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A finite fracture mechanics model for the
prediction of the notched strength of
composite laminates

P.P. Camanho^{a,*}, G.H. Erçin^b, G. Catalanotti^a, S. Mahdi^c,
P. Linde^d

^a*DEMec, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto
Frias, 4200-465, Porto, Portugal*

^b*INEGI, Instituto de Engenharia Mecânica e Gestão Industrial, Rua Dr. Roberto
Frias 400, 4200-465, Porto, Portugal*

^c*AIRBUS Operations SAS, 316, Route de Bayonne, 31060 Toulouse Cedex 03,
France*

^d*AIRBUS Operations GmbH, Kreetzlag 10, 21129 Hamburg, Germany*

Abstract

A new model based on finite fracture mechanics is proposed to predict the open-hole tensile strength of composite laminates. Failure is predicted when both stress-based and energy-based criteria are satisfied. The material properties required by the model are the laminate unnotched strength and fracture toughness. No empirical adjusting parameters are required. Using experimental data obtained in quasi-isotropic carbon-epoxy laminates it is concluded that the model predictions are very accurate, resulting in improvements over the traditional strength prediction methods. It also is shown that the proposed finite fracture mechanics model can be used

to predict the brittleness of different combinations of materials and geometries.

Key words: A. Polymer-matrix composites (PMCs), B. Fracture, C. Analytical modelling.

1 Introduction

The aerospace industry strives for accurate, physically-based and fast strength prediction methods for composite laminates with stress concentrations. While the two first conditions can be satisfied using appropriate non-linear finite element (FE) models [1]–[5], the third one cannot. Implicit non-linear FE models that include material instabilities result in severe convergence difficulties and require very fine meshes with element sizes typically smaller than 1mm. The computing time of explicit non-linear FE models is partially defined by the stable time increment that decreases with the element size. Both approaches normally result in long computing times that are not acceptable for preliminary sizing and for the optimization of aircraft structural details.

The most widely used design method for composite laminates with stress concentrations that is suitable for preliminary sizing and optimization is the point stress or the average stress models proposed by Whitney and Nuismer [6], or variations of thereof [7]. The point stress model assumes that failure takes place when the stress at a given distance from the notch boundary (the 'characteristic distance') reaches the unnotched strength of the laminate, whereas the average stress model predicts failure when the average stress over a

* Corresponding author

Email address: `pcamanho@fe.up.pt` (P.P. Camanho).

characteristic distance is equal to the unnotched strength of the laminate. The characteristic distance must be identified from a test in a notched laminate. While the point or average stress models provides reasonable predictions for hole diameters close to that used for the model calibration, they have one main problem: the 'characteristic distance' is not a material property, it depends on both the material/lay-up and on the geometry [8]. As a result, large and expensive experimental programmes are required to identify the characteristic distances for the different materials and geometries.

An alternative method for the strength prediction of composite laminates loaded in tension containing notches or cracks based on the numerical implementation of cohesive formulations were developed Bäcklund et al. [9]–[12]. The damage mechanisms that occur at the vicinity of a crack or hole are lumped into a damage zone, where a linear relation between the cohesive traction and the crack opening is assumed. A traction-free crack develops when the dissipated energy equals the fracture toughness of the laminate. While this method is based on sound mechanical models, the need for a numerical implementation renders it unsuitable for fast predictions.

Based on the previous observations, the objective of this paper is to develop a fast strength prediction method for composite laminates with circular holes loaded in tension. The strength prediction method should be based on independently measured material properties, and it must not require any type of calibration for different hole sizes or specimen widths.

The model developed is based on the concept of finite fracture mechanics that was originally introduced by Leguillon [13]. Finite fracture mechanics models assume that crack propagation results from the simultaneous fulfilment

of a stress-based criterion and an energy-based criterion. In addition, it is considered that failure occurs by the propagation of kinematically admissible cracks with finite sizes.

