a given distance (PS model) or the stress averaged over a characteristic distance (AS model) reach the unnotched strength of the laminate, i.e., respectively:

$$\sigma_{yy}(x, 0)|_{x=a+r_{ot}} = X_T^L \quad (10)$$

$$\frac{1}{r_{ot}} \int_a^{a+r_{ot}} \sigma_{yy}(x, 0) dx = X_T^L \quad (11)$$

Hence, each model requires that the unnotched laminate strength X_T^L and a characteristic distance r_{ot} (without any physical relation between them) must be determined from an unnotched and from a notched specimen, respectively. The characteristic distances r_{ot} are not material properties, and need to be identified every time the material/lay-up or the geometry are modified. Therefore, a large amount of empirical data from expensive experimental programs to account for the different geometries and lay-ups that are typically used in the aerospace and aircraft industries is required.

Approximating the central notch by an elliptical opening of major axes $(2a)$ equal to the notch length, and minor axes $(2b)$ such that the ellipse's curvature in the major axis is equal to the notch tip radius, as proposed in Ref. [2], and assuming that the normal stress profile for a finite plate, $\sigma_{yy}(x, 0)$, is identical to that for an infinite plate, $\sigma_{yy}^\infty(x, 0)$, except for an FWC factor, the stress distribution along the x -axis is given as [20]:

$$\sigma_{yy}(x, 0) = \frac{K_T}{K_T^\infty} \sigma_{yy}^\infty(x, 0) \quad (12)$$

with:

$$\begin{aligned} \frac{\sigma_{yy}^\infty(x, 0)}{\sigma^\infty} = & \frac{\lambda^2}{(1-\lambda)^2} + \frac{(1-2\lambda)\gamma}{(1-\lambda)^2 \sqrt{\gamma^2 - 1 + \lambda^2}} + \frac{\lambda^2 \gamma}{(1-\lambda)(\gamma^2 - 1 + \lambda^2)^{\frac{3}{2}}} - \\ & - \frac{\lambda^7}{2} \left(K_T^\infty - 1 - \frac{2}{\lambda} \right) \left[\frac{5\gamma}{(\gamma^2 - 1 + \lambda^2)^{\frac{7}{2}}} \frac{7\lambda^2 \gamma}{(\gamma^2 - 1 + \lambda^2)^{\frac{9}{2}}} \right] \end{aligned} \quad (13)$$

where:

$$\lambda = \frac{b}{a} \quad (14)$$

$$\gamma = \frac{x}{a} \quad (15)$$

and K_T^∞ and K_T are, respectively, the stress concentration factors at the edge on the axis normal to the applied load of the central elliptical notch of an infinite orthotropic plate and a finite orthotropic plate, given by [20]:

$$K_T^\infty = 1 + \frac{1}{\lambda} \sqrt{2 \left(\sqrt{\frac{E_y}{E_x}} - \nu_{yx} \right) + \frac{E_y}{G_{xy}}} \quad (16)$$

$$\begin{aligned} \frac{K_T^\infty}{K_T} = & \frac{\lambda^2}{(1-\lambda)^2} + \frac{1-2\lambda}{(1-\lambda)^2} \sqrt{1 + (\lambda^2 - 1) (2a/W)^2} - \\ & - \frac{\lambda^2}{(1-\lambda)^2} \frac{(2a/W)^2}{\sqrt{1 + (\lambda^2 - 1) (2a/W)^2}} + \\ & + \frac{\lambda^7}{2} \left(\frac{2a}{W} \right)^6 \left(K_T^\infty - 1 - \frac{2}{\lambda} \right) \left\{ \left[1 + (\lambda^2 - 1) \left(\frac{2a}{W} \right)^2 \right]^{-\frac{5}{2}} - \right. \\ & \left. - \left(\frac{2a}{W} \right)^2 \left[1 + (\lambda^2 - 1) \left(\frac{2a}{W} \right)^2 \right]^{-\frac{7}{2}} \right\} \end{aligned} \quad (17)$$

For quasi-isotropic laminates, $K_T^\infty = 1 + 2/\lambda$, and equations (17) and (13) are considerably simplified. Using equations (12) to (17) in (10) and (11), the PS and AS failure models for a center-notched laminate read, respectively [20]:

