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Catalanotti, G., & Camanho, P. P. (2013). A semi-analytical method to predict net-tension failure of mechanically fastened joints in composite laminates. *Composites Science and Technology*, 76, 69-76.
<https://doi.org/10.1016/j.compscitech.2012.12.009>

Published in:
Composites Science and Technology

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
[Link to publication record in Queen's University Belfast Research Portal](#)

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A semi-analytical method to predict net-tension failure of mechanically fastened joints in composite laminates

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Abstract

A Finite Fracture Mechanics model is proposed to predict the net-tension strength of composite mechanically fastened joints. A modification of coupled stress–energy criteria is suggested to take properly into account the effect of the R-curve in determining the crack length corresponding to the unstable crack propagation. The proposed model works for quasi-isotropic laminates and it requires as input the longitudinal strength and the fracture toughness of the laminate. The predictions obtained show good agreement when compared with the experimental results.

Key words:

A. Polymer-matrix composites (PMCs), B. Fracture, B. Strength

1 Introduction

Mechanically fastened joints are widely used in the aeronautical industry and, despite the fact that fasteners cause an increase of weight of aircraft structures, such joints are often the only feasible solution. For example, bonded joints should not be used in thick composite parts or when the joint needs to be disassembled for the maintenance or inspection of the structure.

Several studies on mechanically fastened joints in polymer composites have been conducted over the past 40 years. An extensive review of all the relevant studies in the field is reported in [1–3].

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Figure 1 shows the typical geometry of the specimen and the relevant parameters: the width, w , the end distance, e , the diameter of the hole, d . The effects of these parameters on the strength of composite mechanically fastened joints were discussed by several authors [4–9].

[Fig. 1 about here.]

Hart-Smith [4] showed that when the diameter is large compared with the width of the specimen net-tension failure mode occurs. The critical value of the w/d ratio that defines the change in the failure modes from net-tension to bearing depends on the material and on the lay-up used. For quasi isotropic CFRP laminates the w/d ratio corresponding to the change in failure mode is about 3–4. Modifications of the geometry and of the layup of the laminate change the failure mode of the joint and consequently the strength of the joint.

The strength prediction methods developed in previous studies for the fast design and optimization of composite bolted joints use empirical parameters: the tensile strength of the laminate, X_L , and one or more characteristic dimensions. The strength prediction methods used are based on modifications of the point- [10] and average-stress [11] models proposed by Whitney and Nuismer [12].

A combination of a characteristic distance [12] and a failure theory were used by several authors [13,15–19]. Chang *et al.* [13] used the Yamada-Sun [14] failure criterion together with a proposed characteristic curve in a bi-dimensional Finite Element (FE) model. The Yamada-Sun [14] failure criterion is expressed as:

$$\left(\frac{\sigma_{11}}{X_L}\right)^2 + \left(\frac{\sigma_{12}}{S_c}\right)^2 = e_f^2 \quad (1)$$

where σ_{11} and σ_{12} are respectively the longitudinal and in-plane shear stress in a ply, X_L is the ply longitudinal strength and S_c the ply shear strength measured from a cross-ply laminate. Failure occurs when $e_f \geq 1$. The characteristic curve was defined as:

$$r_c(\theta) = \frac{d}{2} + d_{0t} + (d_{0c} + d_{0t}) \cos \theta \quad (2)$$

where d_{0t} and d_{0c} are the characteristic dimensions for tension and compression, respectively. Both parameters must be obtained experimentally. The non-linear shear stress-shear strain was also taken into account in [19].

Camanho and Lambert [20] using the concept of characteristic curve together with the LaRC04 failure criteria [21] developed a methodology able to predict

42 the elastic limit of the joint and the ultimate failure load of the joint.

43 All the previously mentioned methods are able to predict the strength of
44 composite mechanically fastened joints. The drawback of those methods is that
45 the empirical characteristic dimensions have to be experimentally determined
46 for each lay-up and the accuracy of the solution depends on the particular
47 geometry used to identify the model. When the material is changed, also the
48 characteristic dimensions change and a further recalibration of the model is
49 required. Moreover the physical meaning of the characteristic dimension is not
50 clear.

51 Therefore the objective of this paper is to present a new model that can predict
52 the net-tension strength of a mechanically fastened joint without the use of a
53 characteristic dimension. The model proposed is developed in the framework
54 of the Finite Fracture Mechanics (FFM) [22–30].