The finite fracture mechanics criteria predict failure based on two conditions [14]: the stress ahead of a crack tip averaged over a distance l reaches the material strength X_T^L , i.e. $\int_a^{a+l} \sigma(x)dx = X_T^L l$. In addition, the energy available to propagate the crack a finite distance l must be equal to a critical value that is defined by the fracture toughness of the material. This condition, established using the stress intensity factor $\mathcal{K}$ and its critical value $\mathcal{K}_{Ic}$, reads [14]: $\int_a^{a+l} \mathcal{K}_I^2(a)da = \mathcal{K}_{Ic}^2 l$.

Finite fracture mechanics models have been applied to the prediction of fracture of unidirectional composites under off-axis tension [15], sharp V-notches in isotropic materials [16]–[17], three-point bending tests in notched and unnotched specimens [14], [18], bi-material joints [19], and free-edge delamination [20].

2 Finite fracture mechanics model for notched composites

Consider a composite laminate with a central circular hole with a diameter d , radius R and width W loaded in tension (Figure 1).

[Fig. 1 about here.]

For the loading conditions shown in Figure 1, and assuming that the lay-up of the laminate leads to either the brittle or pull-out failure modes defined by Green et al. [21], the propagation of the macro-crack that leads to final

failure occurs along the x -direction. Using the coupled stress and energy finite fracture mechanics criterion proposed by Cornetti et al. [14] for the particular case of a specimen with an uncracked central circular hole, fracture occurs when the following system of equations is satisfied:

$$\left\{ \begin{array}{l} \frac{1}{l} \int_R^{R+l} \sigma_{yy}(x, 0) dx = X_T^L \\ \frac{1}{l} \int_R^{R+l} \mathcal{K}_I^2(a) da = \mathcal{K}_{Ic}^2 \end{array} \right. \quad (1)$$

where X_T^L is the unnotched strength of the laminate, $\mathcal{K}_{Ic}$ is the mode I fracture toughness of the laminate, and l is the crack extension at failure. The first equation in (1) corresponds to the average-stress model [6]; therefore, the proposed finite fracture model enriches the average-stress model using a second equation that represents an energy balance that must be satisfied during crack propagation. Taking into account that the system of equations (1) yields the remote notched strength and the crack extension at failure, there is no need to use an empirical 'characteristic distance' identified from one notched test specimen [6].

The stress distribution along the x -axis, $\sigma_{yy}(x, 0)$, is obtained as [22]:

$$\sigma_{yy}(x, 0) = R_K \frac{\sigma^\infty}{2} \left[2 + \xi^2 + 3\xi^4 - (K_T^\infty - 3) (5\xi^6 - 7\xi^8) \right], \quad \xi = \frac{R}{x} \quad (2)$$

where σ^∞ is the remote stress, K_T^∞ is the stress concentration factor of an infinite plate containing a circular hole, and R_K is the finite width correction

factor. These parameters are defined as [22]:

$$K_T^\infty = 1 + \sqrt{\frac{2}{A_{22}} \left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}} \right)} \quad (3)$$

$$R_K = \frac{K_T}{K_T^\infty} = \left\{ \frac{3(1-2R/W)}{2+(1-2R/W)^3} + \frac{1}{2} \left(\frac{2R}{W} M \right)^6 (K_T^\infty - 3) \left[1 - \left(\frac{2R}{W} M \right)^2 \right] \right\}^{-1} \quad (4)$$

where A_{ij} are the components of the laminate in-plane stiffness matrix [24],

and M is calculated as:

$$M^2 = \frac{\sqrt{1 - 8 \left[\frac{3(1-2R/W)}{2+(1-2R/W)^3} - 1 \right]} - 1}{2(2R/W)^2} \quad (5)$$