$$\begin{aligned} \frac{\bar{\sigma}^\infty}{X_T^L} = & \frac{K_T^\infty}{K_T} \left\{ \frac{\lambda^2}{(1-\lambda^2)} + \frac{(1-2\lambda)\xi_1^{-1}}{(1-\lambda)^2 \sqrt{\xi_1^{-2} - 1 + \lambda^2}} + \frac{\lambda^2 \xi_1^{-1}}{(1-\lambda) (\xi_1^{-2} - 1 + \lambda^2)^{3/2}} - \right. \\ & \left. - \frac{\lambda^7}{2} \left(K_T^\infty - 1 - \frac{2}{\lambda} \right) \left[\frac{5\xi_1^{-1}}{(\xi_1^{-2} - 1 + \lambda^2)^{7/2}} - \frac{7\lambda^2 \xi_1^{-1}}{(\xi_1^{-2} - 1 + \lambda^2)^{9/2}} \right] \right\}^{-1} \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\bar{\sigma}^\infty}{X_T^L} = & \frac{K_T^\infty}{K_T} (\xi_2^{-1} - 1) \left\{ \frac{\lambda^2}{(1-\lambda^2)} \xi_2^{-1} + \frac{1-2\lambda}{(1-\lambda)^2} \sqrt{\xi_2^{-2} - 1 + \lambda^2} - \frac{\lambda^2}{(1-\lambda) \sqrt{\xi_2^{-2} - 1 + \lambda^2}} + \right. \\ & \left. + \frac{\lambda^7}{2} \left(K_T^\infty - 1 - \frac{2}{\lambda} \right) \left[(\xi_2^{-2} - 1 + \lambda^2)^{-5/2} - \lambda^2 (\xi_2^{-2} - 1 + \lambda^2)^{-7/2} \right] \right\}^{-1} \end{aligned} \quad (19)$$

where ξ_1 and ξ_2 are given as [20]:

$$\xi_1 = \xi_2 = \frac{a}{a + r_{ot}} \quad (20)$$

3.4. Finite Fracture Mechanics model

In the present work, the FFMs model, proposed by Camanho et al. [7] for the prediction of the notched response of laminated composites with open-holes, is extended to address the notched response of laminates with center through cracks. The FFMs model predicts fracture of a laminate with a central notch when the following system of equations is satisfied:

$$\begin{cases} \frac{1}{l} \int_a^{a+l} \sigma_{yy}(x, 0) dx = X_T^L \\ \frac{1}{l} \int_a^{a+l} \mathcal{K}_I^2(a) da = \mathcal{K}_{Ic}^2 \end{cases} \quad (21)$$

where l is the crack extension at failure, $\sigma_{yy}(x, 0)$ is the stress distribution along the x -axis, X_T^L is the laminate tensile unnotched strength, $\mathcal{K}_I(a)$ is the mode I SIF, and $\mathcal{K}_{Ic}$ is the mode I laminate fracture toughness.

Approximating the central notch by an elliptical opening of major axes ($2a$) equal to the notch length, and minor axes ($2b$) such that the ellipse's curvature in the major axis is equal to the notch tip radius [2], the stress distribution along the x -axis, $\sigma_{yy}(x, 0)$, is given by equation (12). Integrating equation (12) and using it in the system of equations (21), the first member of the first equation reads [20]:

$$\frac{1}{l} \int_a^{a+l} \sigma_{yy}(x, 0) dx = \frac{K_T}{K_T^\infty} \sigma^\infty f_\sigma \quad (22)$$

with:

$$\begin{aligned} f_\sigma = & \frac{1}{\xi^{-1} - 1} \left\{ \frac{\lambda^2}{(1 - \lambda^2)} \xi^{-1} + \frac{1 - 2\lambda}{(1 - \lambda)^2} \sqrt{\xi^{-2} - 1 + \lambda^2} - \frac{\lambda^2}{(1 - \lambda) \sqrt{\xi^{-2} - 1 + \lambda^2}} + \right. \\ & \left. + \frac{\lambda^7}{2} \left(K_T^\infty - 1 - \frac{2}{\lambda} \right) \left[(\xi^{-2} - 1 + \lambda^2)^{-5/2} - \lambda^2 (\xi^{-2} - 1 + \lambda^2)^{-7/2} \right] \right\} \end{aligned} \quad (23)$$

where σ^∞ is the applied remote stress, λ is the aspect ratio, given by equation (14), K_T^∞ is the stress concentration factor at the edge of the elliptical opening of an infinite plate, given by equation (16), K_T is the stress concentration factor at the edge of the elliptical opening of a plate of finite width, given by equation (17), and ξ is given as:

$$\xi = \frac{a}{a + l} \quad (24)$$

The SIF solution $\mathcal{K}_I(a)$ required to solve the energy-based criterion corresponds to a central notch that is extended by two cracks of finite dimension, which can be reasonably approximated by a plate with a central crack. For an orthotropic plate, $\mathcal{K}_I(a)$ is given by equation (2). Replacing the SIF solution, $\mathcal{K}_I(a)$, in the system of equations (21), the first member of the second equation reads:

$$\frac{1}{l} \int_a^{a+l} \mathcal{K}_I^2(a) da = (\sigma^\infty)^2 \frac{\pi}{8} \chi^2 \frac{W^2}{l} f_K \quad (25)$$

with:

$$f_K = \ln(W + 2a) + \ln(W - 2a) - \ln(W + 2a + 2l) - \ln(W - 2a - 2l) \quad (26)$$