55 A simple analytical model to predict the strength of a mechanically fastened
56 joint using only the strength X_L^T and the fracture toughness $\mathcal{K}_C$ of the laminate
57 as input parameters is proposed. The model is valid for the entire family of
58 quasi-isotropic laminates; therefore, it can be used as a quick-design tool in
59 an industrial environment for preliminary design and optimization.

60 **2 Finite Fracture Mechanics Model for mechanically fastened joints**

61 The following assumptions are made:

- 62 • only quasi-isotropic laminates will be addressed. This is not a severe lim-
63 itation because even if in theory the layup of the composite joint can be
64 a different one the common practice in the aeronautical industry is to use
65 quasi-isotropic laminates to enhance the joint performance under different
66 loading cases and to minimize the stress concentration factor. However the
67 method is general and it can be extended to different layups and stacking
68 sequences with some modifications.
- 69 • only the net-tension failure mechanism is investigated. This failure mecha-
70 nism occurs in highly optimized composite joints and in relevant structural
71 details such as composite lugs.

72 It should be noted that for the case of net-tension failure there is a clear macro-
73 crack and therefore Finite Fracture Mechanics is applicable. This is not the
74 case of bearing failure mode where permanent deformation of the hole caused
75 by ply-level failure mechanisms and delamination occur [20,31].

76 The problem to solve in the Finite Fracture Mechanics framework is shown
 77 in Figure 1 where a mechanically fastened joint under tensile loading and
 78 the associated coordinate system are shown. The contact pressure around the
 79 pin is assumed to be cosinusoidal, as it has been suggested by Waszczak and
 80 Cruse [32] for both isotropic and anisotropic materials. Moreover, it is assumed
 81 that the x and y axes correspond to the preferred axes of the material and that
 82 the end distance, e , is sufficiently large to prevent shear-out failure. Finally, all
 83 the variables used are defined per unit thickness of the laminate. The contact
 84 pressure is given as:

$$p = p_{max} \cos \vartheta \quad (3)$$

85 where p_{max} is the maximum value of the contact pressure and ϑ is the angle
 86 shown in Figure 1. The load applied to the joint along the y -direction is simply
 87 obtained as:

$$F_y = \frac{1}{2} d p_{max} \int_{-\pi/2}^{\pi/2} \cos^2 \vartheta d\vartheta = \frac{\pi}{4} d p_{max} \quad (4)$$

88 The remote stress, σ_∞ , and the bearing stress, σ_b , are given as:

$$\sigma_\infty = \frac{F_y}{w} = \frac{\pi d p_{max}}{4 w} \quad (5)$$

$$\sigma_b = \frac{F_y}{d} = \frac{\pi}{4} p_{max} \quad (6)$$

89 It is noted that the bearing strength depends only on the maximum contact
 90 pressure applied and not on the dimension of the joint.

91 A modification of the coupled stress-energy criterion proposed in [26] and
 92 applied in [30] for the prediction of open-hole strength of composite laminates
 93 is used to predict the strength of the joint. However, the knowledge of both
 94 the longitudinal stress distribution along the net-tension plane (Figure 2) and
 95 the stress intensity factor for the case of two cracks emanating from a hole for
 96 a composite joint loaded in tension (see Figure 3) is required.

97 [Fig. 2 about here.]

98 [Fig. 3 about here.]

99 It is convenient to define a new coordinate system (ξ, y) , with the origin at
 100 the edge of the hole where ξ is defined as:

$$\xi = \frac{2}{w-d} \left(x - \frac{d}{2} \right) \quad (7)$$

It should be noted that ξ is dimensionless and takes the value of $\xi = 0$ at the hole edge and $\xi = 1$ at the outer edge of the specimen. In this coordinate system the coupled stress-energy criterion [26] reads:

$$\begin{cases} \frac{1}{l} \int_0^l \sigma_y(\xi) d\xi = X_L^T \\ \frac{1}{l} \int_0^l \mathcal{K}_I^2(\xi) d\xi = \mathcal{K}_{IC}^2 \end{cases} \quad (8)$$

where l is the crack extension just before its unstable propagation. It should be noted that the fracture toughness $\mathcal{K}_{IC}$ in the right-hand side of the second equation in (8) is considered constant in [26] and it can be considered equal to the toughness propagation value when the length of process zone, l_{pz} , is small with respect to the characteristic dimensions of the specimen. In that case the actual shape of the R-curve can be neglected and a constant value of the fracture toughness $\mathcal{K}_{IC}$ is considered.