The stress intensity factor $\mathcal{K}_I$ corresponding to two symmetric cracks emanating from a plate with a central circular hole is given for an isotropic plate as [23]:

$$\mathcal{K}_I = \sigma^\infty F_h F_w \sqrt{\pi a} \quad (6)$$

with:

$$F_h = \sqrt{1 - \frac{R}{a}} f_n \quad (7)$$

$$f_n = 1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4, \quad \lambda = R/a \quad (8)$$

and:

$$F_w = \sqrt{\sec\left(\frac{\pi R}{W}\right) \sec\left(\frac{\pi a}{W}\right)} \quad (9)$$

Observing equations (1), (2) and (6) it becomes clear why the 'characteristic distance' l used in the average-stress model [6], which corresponds to the first equation in (1), cannot be a material property: the geometric terms included in the solution for the stress distribution, equation(2), and in the solution for the stress intensity factor, equation (6), imply that the solutions of the system of equations (1) are functions of both the geometry and the material properties. The fact that the 'characteristic distance' is a function of the geometry of the specimen has been demonstrated by several experimental results, see for example [8].

Using (2) and (6) in (1), and dividing the second equation (1) by the square of the first one yields:

$$\frac{4l\pi \int_R^{R+l} (F_h F_w)^2 ada}{R_K^2 \left\{ \int_R^{R+l} [2 + \xi^2 + 3\xi^4 - (K_T^\infty - 3)(5\xi^6 - 7\xi^8)] dx \right\}^2} = \left(\frac{\mathcal{K}_{Ic}}{X_T^L} \right)^2 \quad (10)$$

The integral in the denominator of equation (10) can be solved analytically, whereas the integral in the numerator cannot. Using Simpson's rule [25] to numerically integrate the numerator of (10) the resulting non-linear equation can be solved for l . Once l is known, it is possible to calculate the remote stress at failure, $\bar{\sigma}^\infty$, using one of the equations (1).

It should be noted that the correction factors applied to the stress intensity factor should account for the orthotropy of the composite material [26]. However, for quasi-isotropic laminates the stress intensity factor calculated using (6) is accurate and no additional correction factors are required.

3 Material selection and characterization

The material selected for this work is the Hexcel IM7-8552 carbon epoxy uni-directional laminate. After laying-up, the material was cured according to the manufacturer's specifications, with temperature stages of 110°C during 1 hour, followed by 180°C for 2 hours using heating and cooling rates of 3°C/min. The pressure of 7 bar was applied during the duration of the curing cycle. To validate the finite fracture mechanics model using previously obtained experimental data [5] the $[90/0/\pm 45]_{3s}$ lay-up is selected, corresponding to laminates with a nominal thickness of 3mm.

The finite fracture mechanics model proposed requires information about the laminate lay-up, the ply elastic properties, the laminate unnotched strength and the laminate mode I fracture toughness.

The ply elastic properties were measured in a previous investigation [5] using ASTM standards [27]-[28]. The results are shown in Table 1, where E_1 and E_2 are respectively the ply longitudinal and transverse Young's modulus, G_{12} is the ply shear modulus, and ν_{12} is the ply major Poisson's ratio. Table 1 also shows the standard used in each test, as well as the standard deviation (STDV).

[Table 1 about here.]

The unnotched tensile strength of the laminate, X_T^L , was measured using five test specimens. The tests were performed following the ASTM standard D-3039 [27], and the average value obtained was $X_T^L = 845.1\text{MPa}$ (standard deviation of 47MPa).

133 There are no standard test methods to measure the fracture toughness of the
134 laminate. Taking into account the simplicity of the geometry and of the data
135 reduction method, specimens with a central crack are selected to measure this
136 property.

137 Four specimens with central cracks with a length $2a = 15\text{mm}$ were tested.
138 Two of the test specimens are 45mm wide and the other two are 48mm wide.
139 The specimens were loaded in tension at a rate of 2mm/min until final failure.
140 All the specimens tested failed by net-tension, with crack propagation from
141 the original central crack towards the edges of the specimen.