Using equations (22) and (25) in (21), and dividing the second equation in (21) by the square of the first one yields:

$$\frac{\pi \chi^2 W^2 f_K}{8l \left(\frac{K_T}{K_T^\infty} \right)^2 f_\sigma^2} = \left(\frac{\mathcal{K}_{Ic}}{X_T^L} \right)^2 \quad (27)$$

which is a nonlinear equation with only one unknown, the crack extension at failure l . After solving equation (27) for l , the residual strength $\bar{\sigma}^\infty$ of a center-notched laminate can be calculated using, for example, the first equation in (21), i.e.:

$$\bar{\sigma}^\infty = \frac{K_T^\infty}{K_T} f_\sigma^{-1} X_T^L \quad (28)$$

Equations (27) and (28) can be used to calculate the residual strength of center-notched laminates with a variety of geometries using only two material properties: the unnotched strength X_L^T and the fracture toughness $\mathcal{K}_{Ic}$. The latter can be calculated by means of a FFMs analysis of a specimen with a central notch using the same set of equations, as described in Ref. [2], or from a specimen with a different geometry. No “*characteristic distance*” calculated from a baseline specimen is necessary. In the following section, the FFMs predictions of the residual strength and LDC of the tested NCF thin-ply laminate is presented, as well as a comparison with the alternative analysis methods and experimental data.

4. Predictions and discussion

The test results obtained in Ref. [1] for the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ tensile specimens with a 4 mm long central notch are used to calculate the fracture toughness ($\mathcal{K}_{Ic}$) in the LEFM and FFMs models, and calibrate the IFM and the PS and AS models by calculating the respective length of the inherent flaw (a_0) and characteristic distances (r_{ot}). The tensile unnotched strength (X_L^T), required in all the models except the LEFM model, was also obtained in Ref. [1]. The values of the properties required by each macromechanical model are shown in table 2.

[Table 2 about here.]

Figure 9 shows the predictions of the tensile residual strength of a center-notched thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate obtained using the macromechanical models described in section 3. The height of the central notch (h) and the $W/(2a)$ ratio are kept constant and equal to 1 mm and 7.5, respectively. Figure 9 also shows the experimental results for the center-notched specimen ($2a = 4$ mm) [1], and for the center-notched plate ($2a = 36$ mm), whose results were presented in section 2. For the latter, both stresses at the initiation of the fracture process and at the final fracture are shown. *Note that subcritical damage mechanisms such as delamination and splitting are neglected when using the models presented in section 3, an assumption acceptable for laminates whose failure is dominated by intralaminar damage, as the one tested in the present work (see section 2 and Refs. [21, 22] for similar or lower grades).*

[Figure 9 about here.]

From figure 9, it becomes clear that the LEFM model is not able to capture the right trend of the residual strength with changing notch length. Regarding the remaining analysis models, they all show similar trends, with slightly different predictions for the smaller and larger notch lengths. In order to better distinguish these differences, in particular with respect to LDC, table 3 shows the predictions and relative errors (ϵ_r) for the LDC of the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate. It can be seen that all models fail to predict the residual strength of laminated plates with large through-thickness cracks (or notches). However, the predictions of these models (with the exception of the LEFM model) match with reasonable accuracy the stress at the initiation of the fracture process, with errors below 1% for the IFM, AS model and FFMs model. Therefore, it seems

that these models are able to capture the onset of the fracture process, which, for small crack lengths, is close to final fracture [1], yielding, in this case, good predictions. For large crack lengths, though, the onset of the fracture process occurs considerably before final fracture, consequently yielding conservative results.

[Table 3 about here.]

From the LDC experimental analysis (section 2), it was observed that the NCF thin-ply laminate studied in this work is characterized by a remarked $\mathcal{R}$ -curve effect, with a large, stable fracture process zone developing before final fracture. However, the analysis methods described in section 3 were presented assuming that fracture occurs unstably, as a singular event, regardless of the crack length. For small crack lengths, this assumption is valid, because the fracture process zone develops just before final fracture [1], and the predictions typically yield good results. However, for large crack lengths, this assumption is no longer valid, and, from a macromechanical point of view, the bridging mechanisms that prompt the development of a stable fracture process zone must be lumped into the crack tip in a physically admissible way, to take them into account in the macromechanical model. One possibility is to take into account the effect of the $\mathcal{R}$ -curve in the analysis method. This has already been done to predict net-tension failure of mechanically fastened joints in composite laminates using the FFMs model [23], and no other analysis method seems to be able to account for this effect.