Figure 4(a) shows qualitatively the R-curve of the material and the energies associated with the crack propagation. Considering the fracture toughness constant and equal to $\mathcal{K}_{IC}^p$, the energy associated to the crack propagation will be proportional to the area of the rectangle ($\Delta a_c \times (\mathcal{K}_{IC}^p)^2$). In the presence of a R-curve, the energy is proportional to the area under the R-curve (striped area in Figure 4(a)). When $\Delta a_c \gg l_{pz}$ this difference can be neglected.

However, in the case of bolted joints that fail by net-tension, and in particular in the presence of small diameters, the length of process zone, l_{pz} and the critical crack length, Δa_c , may be of the same order than the characteristic dimension of the specimen ($l_{pz} \sim \Delta a_c \sim (w-d)/2$) and in that case the actual shape of the R-curve must not be neglected (see Figure 4(b)).

[Fig. 4 about here.]

Therefore, to take into account the R-curve of the material a modification of the coupled stress-energy criterion, equation (8), is proposed:

$$\begin{cases} \frac{1}{l} \int_0^l \sigma_y(\xi) d\xi = X_L^T \\ \int_0^l \mathcal{K}_I^2(\xi) d\xi = \int_0^l \mathcal{K}_{IC}^2(\Delta\xi) d\Delta\xi \end{cases} \quad (9)$$

where $\mathcal{K}_{IC}(\Delta\xi)$ is the R-curve expressed in the (ξ, y) coordinate system. Assuming that the R-curve, $\mathcal{K}_{IC}(\Delta a)$ where Δa the crack increment, is known,

127 the fracture toughness $\mathcal{K}_{IC}(\Delta\xi)$ is obtained taking into account that $\Delta a =$
 128 $\Delta\xi(w-d)/2$.

129 For the elastic orthotropic case the stress σ_y and the stress intensity factor $\mathcal{K}_I$
 130 of equation (9) depend, for a given bearing strength, σ_b , only on the dimensions
 131 and on the orthotropy of the material.

132 For quasi-isotropic laminates the dependence of the stress intensity factor on
 133 the material can be neglected. In fact the response of an orthotropic solid in
 134 plane stress only depends on two dimensionless parameters [33,34]:

$$\lambda = \frac{E_y}{E_x} \quad (10)$$

$$\rho = \frac{\sqrt{E_x E_y}}{2 G_{xy}} - \sqrt{\nu_{xy} \nu_{yx}} \quad (11)$$

135 where E_x and E_y are respectively the laminate longitudinal and transverse
 136 Young's modulus, G_{xy} is the laminate shear modulus, and ν_{xy} and ν_{yx} are
 137 the laminate Poisson's ratios. For the quasi-isotropic laminate and for a crack
 138 running along a preferred axis of the material λ and ρ take the values $\lambda =$
 139 $\rho = 1$. Therefore the dependence on the material orthotropy can be eliminated
 140 from the calibration of the model.

141 Therefore the stress distribution and the stress intensity factor only depend
 142 on the two dimensionless variables ξ and $\omega = w/d$ and on the diameter d . It
 143 should be noted that ξ and ω define the relations between the diameter, the
 144 width and the material point on the net-tension plane where the stress and
 145 stress intensity factors are calculated.

146 To compute the stress distribution $\sigma_y(\xi, \omega)$ it is convenient to use the super-
 147 position of the stress field. It is possible to compute numerically, using the
 148 Finite Element Method, the stress distribution when a unitary remote load is
 149 applied at a joint with a unitary diameter of the hole:

$$\sigma_y(\xi, \omega)|_{\substack{\sigma_b=1 \\ d=1}} = \phi(\xi, \omega) \quad \sigma_b|_{\sigma_b=1} = \phi(\xi, \omega) \quad (12)$$

150 The polynomial fitting function ϕ is calculated using the commercial software
 151 Abaqus 6.8-3 [35]. The two variables to investigate are ω ($1 < \omega < \infty$) and ξ
 152 ($0 \leq \xi < 1$). However, for practical application $1.5 < \omega \leq 4$. In fact for $\omega > 4$
 153 bearing failure occurs [4] instead of net-tension failure.

154 A parametric Finite Element model was created using Python [36] together
 155 with Abaqus 6.8-3 CAE [35]. The 8-node biquadratic element, CPS8, was used
 156 to mesh the models and a unitary bearing stress $\sigma_b = 1$ was applied.