142 The calculation of the fracture toughness is based on the finite fracture me-
143 chanics analysis of the specimen with a central crack. Failure occurs when the
144 following system of equations is satisfied:

$$\left\{ \begin{array}{l} \frac{1}{l} \int_a^{a+l} \sigma_{yy}(x, 0) dx = X_T^L \\ \\ \frac{1}{l} \int_a^{a+l} \mathcal{K}_I^2(a) da = \mathcal{K}_{Ic}^2 \end{array} \right. \quad (11)$$

145 For sufficiently large W/a ratios, the stress distribution used in equation (11)
146 reads:

$$\sigma_{yy}(x, 0) = \frac{\sigma^\infty x}{\sqrt{x^2 - a^2}} \quad (12)$$

147 and the stress intensity factor is given by:

$$\mathcal{K}_I = \sigma^\infty \sqrt{\pi a} \quad (13)$$

Using the remote stress at failure measured in the experimental tests of the cracked specimens, $\bar{\sigma}^\infty$, and knowing the laminate unnotched strength, $X_T^I = 845.1\text{MPa}$, it is possible to solve (11) for $\mathcal{K}_{Ic}$ and for l . Following this procedure, the mean value of the fracture toughness is $48.0\text{MPa}\sqrt{\text{m}}$ (standard deviation of $1.6\text{MPa}\sqrt{\text{m}}$).

It is interesting to note that this value is quite close to that obtained from the direct application of Linear-Elastic Fracture Mechanics ($45.1\text{MPa}\sqrt{\text{m}}$). This means that it is possible to use models that relate the fracture toughness of the 0° ply to the fracture toughness of a multidirectional laminate [29] in the finite fracture mechanics model without incurring in significant errors.

4 Model validation

The model proposed in this paper is validated by comparing its predictions with experimental data obtained in IM7-8552 CFRP open-hole tensile tests [5],[21].

Open-hole tensile tests were performed in a previous investigation using the $[90/0/\pm 45]_{3s}$ lay-up [5]. Specimens with five different hole diameters, $d=2\text{mm}$, 4mm , 6mm , 8mm , 10mm and with a constant width-to-diameter ratio (W/d) equal to 6 were tested following the ASTM D-5766 standard [30]. All the details of the tests performed and the test results are presented in [5].

The comparison between the experimental results and the predictions of the

finite fracture mechanics (FFM) previously described is shown in Figure 2 and in Table 2. The comparison includes the predictions obtained using the point stress (PS) model [6], which requires the inverse identification of the 'characteristic distance'. Using the test results of the specimen with a 6mm diameter hole the characteristic distance is calculated as 0.83mm.

[Fig. 2 about here.]

[Table 2 about here.]

The results shown in Figure 2 and in Table 2 indicate that the finite fracture mechanics model provides more accurate predictions than the point stress model. In addition, it should be stressed that, unlike the point stress model, the finite fracture mechanics model does not require any inverse identification from one of the tests.

To further validate the model proposed in this paper, additional experimental information on the open hole tensile strength of laminates manufactured using the same material system, Hexcel IM7-8552, is used. Green et al. [21] performed tests using the $[45/90/-45/0]_{4s}$ lay-up, with $W/d=5$ and hole diameters equal to 3.175mm, 6.35mm, 12.7mm, and 25.4mm. The unnotched strength reported for this lay-up is $X_T^L = 929\text{MPa}$ [31]. However, there is no information regarding the mode I fracture toughness of this lay-up, a property required for the finite fracture mechanics model.

To estimate the value of $\mathcal{K}_{Ic}$ the results of the test performed in the specimen with a 3.175mm hole is used. Knowing the unnotched strength of the material and the notched strength of this test specimen it is possible to solve equation (1) for l and $\mathcal{K}_{Ic}$, which results in $\mathcal{K}_{Ic} = 42.3\text{MPa}\sqrt{\text{m}}$. This value is close to

192 that experimentally obtained for the $[90/0/\pm 45]_{3s}$ lay-up.