4.1. Finite Fracture Mechanics predictions taking into account the $\mathcal{R}$ -curve of the material

The effect of the $\mathcal{R}$ -curve in determining the crack length corresponding to unstable fracture of a center-notched laminate can be taken into account using the FFMs model by rewriting the system of equations (21) as:

$$\begin{cases} \frac{1}{l} \int_a^{a+l} \sigma_{yy}(x, 0) dx = X_T^L \\ \int_a^{a+l} \mathcal{G}_I(a) da = \int_0^l \mathcal{R}(\Delta a) d\Delta a \end{cases} \quad (29)$$

where $\mathcal{G}_I(a)$ is the energy release rate (ERR) and $\mathcal{R}(\Delta a)$ is the $\mathcal{R}$ -curve, expressed in terms of a critical ERR, of the laminate. To solve this system of equations, the $\mathcal{R}$ -curve of the material must

be known. However, no standard test method exists to characterize the $\mathcal{R}$ -curve associated with intralaminar fracture of composite laminates.

Compact tension (CT) test specimens are normally used to measure this property in composite materials reinforced by unidirectional fibers [24–27]. Over-height compact tension (OCT) specimens [24], complemented with an anti-buckling support, as proposed by Li et al. [26], were employed to measure the $\mathcal{R}$ -curve of the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate. Unfortunately, significant overall buckling of the unnotched end of the OCT specimen was observed, despite the use of the anti-buckling support, and no crack propagation occurred — see figure 10. This buckling phenomenon resulted from the low thickness of this laminate (about 1.3 mm), which easily promotes such instabilities, and from the reasonably high resistance of this material to unstable fracture (as observed in section 2), despite the brittle type of net-section failure mode exhibited in the experimental program [1]. Accordingly, a new test methodology should be developed to characterize the $\mathcal{R}$ -curve of (thin) laminated composites that show improved fracture resistance.

[Figure 10 about here.]

Recently, a methodology based on Bažant’s size effect law [28] to measure the fracture toughness and the $\mathcal{R}$ -curve of composite laminates, in both tension and compression, was proposed [11, 15]. According to this methodology, the $\mathcal{R}$ -curve of a composite laminate can be measured taking into account that, for different characteristic sizes w_n , assuming that the size effect law, $\bar{\sigma}^\infty = \bar{\sigma}^\infty(w)$, is known, the driving force curves $\mathcal{G}_I$ at the ultimate remote stresses $\bar{\sigma}^\infty(w_n)$ are tangent to the $\mathcal{R}$ -curve. In other words, the $\mathcal{R}$ -curve can be determined as the envelope of the driving force curves at the ultimate remote stresses (see, e.g., Ref. [29]). Mathematically, this means that the ultimate remote stresses can be obtained solving the following system of equations [11]:

$$\begin{cases} \mathcal{G}_I(\Delta a) = \mathcal{R}(\Delta a) \\ \frac{\partial \mathcal{G}_I(\Delta a)}{\partial \Delta a} = \frac{\partial \mathcal{R}(\Delta a)}{\partial \Delta a} \end{cases} \quad (30)$$

Taking x and y as the preferred axes of the material, for a two-dimensional orthotropic body the ERR in mode I, $\mathcal{G}_I$, for a crack propagating in the x -direction is [12, 13]:

$$\mathcal{G}_I = \frac{1}{E} \mathcal{K}_I^2 \quad (31)$$

where the equivalent modulus $\acute{E}$ is given as:

$$\acute{E} = \left(\frac{1 + \rho}{2E_x E_y} \right)^{-1/2} \mu^{1/4} \quad (32)$$

In equation (32), ρ and μ are two dimensionless elastic parameters, given, respectively, by equation (5) and by:

$$\mu = \frac{E_y}{E_x} \quad (33)$$

Using the orthotropy rescaling technique [12, 13], the SIF in equation (31) can be written in a generic form as:

$$\mathcal{K}_I = \sigma^\infty \sqrt{w} \kappa(\alpha, \rho, \varphi) \quad (34)$$

where σ^∞ is the remote stress, w is a characteristic dimension, $\varphi = \mu^{-1/4} \varpi$, $\alpha = a/w$ and $\varpi = w/L$ are the shape parameters, and κ is a generic correction factor that accounts for both geometry and orthotropy of the material. Using equation (34) in equation (31), and defining $\alpha_0 = a_0/w$ as the initial value of the shape parameter (where a_0 is the initial crack length), the driving force curves read:

$$\mathcal{G}_I(\Delta a) = \frac{1}{\acute{E}} w (\sigma^\infty)^2 \kappa^2 \left(\alpha_0 + \frac{\Delta a}{w}, \rho, \varphi \right) \quad (35)$$

At the ultimate remote stress, $\bar{\sigma}^\infty$, the following expression for $\mathcal{R}(\Delta a)$ can be obtained substituting equation (31) in the first equation of (30):

$$\mathcal{R}(\Delta a) = \frac{1}{\acute{E}} w (\bar{\sigma}^\infty)^2 \kappa^2 \quad (36)$$