157 Finally, the parameters that best fit the data are obtained via a least-squares
 158 curve fitting method in Matlab 7.6.0 [37]. Our calculation shows that $\phi(\xi, \omega)$
 159 can be approximated with high accuracy with a polynomial fitting function:

$$\phi(\xi, \omega) = \sum_{i=1}^M \sum_{j=1}^N \Phi_{ij} \omega^{j-1} \xi^{i-1} \quad (13)$$

160 where Φ_{ij} is the element of the matrix Φ at the row i and at the column j ,
 161 and M and N are the number of rows and columns of Φ respectively. The
 162 matrix Φ is reported in Appendix A. Figure 5 shows the numerical results
 163 obtained and the corresponding fitting surface, $\phi(\xi, \omega)$. It should be noted
 164 that the only variable that varies in the model is ω ; therefore, in Figure 5
 165 the numerical points corresponding to the same ω are calculated in the same
 166 analysis.

167 [Fig. 5 about here.]

168 For a general bearing stress σ_b , and for any diameter, d , the stress distribution
 169 reads:

$$\sigma_y(\xi, \omega) = \sigma_b \phi(\xi, \omega) \quad (14)$$

170 In an analogous way, for a crack of length ξ the stress intensity factor when a
 171 unitary bearing stress is applied reads:

$$\mathcal{K}_I(\xi, \omega)|_{\sigma_b=1} = \sigma_b|_{\sigma_b=1} \sqrt{d}|_{d=1} \psi(\xi, \omega) = \psi(\xi, \omega) = \frac{\sqrt{\xi}}{1 - \xi} \sum_{i=1}^M \sum_{j=1}^N \Psi_{ij} \omega^{j-1} \xi^{i-1} \quad (15)$$

172 To compute Ψ a parametric Finite Element model was created using Python [36]
 173 together with Abaqus 6.8-3 CAE [35]. The 4-node quadratic, reduced integra-
 174 tion element, CPS4R, was used to mesh the models and a unitary bearing
 175 stress $\sigma_b = 1$ has been applied. In this model the variables that vary are the
 176 diameter over width ratio, ω ($1.5 < \omega \leq 4$), and the crack length l ($0 < l < 1$).
 177 Figure 6 shows the contour plot of σ_y obtained in the model with $\omega = 2.75$
 178 and a crack length $\xi = 0.45$.

179 [Fig. 6 about here.]

180 It should be noticed that the expression of $\psi(\xi, \omega)$ respects the boundary
 181 conditions, i.e. $\psi = 0$ for $\xi \rightarrow 0$ and $\psi = \infty$ for $\xi \rightarrow 1$. Figure 7 shows the
 182 numerical results obtained and the corresponding fitting surface, $\psi(\xi, \omega)$ (each
 183 dot represents one numerical simulation). The matrix Ψ of equation (15) is
 184 reported in Appendix A.

185 [Fig. 7 about here.]

186 Finally, for a general bearing stress, σ_b , and for any diameter, d , the stress
 187 intensity factor reads:

$$\mathcal{K}_I(\xi, \omega) = \sqrt{d} \sigma_b \psi(\xi, \omega) \quad (16)$$

188 Inserting equations (14) and (16) in (9):

$$\begin{cases} \sigma_b \int_0^l \phi \, d\xi = X_L^T l \\ d \sigma_b^2 \int_0^l \psi^2 \, d\xi = \int_0^l \mathcal{K}_{IC}^2 \, d\Delta\xi \end{cases} \quad (17)$$

189 where the bearing stress at failure, σ_b , and the crack length at failure, l , are
 190 the two unknowns. Dividing the first of equation (17) for the squared root of
 191 the second gives:

$$\frac{\int_0^l \phi \, d\xi}{X_L^T} = l \left(\frac{d \int_0^l \psi^2 \, d\xi}{\int_0^l \mathcal{K}_{IC}^2 \, d\Delta\xi} \right)^{1/2} \quad (18)$$

192 Equation (18) can be solved for l ($0 < l < 1$). Substituting the value of l in
 193 one of (17) enables the calculation of the bearing stress at failure, σ_b .

194 3 Model Validation

195 The material chosen for the experimental investigation is the Hexcel IM7-8552
 196 carbon epoxy unidirectional pre-preg. The $[90/0/\pm 45]_{3s}$ lay-up, corresponding
 197 to laminates with a nominal thickness of 3mm, was selected.

198 The material was cured according to the specifications of the manufacturer.
 199 The curing cycle consisted in keeping the laminate at 110°C for 1 hour, in-
 200 creasing the temperature up to 180°C, keeping this temperature for 2 hours,
 201 and cooling at 3°C/min. The pressure of 7 bar was used during all the curing
 202 cycle.