193 Equipped with this value, it is now possible to predict the notched strength for
194 the other geometries. The comparison between the experimental results and
195 the predictions is shown in Figure 3 and in Table 3. The comparison includes
196 the predictions obtained using the point stress (PS) model with a characteristic
197 distance of 0.45mm calculated using the specimen with a 3.175mm diameter
198 hole.

199 [Fig. 3 about here.]

200 [Table 3 about here.]

201 As before, improved results are obtained using the finite fracture mechanics
202 model.

203 Taking into account that the finite fracture mechanics model provides predic-
204 tions of notched strength in a few of seconds, it can be used to generate design
205 charts for notched laminates. Figure 4 shows the predicted normalized notched
206 strength of the $[90/0/\pm 45]_{3s}$ laminate, defined as $\bar{\sigma}_N = \bar{\sigma}^\infty/X_T^L$, as a function
207 of the d/W ratio for different hole sizes. This Figure also includes the predic-
208 tions obtained for a notch-sensitive material, whose normalized strength is a
209 function of the stress concentration factor K_T ($\bar{\sigma}_N = 1/K_T$), and the predic-
210 tions for a notch-insensitive material, whose normalized strength is a function
211 of the geometry ($\bar{\sigma}_N = 1 - d/W$).

212 [Fig. 4 about here.]

213 Figure 4 shows that, for a constant d/W ratio, the finite fracture mechanics
214 model predicts that the mechanical response of a notched composite laminate

215 moves from notch sensitivity to notch insensitivity for decreasing hole sizes.

216 This result is consistent with the experimental results previously presented.

217 In addition, the finite fracture mechanics model predicts that for large d and

218 d/W ratios the response becomes brittle and a simple analysis based on stress

219 concentration factors would yield sufficiently accurate predictions.

220 To further illustrate how the finite fracture mechanics model can be used to

221 assess the inherent brittleness of a given material and geometry, the following

222 notch sensitivity factor is introduced:

$$\eta_N = \frac{d}{l} \quad (14)$$

223 where l is calculated from the non-linear equation (10). This notch sensitivity

224 factor is equivalent to the dimensionless group introduced by Suo et al. [32] in

225 terms of the notch size, the material Young's modulus and unnotched strength,

226 and the crack opening displacement.

227 The finite fracture mechanics model is used to predict the notch sensitivity

228 factor of the IM7-8552 $[90/0/\pm 45]_{3s}$ laminate with $W/d = 6$ using two ad-

229 ditional extreme cases. The first one correspond to a ductile material with

230 a fracture toughness corresponding to twice that calculated for IM7-8552

231 $[90/0/\pm 45]_{3s}$ ($\mathcal{K}_{Ic}=2\times 48.0\text{MPa}\sqrt{\text{m}}$); the second case corresponds to a brittle

232 material with a fracture toughness corresponding to one-half of that previously

233 used ($\mathcal{K}_{Ic}=1/2\times 48.0\text{MPa}\sqrt{\text{m}}$).

234 Figure 5 shows the relation between the notch sensitivity factor and the hole

235 diameter for the three materials considered and Figure 6 shows the corre-

236 sponding predicted relation between the hole diameter and the normalized

237 strength.

238 [Fig. 5 about here.]

239 [Fig. 6 about here.]

240 The previous figures demonstrate that the model developed in this work is use-
241 ful to assess the inherent brittleness of a given material/geometry combination.
242 The notch sensitivity factor increases with the hole size, specially for materials
243 with a low fracture toughness. For the materials and geometries used in this
244 example it is concluded that when $\eta_N \geq 22.5$ (brittle material/structure) it
245 is possible to predict the notched strength with a good accuracy simply using
246 the stress concentration factor as: $\bar{\sigma}^\infty = X_T^L/K_T$. Figure 6 shows that in such
247 conditions the finite fracture mechanics prediction tends to the solution ob-
248 tained for a perfectly brittle, notch sensitive material. When $\eta_N \leq 0.4$ (ductile
249 material/structure) it is possible to predict the notched strength simply using
250 the geometry of the specimen as: $\bar{\sigma}^\infty = X_T^L (1 - d/W)$. More complex analysis
251 methods are required for the intermediate, quasi-brittle, material response.