Differentiating equation (36) with respect to w , assuming that *geometrically similar specimens* (i.e., with α_0 and ϖ independent of w) with *positive geometry* (i.e., where the correction factor κ is an increasing function of the crack length) are tested, and taking into account that, by definition, the $\mathcal{R}$ -curve is size independent (i.e., $\partial \mathcal{R} / \partial w = 0$), the following equation is obtained:

$$\frac{\partial}{\partial w} [w (\bar{\sigma}^\infty)^2 \kappa^2] = 0 \quad (37)$$

Knowing the size effect law, $\bar{\sigma}^\infty = \bar{\sigma}^\infty(w)$, equation (37) can be solved for $w = w(\Delta a)$. Using $w = w(\Delta a)$ in equation (36), the $\mathcal{R}$ -curve, $\mathcal{R}(\Delta a)$, is obtained. Thus, unlike other experimental techniques used to determine the $\mathcal{R}$ -curve, such as the CT test, no measurement of the crack length at different load values is needed when using this methodology. In fact, only the ultimate remote stress (or peak load) values for a set of *geometrically similar specimens* of different sizes with *positive geometry* are needed to calibrate the size effect law. This is done applying one of the regression methods proposed in Ref. [28].

Catalanotti et al. [11] measured the $\mathcal{R}$ -curve of the $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ NCF thin-ply laminate, investigated in the present work, using scaled double edge-notched specimens (width (W) varying between 10 mm and 35 mm, and an edge notch length-to-width ratio $a/W = 0.3$). The size effect law that best fitted the experimental data was the *linear regression I* proposed in Ref. [28]:

$$\frac{1}{(\bar{\sigma}^\infty)^2} = A w + C \quad (38)$$

where A and C are the fitting parameters shown in figure 11. Using these fitting parameters, it is possible to calculate the length of the fracture process zone, l_{fpz} , and the fracture toughness at propagation, $\mathcal{R}_{SS}$, as [11]:

$$l_{fpz} = \frac{\kappa_0}{2\dot{\kappa}_0} \frac{C}{A} \quad (39)$$

$$\mathcal{R}_{SS} = \frac{\kappa_0^2}{\dot{E}} \frac{1}{A} \quad (40)$$

where $\kappa_0 = \kappa|_{\alpha=\alpha_0}$ and $\dot{\kappa}_0 = \partial\kappa/\partial\alpha|_{\alpha=\alpha_0}$ can be obtained following one of the procedures proposed by Catalanotti et al. [11], and $\dot{E}$ is the equivalent modulus given in equation (32). A length of the fracture process zone $l_{fpz} = 1.92$ mm and a fracture toughness at propagation $\mathcal{R}_{SS} = 101.5$ kJ/m² were reported in Ref. [11] for the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate.

[Figure 11 about here.]

Figure 12 shows the $\mathcal{R}$ -curve of the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate, obtained as the envelope of the crack driving force curves calculated using the size effect law of figure

11. To simplify the implementation of the $\mathcal{R}$ -curve in numerical or analytical models, Catalanotti et al. [11] used the following equation to fit the $\mathcal{R}$ -curve of figure 12:

$$\mathcal{R}(\Delta a) = \begin{cases} \mathcal{R}_{SS} [1 - (1 - \zeta \Delta a)^\eta] & , \Delta a \leq l_{fpz} \\ \mathcal{R}_{SS} & , \Delta a > l_{fpz} \end{cases} \quad (41)$$

where ζ and η are the parameters that best fit the formula to the $\mathcal{R}$ -curve. For the quasi-isotropic thin-ply NCF [(0/-45)/(45/0)/(90/45)/(-45/90)]_S laminate, these parameters are $\zeta = 0.4226 \text{ mm}^{-1}$ and $\eta = 4.004$.

[Figure 12 about here.]

Unlike most of the experimental results available in the literature, which consider the $\mathcal{R}$ -curve to start from some initial nonzero value of $\mathcal{R}$, the $\mathcal{R}$ -curve presented in figure 12 starts from zero. However, as stressed by Bažant (see, e.g., Ref. [30]), an $\mathcal{R}$ -curve that starts from some initial nonzero value implies that the crack tip can sustain a singular stress field without showing any damage, which is unreasonable. The fact that the $\mathcal{R}$ -curve starts from zero means that the process zone forms right at the beginning of loading and that there is never any singularity at the crack tip, which is more physically reasonable. For FRPs exhibiting a relatively large bridging zone, however, it is observed that the initial rising part of the $\mathcal{R}$ -curve is considerably steep, which is the reason why, in practice, a nonzero initial value of $\mathcal{R}$ is used. According to Bažant and Kazemi [30], this relatively large initial slope can be explained by assuming that the matrix itself will, on a smaller scale, exhibit a microscopic, relatively fast-rising $\mathcal{R}$ -curve.