203 After curing, the laminate was cut using a diamond saw and the specimens
 204 were drilled clamping sacrificial plates at the insertion and exit points of the
 205 drill to avoid damage at the hole edge.

206 The bolted joints were tested according to ASTM D-5961 standard [38] with
 207 a 2.2Nm torque applied to the bolt. The relevant dimension of the specimens
 208 are shown in Table 1.

209 [Table 1 about here.]

210 The Finite Fracture Mechanic model described in the previous section was
 211 implemented in Maple 13.0 [39]. Knowing ω the functions ϕ and ψ defined in
 212 equations (13) and (15) respectively can be calculated. Figures 8 and 9 show
 213 ϕ and ψ as a function of ξ for $\omega = 2$.

214 [Fig. 8 about here.]

215 [Fig. 9 about here.]

216 The strength of the laminate, $X_T^L = 845.1\text{MPa}$, and the fracture toughness of
 217 the laminate $\mathcal{K}_{IC}$ are reported respectively in [40] and [41]. Without loss of
 218 generality it is proposed to use a Gompertz function [42] to obtain an analytical
 219 expression of the R-curve, $\mathcal{K}_{IC}(\Delta a)$, to be inserted in equation (18). Therefore
 220 the R-curve can be expressed as:

$$\mathcal{K}_{IC} = \mathcal{K}_{IC}^p e^{\log(\mathcal{K}_{IC}^i/\mathcal{K}_{IC}^p)e^{b\Delta a}} \quad (19)$$

221 where $\mathcal{K}_{IC}^i$ and $\mathcal{K}_{IC}^p$ are respectively the initiation and the propagation values
 222 of the fracture toughness, e is the Euler's number, and b is calculated fitting the
 223 experiments. Figure 10 shows the fracture toughness values obtained in [41]
 224 and the fitting function for the laminate under investigation. The parameters
 225 used in equation (19) are $\mathcal{K}_{IC}^i = 36.7\text{MPa}\sqrt{\text{m}}$, $\mathcal{K}_{IC}^p = 42.8\text{MPa}\sqrt{\text{m}}$, and
 226 $b = -1.27\text{mm}^{-1}$.

227 [Fig. 10 about here.]

228 For a given ω , the length l can be obtained from equation (18). Consequently,
 229 substituting l , for example in equation (14), the bearing stress at failure can
 230 be obtained. Table 2 shows the comparison between the experimental value of
 231 the joint strength and the corresponding prediction. It is concluded that the
 232 proposed method, that is based on only two material properties (the strength
 233 and the fracture toughness), yields predictions that are in good agreement
 234 with the experimental data.

235 [Table 2 about here.]

236 Figure 11 shows the specimen after testing. A straight crack in the net tension
237 plane is observed.

238 [Fig. 11 about here.]

239 Figure 12 shows the predicted failure envelope, the experimental data and the
240 bearing cut-off strength obtained in a previous investigation [20].

241 [Fig. 12 about here.]

242 From an inspection of equation (17) it is concluded that the bearing stress at
243 failure will depend on the diameter of the joint. Using the proposed model it is
244 possible to identify this dependency as shown in Figure 13 where the bearing
245 stress at failure of a joint with $\omega = 2$ is plotted as a function of the diameter d .

246 [Fig. 13 about here.]

247 4 Conclusions

248 A Finite Fracture Mechanics model was proposed to predict the strength of
249 a mechanically fastened joint when net-tension failure occurs. The model is
250 applicable for quasi-isotropic laminates and it requires as input the longitu-
251 dinal strength and the fracture toughness of the laminate. Using the Finite
252 Element Method a parametric study was conducted to determine the stress
253 distribution of a mechanically fastened joint and the stress intensity factor for
254 the joint with two cracks emanating from the hole edge.

255 A modification of the coupled stress-energy criterion proposed in [26] was sug-
256 gested to properly take into account the actual shape of the R-curve of the
257 material. The predictions obtained are in good agreement with the experimen-
258 tal results.

259 It should be pointed out that the proposed methodology can be easily extended
260 to the case of an orthotropic mechanically fastened joint. In that case, the
261 influence of the orthotropy of the material should be taken into account (the
262 parameters ρ and λ of equation (11) will differ from unity) and consequently
263 ϕ and ψ will also be function of ρ and λ (or only ρ is the orthotropic rescaling
264 technique is used [33]). Moreover it should be pointed out that the hypothesis
265 that the stress distribution at the hole be cosinusoidal [32] would be unrealistic
266 for highly orthotropic laminates so a different stress distribution should be
267 used for those laminates.