252 5 Conclusions

253 The finite fracture mechanics model developed in this paper is an economic,
254 fast and accurate method to predict the open-hole tensile strength of com-
255 posite laminates. Economic because it only requires the ply elastic constants
256 and two additional independent material properties: the laminate unnotched
257 strength and the laminate fracture toughness. No inverse identification meth-
258 ods are required. The predictions are obtained in a few seconds because no
259 finite element analysis or complex computational methods are required. Fi-

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260 nally, based on the comparison between the predictions and the experimental
261 results, it is concluded that the finite fracture mechanics model is very accu-
262 rate.

263 These characteristics of the model make it quite suitable for the generation of
264 design charts for notched composite laminates, to predict the notch sensitivity
265 of a given material/structure, and to verify when simple strength of materials
266 analysis are suitable design tools.

267 The finite fracture mechanics model is applicable to notched composite lam-
268 inates that exhibit either brittle or pull-out failure modes. The strength pre-
269 diction of laminates whose main failure mechanism is delamination requires
270 appropriate finite element analysis; the model proposed herein is not appro-
271 priate for this type of laminates.

272 Future work will address the generalization of the model to deal with the
273 strength prediction of notched composite laminates subjected to multi-axial
274 loading.

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Fig. 1. Notched laminate under tensile loading.

(a) Notched strength vs. hole diameter.

(b) Notched strength vs. hole diameter (logarithmic scale).

Fig. 2. Comparison between predictions and experiments - $[90/0/\pm 45]_{3s}$ lay-up.

(a) Notched strength vs. hole diameter.

(b) Notched strength vs. hole diameter (logarithmic scale).

Fig. 3. Comparison between predictions and experiments - $[45/90/-45/0]_{4s}$ lay-up.

Fig. 4. Design chart for IM7-8552 [90/0/±45]_{3s} laminates.

Fig. 5. Notch sensitivity factor calculated using finite fracture mechanics.

Fig. 6. Predicted normalized strength for different materials.

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Table 1
 Ply elastic properties.

Property	Standard	Mean value	STDV
E_1 [GPa]	Ref. [27]	171.4	2.38
E_2 [GPa]	Ref. [27]	9.1	0.09
G_{12} [GPa]	Ref. [28]	5.3	0.13
ν_{12}	Ref. [27]	0.3	0.02

Table 2

Comparison between predictions and experiments for the $[90/0/\pm 45]_{3s}$ lay-up.

d [mm]	$\bar{\sigma}^\infty$ (Exp.) [MPa]	$\bar{\sigma}^\infty$ (FFM) [MPa]	Error (%)	$\bar{\sigma}^\infty$ (PS) [MPa]	Error (%)
2	555.7	560	+0.7	639	+15.0
4	480.6	470	-2.2	506	+5.3
6	438.7	420	-4.1	n/a	n/a
8	375.7	390	+3.8	401	+6.7
10	373.7	370	-1.1	376	+0.6

Table 3
Comparison between predictions and experiments for the $[45/90/-45/0]_{4s}$ lay-up.

d [mm]	$\bar{\sigma}^\infty$ (Exp.) [MPa]	$\bar{\sigma}^\infty$ (FFM) [MPa]	Error (%)	$\bar{\sigma}^\infty$ (PS) [MPa]	Error (%)
3.175	478	n/a	n/a	n/a	n/a
6.35	433	400	-7.6	392	-9.5
12.7	374	351	-6.1	344	-8.0
25.4	331	324	-2.1	320	-3.3