Equation (41) can be used in the system of equations (29) to take into account the $\mathcal{R}$ -curve of the laminate in the FFM's predictions. The first equation in (29) (FFM's stress criterion) is equal to the first equation of the system of equations (21), and no modifications are necessary. However, the second equation in (29) (FFM's energy criterion) is now written in terms of the ERR $\mathcal{G}_I(a)$. Using equation (31) in the system of equations (29), and using equation (2) to compute the SIF, the first member of the second equation of (29) reads, after integration:

$$\int_a^{a+l} \mathcal{G}_I(a) da = (\sigma^\infty)^2 \frac{\pi}{8} \frac{1}{E} \chi^2 W^2 f_K \quad (42)$$

where f_K is given by equation (26).

The second member of the second equation in (29) can be obtained using the formula in equation (41), yielding, after integration:

$$\begin{aligned} \int_0^l \mathcal{R}(\Delta a) d\Delta a &= \mathcal{R}_{0-l} = \\ &= \begin{cases} \mathcal{R}_{SS} \frac{\zeta l + \eta \zeta l + (1 - \zeta l)^\eta - (1 - \zeta l)^\eta \zeta l - 1}{\zeta(\eta + 1)} & , l \leq l_{fpz} \\ \mathcal{R}_{SS} \left[\frac{\zeta l_{fpz} + \eta \zeta l_{fpz} + (1 - \zeta l_{fpz})^\eta - (1 - \zeta l_{fpz})^\eta \zeta l_{fpz} - 1}{\zeta(\eta + 1)} + l - l_{fpz} \right] & , l > l_{fpz} \end{cases} \end{aligned} \quad (43)$$

Using equations (22), (42) and (43) in (29), and dividing the second equation in (29) by the square of the first one yields:

$$\frac{\pi}{8} \left(\frac{K_T^\infty}{K_T} \right)^2 \frac{1}{E} \chi^2 W^2 \frac{f_K}{f_\sigma^2} = \frac{\mathcal{R}_{0-l}}{(X_T^L)^2} \quad (44)$$

which is a nonlinear equation with only one unknown, the crack extension at failure l . After solving equation (44) for l , the residual strength $\bar{\sigma}^\infty$ of a center-notched laminate can be calculated using, for example, equation (28).

It is worth noting that the $\mathcal{R}$ -curve determined from Bažant's size effect law is defined such that the fracture energy at propagation, $\mathcal{R}_{SS}$, corresponds to the energy required for crack growth in an infinitely large specimen, which, by definition, is size and geometry independent. However, because the size effect law is not exact, this should be regarded as an approximation. Yet, for structures until one order of magnitude larger than the end of the size range for which the size effect law has been calibrated (which is the case here) the approximation is acceptable – see, e.g., Refs. [31, 32]. On the other hand, for some materials, the $\mathcal{R}$ -curve may change slightly with specimen shape (see, e.g., Refs. [29–32]). However, the differences in the $\mathcal{R}$ -curve may result either from the statistical scatter of the experimental tests carried out to calibrate the size effect law (see, e.g., Ref. [29]), or due to the effect of the shape correction functions κ , which vary with the different specimen shapes (notice that the length of the fracture process zone, l_{fpz} , is dependent on κ and on its derivative, making l_{fpz} more sensitive to experimental or numerical error than $\mathcal{R}_{SS}$ — see, e.g., Refs. [31, 32]). Although the authors did not assess the effect of coupon shape on the determination of the $\mathcal{R}$ -curve of CFRPs using the size effect law, based on the previous discussion, the hypothesis of a unique $\mathcal{R}$ -curve is indeed acceptable.

Figure 13 shows the predictions of the tensile residual strength of a center-notched thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ plate for a wide range of crack lengths ($2a$), obtained using the FFMs model taking into account the $\mathcal{R}$ -curve of the laminate. The height of the central notch (h) and the $W/(2a)$ ratio are kept constant and equal to 1 mm and 7.5, respectively. Figure 13 also shows the experimental results for the center-notched specimen ($2a = 4$ mm) [1] and for the center-notched plate ($2a = 36$ mm), whose results were presented in section 2, and the corresponding relative errors.

[Figure 13 about here.]

Observing table 3, and comparing with the results shown in figure 13, it is clear that the introduction of the $\mathcal{R}$ -curve in the FFMs formulation results in a significant improvement of the predictions of large damage capability. Using the traditional analysis methods, the relative errors of the predictions range from -41.2% to -21.6% for a center-notched plate with a crack length $2a = 36$ mm, whereas the FFMs prediction that takes into account the $\mathcal{R}$ -curve of the laminate has a relative error of -9.7%, which is within the range typically obtained by the traditional analysis methods for small damage coupons (see, for example, references [2, 7, 14]).