268 Acknowledgments

269 The first author would like to acknowledge the support of the Fundação para
270 a Ciência e a Tecnologia under the grant FCT-DFRH-SFRH-BPD-78104-
271 2011.

272 The authors acknowledge the support of AIRBUS under the project iComp –
273 integrated method for the structural design of Composite components.

A Polynomial fitting functions calculation

The matrices Φ and Ψ of equations (13) and (15) respectively are:

$$\Phi = \begin{bmatrix} 88.47259368 & -170.4688173 & 142.1541230 & -63.66745751 & 16.02655846 & -2.141735655 & 0.1184734754 \\ -66.06806270 & 130.3642184 & -115.6511798 & 53.44517736 & -13.76218368 & 1.869898039 & -0.1045676693 \\ -122.1958288 & 153.5379000 & -62.54369626 & 12.14942145 & -0.4008873416 & -0.1271694776 & 0.007169337386 \\ 363.2685871 & -209.2179324 & -139.5989080 & 138.6183273 & -45.43450856 & 6.315612158 & -0.2992400294 \\ -606.3183747 & -78.63062116 & 799.2229915 & -521.2745009 & 146.4340404 & -18.11785833 & 0.7364093402 \\ 816.2118494 & -19.49098364 & -834.3668216 & 527.9131694 & -132.9112667 & 12.59320117 & -0.1400671546 \\ -746.1678535 & 513.6516665 & 33.30630791 & -64.69929108 & 0.5204533361 & 5.777467829 & -0.8363149246 \\ 284.0865968 & -340.6938503 & 194.6007013 & -90.06655857 & 31.42283172 & -6.421597015 & 0.5320569763 \end{bmatrix} \quad (\text{A.1})$$

$$\Psi = \begin{bmatrix} 22.83435806 & -29.81479531 & 18.40143327 & -5.758140163 & 0.9066311202 & -0.05717860696 \\ -37.50380850 & 23.07360348 & -8.222158301 & 0.6293260657 & 0.09732446130 & -0.01140889238 \\ 301.6521309 & -222.6945915 & 84.43828034 & -14.02728352 & 1.448093939 & -0.1084130419 \\ -1273.889397 & 884.2488556 & -273.6088052 & 28.35795338 & -0.7156219526 & 0.07507960542 \\ 2843.096095 & -1803.563243 & 416.1988366 & 15.03106666 & -13.55753986 & 0.9925783371 \\ -3669.049920 & 2346.185565 & -529.1567783 & -27.31814770 & 20.82634120 & -1.697989115 \\ 2518.166947 & -1714.235036 & 452.2469504 & -13.23375312 & -9.695981378 & 0.961030382 \\ -704.7412797 & 516.0148956 & -159.8012277 & 16.15772934 & 0.7170582248 & -0.1554093946 \end{bmatrix} \quad (\text{A.2})$$

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Fig. 1. Mechanically fastened joint under tensile loading

Fig. 2. Model for the stress distribution on the net-tension plane

Fig. 3. Model for the stress intensity factor of two cracks emanating from a hole

Fig. 4. R-curve and energies associated to the crack propagation

Fig. 5. FE results and fitting surface $\phi(\xi, \omega)$

Fig. 6. Contour plot of σ_y

Fig. 7. FE results and fitting surface $\psi(\xi, \omega)$

Fig. 8. ϕ as a function of ξ for $\omega = 2$

Fig. 9. ψ as a function of ξ for $\omega = 2$

Fig. 10. R-curve of the laminate: experiments and fitting

Fig. 11. Failed specimens for the three geometries tested

Fig. 12. Bearing strength as a function of ω for the 6mm diameter joint

Fig. 13. Predicted value of the bearing strength for different diameters d ($\omega = 2$)

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Specimen	d [mm]	ω	w [mm]	e [mm]	length [mm]
NT1	6	1.5	9	35	215
NT2	6	1.75	10.5	35	215
NT3*	6	2	12	25	170

*Experiments presented in [20]

Table 1
Relevant dimension of the specimen

Specimen	# samples	Strength [MPa]	STDV [MPa]	Predicted strength [MPa]	ϵ [%]
NT1	3	322.3	0.26	310.2	-3.8
NT2	3	466.2	2.41	434.8	-6.7
NT3*	5	526.7	20.7	549.5	+4.3

*Experiments presented in [20]

Table 2

Comparison between experimental and predicted strength of the joint