For the smaller notch sizes, the FFMs analysis that takes into account the $\mathcal{R}$ -curve of the laminate seems to overpredict the residual strength, with a relative error of 7.4% for the center-notched specimen with a crack length $2a = 4$ mm. However, such error is again within the range typically obtained by the traditional analysis methods for specimens with small notches that are not used for calibration. On the other hand, the center-notched specimens with this geometry were used to calibrate the IFM and the PS and AS models, and calculate the fracture toughness required for the LEFM and FFMs models, whereas an independent test was used to determine the $\mathcal{R}$ -curve; therefore, a fair comparison between the predictions for the small notch range of the traditional analysis methods and the FFMs predictions that take into account the $\mathcal{R}$ -curve of the laminate is not possible, unless more specimens with geometries within this range (whose results are not used for calibration purposes) are tested.

Finally, figure 14 shows a comparison between the FFMs predictions that do not take and that take into account the $\mathcal{R}$ -curve of the laminate. It is clear from this figure that, by including the $\mathcal{R}$ -curve in the formulation of the FFMs model, and consequently taking into account the bridging

process that delays unstable fracture and increases the fracture toughness as the fracture process zone advances, more reliable analyses of large through-penetration damage of composites are obtained without requiring fitting parameters (and the associated extensive test programs) or complex finite element analyses. Taking into account that the FFMs model relies on a closed-form, macromechanical solution, it can be used in preliminary design and implemented in optimization codes, without the need of verification tests or fine tuning parameters that account for scale modifications. Furthermore, because it is physically-based, only independently measured material properties are necessary (the laminate strength and the $\mathcal{R}$ -curve), which can be easily included in the experimental characterization programs that are often carried out in the composites industry.

[Figure 14 about here.]

5. Conclusions

Regarding the LDC of the NCF thin-ply laminates, a linear longitudinal displacement (u_y)-remote longitudinal stress (σ_y^∞) relation was observed up to approximately 75% of the peak load. Small load drops were observed, corresponding to the development of a fracture process zone, which had some effect on the stiffness of the center-notched plate, causing the nonlinear behavior observed above 75% of the peak load. An extensive, stable fracture process zone developed before final fracture. From a macromechanical point of view, the development of the fracture process zone took place in discrete, finite steps, which resulted from a bridging process at the crack tip. This bridging process blocked unstable fracture along the ligament width, increasing the fracture toughness of the laminate as the crack length increased, until a maximum was reached, after which final fracture occurred — $\mathcal{R}$ -curve effect. Near the notch tips, some fiber splitting was observed in both 0° and -45° surface-ply, as well as some separated plies due to delamination, which occurred after unstable fracture. Still, the extent of such damage mechanisms was reduced. Similarly to what was observed in the fractured small notch specimens [1], a brittle type of net-section failure mode was identified. In spite of this brittle type of failure, NCF thin-ply laminates exhibit stable (although not regular) intralaminar damage, typical of tougher materials, where subcritical damage mechanisms such as delamination, transverse cracking or fiber splitting [apparently](#) do not play an important role. [In the future, related studies should include non-destructive inspection data of the coupons during testing \[16\] to verify this last conclusion.](#)

Several analysis methods were presented and applied to estimate the residual strength and LDC of NCF thin-ply laminates, namely the LEFM model, IFM, PS and AS models, and FFMs model. It was observed that the classical LEFM model is not able to capture the right trend of the residual strength with changing notch length. Regarding the remaining analysis models, they all showed similar trends, with slightly different predictions for the smaller and larger notch lengths. However, all models failed to predict the residual strength of laminated plates with large through-thickness cracks. On the other hand, the predictions matched with reasonable accuracy the stress at the initiation of the fracture process (except those of the LEFM model), with errors below 1% for the IFM, AS model and FFMs model. So, these models were able to capture the onset of the fracture process, which, for small crack lengths, is close to final fracture [1], yielding good predictions. Nevertheless, for large crack lengths, the onset of the fracture process occurs considerably before final fracture, consequently yielding conservative results.

The previously developed analysis methods assume that fracture occurs unstably, as a singular event, regardless of the crack length. However, the NCF thin-ply laminate studied in this work is characterized by a remarked $\mathcal{R}$ -curve effect, developing an extensive, stable fracture process zone before final failure. Therefore, to predict LDC of laminated composites with a non-negligible fracture process zone, the bridging mechanisms associated with the fracture process must be lumped into the crack tip in a physically admissible way, in order to be able to take them into account in the macromechanical model. One possibility is to take properly into account the effect of the $\mathcal{R}$ -curve in the analysis method.

Based on a new test methodology [11], which is able to characterize the $\mathcal{R}$ -curve of (thin) laminated composites that show improved resistance to the propagation of intralaminar damage, such as the NCF thin-ply laminate studied in the present work, an analytical formula for the $\mathcal{R}$ -curve can be obtained. This formula was used in the FFMs model to take into account the $\mathcal{R}$ -curve of the material. It was observed that the introduction of the $\mathcal{R}$ -curve in the FFMs formulation clearly has a significant positive effect in the predictions of large damage capability. More reliable analyses of large through-penetration damage of composites can be obtained without fitting parameters or complex finite element analyses. The analytical FFMs model can be used in preliminary design and implemented in optimization codes. Furthermore, only independently measured material properties are necessary (the laminate strength and the $\mathcal{R}$ -curve), which can be easily included in the experimental

characterization programs that are often carried out in the composites industry.

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Figure 1: Center-notched plate. Position of the strain gages ($d_1 = 11.5$ mm, $d_2 = 21.5$ mm, and $d_3 = 63.5$ mm) and DIC observation area.

(a) Upper tool of the load introduction device.

(b) Specimen's drawing.

Figure 2: Load introduction mechanism and specimen's drawing.

(a) Longitudinal displacement (u_y)-remote longitudinal stress (σ_y^∞) relation.

(b) Far-field ($SG0$) longitudinal strain (ε_y)-remote longitudinal stress (σ_y^∞) relation.

Figure 3: Center-notched plate test results.

Figure 4: Post-processing of the DIC: (a) time-stress relation showing some points corresponding to the development of the fracture process zone; surface longitudinal strain (ε_y) fields, and surface longitudinal strain (ε_y) measured along a cutting line centered along the notch, for the stages corresponding to (b) the onset of the fracture process, (c) and (d) its development, and (e) just before final failure.

(a) Physical strain gages, notch left side.

(b) Physical strain gages, notch right side.

(c) Virtual strain gages, notch left side.

(d) Virtual strain gages, notch right side.

Figure 5: Longitudinal strain (ε_y)-remote longitudinal stress (σ_y^∞) relations near the notch tips.

(a) Before testing.

(b) Just before final fracture.

Figure 6: Central notch region of the tested center-notched plate.

Figure 7: Fractured center-notched plate after testing. Notice that the two black vertical stripes in the plate's surface are just two unpainted regions used to help delimitating the DIC observation area.

Figure 8: Composite laminate with a central notch loaded in tension.

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Figure 11: Experimental results and best fitting for the size effect law of the quasi-isotropic thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate, as obtained in Ref. [11]. The parameters giving the best fit are $A = 0.656\text{E-}6 \text{ MPa}^{-2}\text{mm}^{-1}$ and $C = 3.84\text{E-}6 \text{ MPa}^{-2}$.

Figure 12: $\mathcal{R}$ -curve of the quasi-isotropic thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate, as obtained in Ref. [11].

Figure 13: Finite Fracture Mechanics predictions and corresponding relative errors for the tensile residual strength of the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate, taking into account the $\mathcal{R}$ -curve effect.

Figure 14: Finite Fracture Mechanics predictions for the tensile residual strength of the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate, not taking and taking into account the $\mathcal{R}$ -curve of the material.

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Table 1: Configuration of the digital image correlation system.

Camera-lens optical system	
CCD camera	Baumer 138 Optronic FWX20 camera 8-bit Resolution: 1624×1236 pixels ² Pixel size: 4.4 microns Sensor format of 1/1.8"
Lens	Nikon AF Micro-Nikkor 200 mm f/4D IF-ED Lens aperture: $f/11$
Shutter time	25 ms
Working distance	2730 mm
Captured region	100×76 mm ²
Conversion factor	0.0636 mm/pixel
Acquisition frequency	1 Hz
DIC measuring parameters	
Facet size	15×15 pixels ²
Facet step	13×13 pixels ²
Strain base length	5 subsets

Table 2: Properties required by the macromechanical models to predict the tensile notched response of the thin-ply NCF $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ laminate.

Model	a_0 (mm)	r_{ot} (mm)	$\mathcal{K}_{Ic}$ (MPa $\sqrt{\text{m}}$)	X_L^T (MPa)
LEFM	-	-	38.1	-
IFM	1.79	-	-	710.3
PS	-	0.76	-	710.3
AS	-	3.36	-	710.3
FFMs	-	-	53.1	710.3

Table 3: LDC predictions and relative errors for the $[(0/-45)/(45/0)/(90/45)/(-45/90)]_S$ NCF thin-ply laminate.

Model	$\bar{\sigma}^\infty$ (MPa)	Relative error (ϵ_r) (%)	
		init. fract.	ult. stress
LEFM	159	-24.3	-41.2
IFM	211	0.9	-21.6
PS	193	-7.8	-28.4
AS	208	-0.8	-22.9
FFMs	211	0.6	-21.8